

ANSWERS AND EXPLANATIONS

**Multiple-Choice Questions**

- **1. (D) is correct.** The answer is a function of the initial velocity, the acceleration, and the distance the object travels, such that  $v^2 = v_0^2 + 2a(x - x_0)$  so  $v = \sqrt{v_0^2 + 2a(x - x_0)} = \sqrt{[(8.0 \text{ m/s})^2 + 2(0.30 \text{ m/s}^2)(60 \text{ m})]} = 10 \text{ m/s}$ .
- **2. (E) is correct.** The time elapsed when the projectile reaches the maximum height occurs when the velocity reaches zero.  $0 = 98 \text{ m/s} + (-9.8)t$ , so  $t = 10 \text{ s}$ . Next,  $x = x_0 + v_0t + 1/2(at^2) = 210 + 98t + 1/2(-9.8 \text{ m/s}^2)(t^2) = 210 \text{ m} + 98 \text{ m/s}(10 \text{ s}) + 1/2(-9.8 \text{ m/s}^2)(10 \text{ s})^2 = 700 \text{ m}$ .
- For an alternate solution, the maximum height above the starting point can be found using  $v^2 = v_0^2 + 2ad$ , and then this can be added to the height of the building.
- **3. (B) is correct.** Breaking the first vector into its northern component and then summing with the second  $V_y = V_1 \sin \theta + V_2 = 150 \sin 30^\circ + 20 \text{ m} = 75 \text{ m} + 20 \text{ m} = 95 \text{ m}$ .



- **4. (D) is correct.** To determine how long the object will be aloft, the relation  $y = y_0 + v_{y0}t + 1/2(a_y t^2)$  can be used, so  $0 = 500 \text{ m} + (0 \text{ m/s})(t) - 4.9t^2$ , so  $t = 10.1 \text{ s}$ . Next, using the mechanics relation  $x = x_0 + v_x t + 1/2(a_x t^2)$   
 $= 0 + 4.5 \text{ m/s}(10.1 \text{ s}) + 1/2(0 \text{ m/s}^2)(10.1 \text{ s})^2$   
 $= 45.45 \text{ m}$ , here to significant figures.
- **5. (A) is correct.** This question can be solved without a calculator by the trigonometric implications of the question. For vector  $V$ ,  $V_y = V \sin \theta$  and so  $34 = 36 \sin \theta$ . Both values of theta have to be between  $\theta = 0^\circ$  and  $180^\circ$  to produce a positive values, immediately eliminating answers B, D, and E. Further,  $\theta$  must be  $150^\circ > \theta > 30^\circ$  to produce a vector component greater than  $V/2$ . This eliminates answer C.
- **6. (A) is correct.** Using the relation  $\Sigma F/m = a$ , where the sum of force are accounted for using  $\Sigma F = 12.5 \text{ N} + 12.5 \text{ N} - 8 \text{ N} = 17 \text{ N}$ , then  $17 \text{ N}/42.5 \text{ kg} = 0.4 \text{ m/s}^2$ .
- **7. (D) is correct.** Summing the forces in the direction parallel to the surface,  $\Sigma F_x = F - \mu F_N = ma = (20 \text{ kg})(0 \text{ m/s}^2) = 0 \text{ N}$ , so  $F = \mu F_N = \mu(mg)$  because  $\Sigma F_y = F_N - mg = 0$ . So  $\mu = 49 \text{ N}/(20 \text{ kg})(9.8 \text{ m/s}^2) = 0.25$ .
- **8. (C) is correct.** The sum of forces on either box can determine the tension. For the left box, the force is  $\Sigma F_x = T_1 = ma = (15 \text{ kg})(4 \text{ m/s}^2) = 60 \text{ N}$ . This is also true for the right box, in which  $\Sigma F_x = 120 - T_1 = (15 \text{ kg})(4 \text{ m/s}^2) = 60 \text{ N}$ , so  $T_1 = 60 \text{ N}$ .
- **9. (E) is correct.** The normal force is given by  $\Sigma F_y = -mg + F_N + F_R \sin 45^\circ = -(35 \text{ kg})(9.8 \text{ m/s}^2) + F_N + 100 \text{ N} \sin 45^\circ = -(35 \text{ kg})(9.8 \text{ m/s}^2) + F_N + 50\sqrt{2} \text{ N} = 0 \text{ N}$ .  
Therefore,  $-(35 \text{ kg})(9.8 \text{ m/s}^2) + F_N + 50\sqrt{2} \text{ N} = 0 \text{ N}$ .  
So  $343 \text{ N} - 71 \text{ N} = 272 \text{ N} = F_N$ , or  $270 \text{ N}$  to two significant figures.
- **10. (B) is correct.** In the direction parallel to the surface on which the box moves, the lone force acting on the box is given by  $\Sigma F_x = (\sqrt{2}/2)(100) = 71 \text{ N} = ma = (35 \text{ kg})(a)$ , so  $a = 2.0 \text{ m/s}^2$ .
- **11. (E) is correct.** If all motion is strictly vertical, then equation III is true. Further, the components of the tension in the horizontal direction are given by equation II. Finally, the vertical components of tension oppose the acceleration due to gravity of the object.
- **12. (B) is correct.** The work required is a function of height, so  $h_1 = 1000 \sin 30^\circ = 1000(1/2) = 500 \text{ m}$  and  $h_2 = 1000 \sin 45^\circ = 1000(\sqrt{2}/2) = 500\sqrt{2} \text{ m}$ . Since  $W_1 = mgh_1$  and  $W_2 = mgh_2$ ,  $W_1/W_2 = mgh_1/mgh_2 = h_1/h_2 = 1/2/\sqrt{2}/2 = 1/\sqrt{2} = \sqrt{2}/2 = .707$ .
- **13. (A) is correct.** Average power is given by  $P_{\text{avg}} = Fv_{\text{avg}}$   
so  $v_{\text{avg}} = P_{\text{avg}}/F = 120 \text{ W}/mg \sin 60^\circ = 120 \text{ W}/(10 \text{ kg})(9.8 \text{ m/s}^2)(\sqrt{3}/2) = 240/(98\sqrt{3})$ .
- **14. (A) is correct.** Using the conservation of energy, the energy of the ball will be entirely potential when it is compressed  $0.3 \text{ m}$  and entirely kinetic when it passes the equilibrium point,  
so  $1/2(mv_1^2) + 1/2(kx_1^2) = 1/2(mv_2^2) + 1/2(kx_2^2)$  here is  
 $1/2(kx_1^2) = 1/2(mv_2^2)$  and  $v_2 = \sqrt{(kx_1^2/m)} = \sqrt{((k)(0.3 \text{ m})^2)/(M)}$   
 $= 0.3\sqrt{(k/M)}$ .

- **15. (B) is correct.** Using the work energy principle,  $4500 \text{ J} = W = KE_2 - KE_1 = \frac{1}{2} m(v_2^2 - v_1^2) = \frac{1}{2} m([60 \text{ m/s}]^2 - [40 \text{ m/s}]^2)$ . Solving for mass,

$$m = \frac{9000 \text{ J}}{3600 \text{ m}^2/\text{s}^2 - 1600 \text{ m}^2/\text{s}^2} = \frac{9000 \text{ J}}{2000 \text{ m}^2/\text{s}^2} = 4.5 \text{ kg.}$$

- **16. (B) is correct.** This applies the conservation of linear momentum to a situation where the masses stick together, here  $m_1v_1 + m_2v_2 = (m_1 + m_2)v'_1$ , where  $v_2 = 0$  so  $m_1v_1/(m_1 + m_2) = (4 \text{ kg})(5 \text{ m/s})/(4 \text{ kg} + 8 \text{ kg}) = (20 \text{ kg m/s})/(12 \text{ kg}) = 5/3 \text{ m/s} = 1.7 \text{ m/s} = v'_1$ .
- **17. (D) is correct.** Kinetic energy is conserved in microscopic collisions, but conservation equations are used for computation in certain macroscopic collisions, since kinetic energy is approximately conserved when the objects that collide are hard and very little energy is lost as heat.
- **18. (C) is correct.** The units of momentum are  $\text{kg}\cdot\text{m/s}$ , and the units of energy are joules, or  $\text{kg}\cdot\text{m}^2/\text{s}^2$ , so their product is  $(\text{kg}\cdot\text{m/s})(\text{kg}\cdot\text{m}^2/\text{s}^2) = \text{kg}^2\cdot\text{m}^3/\text{s}^3 = \text{J}^2\cdot\text{s/m}$ .
- **19. (E) is correct.** The equation for gravitational force is given by  $F = Gm_1m_2/r^2$ . This means that force is directly proportional to the product of their masses and inversely proportional to the square of the distance of separation. None of the first four answers precisely express this.
- **20. (C) is correct.** Gravitational force is given by  $F = Gm_1m_2/r^2 = (6.67 \times 10^{-11})(49 \times 10^{10} \text{ kg})/(0.25 \text{ m})^2 = 523 \text{ N}$ . Note that estimations based on these values can allow for quick algebraic operations that allow you to narrow down the choices to the correct option.
- **21. (E) is correct.** From the conservation of angular momentum,  $I_i\omega_i = I_f\omega_f$ ,  $\Sigma mr_i^2\omega_i = \Sigma mr_f^2\omega_f$  and  $[(225 \text{ kg})(x)^2 + (225 \text{ kg})(x)^2](15 \text{ rad/s}) = [(225 \text{ kg})(x + 0.4)^2 + (225 \text{ kg})(x + 0.4)^2](10 \text{ rad/s})$ , or  $2(225 \text{ kg})(x)^2(15 \text{ rad/s}) = 2(225 \text{ kg})(x + 0.4)^2(10 \text{ rad/s})$ . This simplifies to  $1.5(x)^2 = (x + 0.4)^2$ ,  $0 = -0.5x^2 + 0.8x + 0.16$ . Applying the quadratic formula, the positive value for  $x$  will be  $x = 0.8 + \sqrt{.96}$ .
- **22. (C) is correct.** The period of a simple pendulum is independent of mass, and if the mass of a pendulum bob doubles, the period is unchanged. If the length of string is quadrupled in the equation  $T = 2\pi\sqrt{l/g} = 0.78 \text{ s}$ , then 1.56 seconds will be the new period.
- **23. (B) is correct.** Using the relation between period, mass, and the spring constant,  $T = 2\pi\sqrt{(m/k)}$ , so  $m = k(T/2\pi)^2 = (2.15 \text{ N/m})([2\pi]\text{s}/2\pi)^2 = 2.15 \text{ kg}$ .
- **24. (A) is correct.** Equating the general expression for force with the equation for gravitational force,  $F = m_1a = Gm_1m_2/r^2$ . On Earth,  $a = g$  and  $m_2$  is the mass of earth, and  $m_1$  is the mass of any object on its surface. Here, we can derive a value for  $g'$ , so  $a = g' = Gm_1/r^2 = GM/(R/2)^2 = 4GM/R^2$ .

GO ON TO THE  
NEXT PAGE 

- **25. (C) is correct.** The expression for gravitational force can be applied along with the circular motion equation. Thus,  $G(0.00001M)(M)/R^2 = (0.00001M)v^2/R$  where  $v = 2\pi R/T$  so  $GM/R = (2\pi R/T)^2$ , so  $T = 2\pi\sqrt{(R^3/GM)}$ .
- **26. (B) is correct.** For an object to sink, the buoyant force must be less than its weight. The weight of displaced liquid is given by  $F = mg = \rho gV = (1.4 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(4\pi(0.4 \text{ m})^3/3) = 3700 \text{ N}$ . The weight of the ball must satisfy  $(x \text{ kg})(9.8 \text{ m/s}^2) > 3700 \text{ N}$ , so the mass would have to be greater than 375 kg.
- **27. (A) is correct.** The ratio of an object submerged is given by the ratio of densities, so  $V_{\text{sub}}/V_O = \rho_o/\rho_F = (0.95 \times 10^3 \text{ kg/m}^3/1.1 \times 10^3 \text{ kg/m}^3) = 0.86$ , so 14% of the height of the ice will be visible.
- **28. (E) is correct.** This is a statement of Bernoulli's principle, which underlies Bernoulli's equation, in which  $P_1 + 1/2(\rho v_1^2) + \rho g y_1 = P_2 + 1/2(\rho v_2^2) + \rho g y_2$ .
- **29. (D) is correct.** Specific gravity is the ratio of a substance to that of water at 4°C, where  $\rho = 1.00 \times 10^3 \text{ kg/m}^3$ , so here  $0.82 = \rho/1.00 \times 10^3 \text{ kg/m}^3$  so  $\rho = 0.82 \times 10^3 \text{ kg/m}^3$  and since  $\rho = m/v$ ,  $m = \rho v = (0.82 \times 10^3 \text{ kg/m}^3)(1.3 \text{ m}^3) = 1066 \text{ kg}$ , or 1100 kg to significant figures.
- **30. (A) is correct.** The universal gas law  $PV = nRT$  can be rearranged so that  $V = nRT/P = (4.0 \times 10^4 \text{ mol})R(250 \text{ K})/(8.1 \times 10^4 \text{ N/m}^2) = 125 \text{ R}$ .
- **31. (B) is correct.** Root-mean-square velocity is given by  $v_{\text{rms}} = \sqrt{3kT/m}$ , so as the temperature quadruples,  $v_{\text{rms}} = \sqrt{3k4T/m} = 2\sqrt{3kT/m}$  and the root-mean-square velocity has doubled.
- **32. (D) is correct.** The relationship between heat and temperature change involves mass and the specific heat of the material, such that  $Q = mc\Delta T$ , so  $c = Q/m\Delta T = 12040 \text{ J}/(0.4 \text{ kg})(70 \text{ }^\circ\text{C}) = 430 \text{ J/kg}\cdot^\circ\text{C}$ .
- **33. (C) is correct.** First, heat has to be added to reach the boiling point, so the temperature has to be raised by 900 °C. This requires  $Q = mc\Delta T = (0.55 \text{ kg})(230 \text{ J/kg}\cdot^\circ\text{C})(900 \text{ }^\circ\text{C}) = 114,000 \text{ J} = 114 \text{ kJ}$ . Next, the change of phase will occur if heat is added in the amount of  $Q = mL_F = (0.55 \text{ kg})(0.88 \times 10^5 \text{ J/kg}) = 48400 \text{ J} = 48 \text{ kJ}$ , so a total of 162 kJ, or 160 kJ to two significant figures.
- **34. (B) is correct.** The first law of thermodynamics defines the change in internal energy  $\Delta U = Q - W$ , where the sign conventions classify heat lost and work done on a system as negative values and work done by a system is a positive value,  $\Delta U = Q - W = -1125 \text{ J} - (-705 \text{ J} + 460 \text{ J}) = -880 \text{ J}$ .
- **35. (D) is correct.** An isobaric process is one in which the pressure is held constant, as represented by curve *D*, the horizontal line.
- **36. (B) is correct.** When charged by induction as described, electrons will leave the rod and enter the earth. After the ground is severed, the rod will have a uniform positive charge. When brought near the negatively charged electroscope, the negative charge of the gold leaves will head toward the knob, and their angle of separation will decrease.
- **37. (C) is correct.** Since force between point charges is given by  $F = kQ_1Q_2/r^2$ , the force in the second situation would be given by  $F_2 = k(2Q_1)(2Q_2)/(3r)^2 = (4/9)(kQ_1Q_2/r^2) = 4F/9$ .

- **38. (E) is correct.** The magnitude of force between an end charge and the closest charge is given by  $F = kQ_1Q_2/l^2$   
 $= (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(3.0 \times 10^{-6} \text{ C})(3.0 \times 10^{-6} \text{ C})/(.2)^2 = 2.0 \text{ N}$ .  
 The force between the end charge and the second closest charge is given by  $F = kQ_1Q_2/l^2 = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(3.0 \times 10^{-6} \text{ C})(3.0 \times 10^{-6} \text{ C})/(0.4)^2 = 0.51 \text{ N}$ . The force between the end charge and the other end charge is given by  $F = kQ_1Q_2/l^2 = (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(3.0 \times 10^{-6} \text{ C})(3.0 \times 10^{-6} \text{ C})/(0.6)^2 = 0.23$ . All of these forces are repulsive, and their magnitude is their sum and can be a positive value, since axes are not established.  $F = F_1 + F_2 + F_3 = 2.7 \text{ N}$ , accounting for significant figures. An alternate method would be to find the force between the closest pair and then add one-fourth of that force plus one-ninth of that force to get the sum of the three forces.
- **39. (D) is correct.** Test charges are defined as positive, so if  $B$  will be attractive and  $A$  will be repulsive, so the magnitude of the electric field will be  
 $E = B - A = kQ_1/r_1^2 - kQ_2/r_2^2$   
 $= (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(3.0 \times 10^{-6} \text{ C})/(0.05 \text{ m})^2$   
 $- (9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(8.0 \times 10^{-6} \text{ C})/(0.15 \text{ m})^2$   
 $= 1.08 \times 10^7 \text{ N/C} - 3.2 \times 10^6 \text{ N/C} = 7.6 \times 10^6 \text{ N/C}$ .
- **40. (B) is correct.** The potential energy of two charges is given by  $kQ_1Q_2/r$ , and the force between them is  $kQ_1Q_2/r^2$ , so their ratio is  
 $(kQ_1Q_2/r)/(kQ_1Q_2/r^2) = r$ .
- **41. (C) is correct.** Equipotential lines, which represent the gradations of electric potential with respect to one or more charged particles or surfaces, are drawn perpendicular to field lines and express magnitude without direction.
- **42. (D) is correct.** Using the definition of work done by an electric field to move a positive charge between two places,  $W = qV_{ba} = kQq/r_2 - kQq/r_1 = (9.00 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(4.0 \times 10^{-5} \text{ C})^2[1/r_2 - 1/r_1]$   
 $= [14.4 \text{ N}\cdot\text{m}^2][1/.05 - 1/0.10] = 144 \text{ J}$ .
- **43. (E) is correct.** During the continuous process of a simple electric cell, after positive ions of the negative electrode enter the electrolyte, electrons from the positive terminal are drawn into the electrolyte, which maintains the potential difference between terminals.
- **44. (E) is correct.** From Ohm's law,  $V = IR$ , so  $R = V/I = 9 \text{ V}/6 \text{ A} = 1.5 \Omega$  that the resistance is  $R = \rho L/A$ , so  $L = AR/\rho$   
 $= (1.5 \Omega)(4 \times 10^{-6} \text{ m}^2)/10^{-6} \Omega\cdot\text{m} = 6 \text{ m} = 600 \text{ cm}$ .
- **45. (C) is correct.** Power is defined by  $P = IV$ , which can be combined with a restatement of Ohm's law,  $V = IR$ , so  $P = V^2/R = (12 \text{ V})^2/4.5 \Omega = 144/4.5 \text{ W} = 32 \text{ W}$ . Note that the current could be determined from Ohm's law and directly applied to  $P = IV$ , as well.
- **46. (B) is correct.** Equivalent resistance must be determined first, in which the middle branch has an equivalent resistance of  $12 \Omega + 12 \Omega + 12 \Omega = 36 \Omega$ , and the parallel branches have an equivalent resistance of  $1/R_{\text{eq}} = 1/36$

GO ON TO THE  
NEXT PAGE 

$\Omega + 1/12 \Omega = 4/36 = 1/9$  so  $R_{\text{eq}} = 9 \Omega$ , and, from Ohm's law,  $V = IR_{\text{eq}}$ , so  $I = 2 \text{ A}$ .

- **47. (D) is correct.** This question requires an application of Kirchhoff's rules, in which the junction rule is accurately expressed in answer *A*, whereas the loop rule governs the others. Note that answer *E* is rearranged from the usual form, but is still correct.
- **48. (B) is correct.** The force on a wire of length,  $l$ , carrying a current of  $I$  in a magnetic field of strength,  $B$ , positioned at  $\theta$  is given by  $F = IlB \sin \theta$ . The maximum force this wire could attain would be when it is positioned  $90^\circ$  from the direction of the magnetic field.  $F_{\text{max}} = IlB = 34 \text{ N}/\sin 60^\circ = 39 \text{ N}$ .
- **49. (B) is correct.** The magnetic field at a point  $B$  is inversely proportional to the distance, where  $B = \mu_0 I / 2\pi r$ , so if  $r$  quadruples,  $B$  is quartered—that is, it is decreased by a factor of 4.
- **50. (E) is correct.** Using the right-hand rule, in which the thumb points in the direction of current and the fingers curling around the wire indicate the direction of the magnetic field caused by the wire, the direction is into the page.
- **51. (A) is correct.** Transformers require AC input to operate, since it is the changing magnetic flux created by the changing input current that induces a secondary voltage. Other answers listed represent erroneous uses of the transformer equation.
- **52. (D) is correct.** From Faraday's law of induction,  $\mathcal{E} = -N\Delta\Phi_B/\Delta t$  where  $\Phi_B = BA \cos \theta$ , here  $\mathcal{E} = -NB(A_2 - A_1)/\Delta t$   
 $= (-400)(4.2 \text{ T})\pi(0.25 \text{ m}^2 - 0.15 \text{ m}^2)/12 \text{ s} = -(400)(4.2)(0.04 \text{ m}^2)\pi/12 \text{ s}$   
 $= -5.6\pi \text{ V}$ .
- **53. (A) is correct.** Lenz's law states that the magnetic field of the current caused by an induced emf is in the opposite direction from the change in the magnetic flux. This justifies the negative sign of Faraday's law.
- **54. (E) is correct.** If identical waves are offset by integer multiples of  $\lambda$  (including zero), the amplitudes will sum such that their net maxima will be twice that of an individual wave. Note that if offset by  $\lambda/2$ , two waves would interfere completely destructively.
- **55. (A) is correct.** The  $n$ th overtone is defined as the  $n + 1^{\text{th}}$  harmonic, so the second overtone is the third harmonic. The nodes describe locations on the wave where the string doesn't move, in this case between the three antinodes and at the endpoints.
- **56. (B) is correct.** The relationship between frequency, wavelength and velocity is given by  $f\lambda = v$ , while the relationship between velocity, string tension, string mass, and length in cases such as this is given by  $v = \sqrt{(F_T/m/L)}$ . Equating these,  $f\lambda = \sqrt{(F_T/m/L)}$ . Next,  $mf^2\lambda^2/F_T = L$ , and from the values given,  $L = MF^2\lambda^2/T$ .
- **57. (B) is correct.** From the law of refraction,  $\sin \theta_2/\sin \theta_1 = v_2/v_1$ ,  $\sin 30^\circ/\sin 45^\circ = v_2/14 \text{ m/s}$ , so  $v_2 = (14 \text{ m/s})(\sin 30^\circ/\sin 45^\circ) = 14 \sqrt{2}/2 = 7\sqrt{2} = 9.9 \text{ m/s}$ .
- **58. (D) is correct.** The product of frequency and wavelength is equal to the speed of light,  $f\lambda = c$ , so in this case,  $f = (3.0 \times 10^8 \text{ m/s})/(4.0 \times 10^{-7} \text{ m}) = 0.75 \times 10^{15} \text{ s}^{-1}$ .

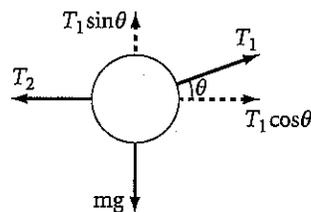
59. (B) is correct. Two Polaroid sheets arranged sequentially such that their axes are perpendicular will be entirely opaque, completely stopping light from an unpolarized light source.
60. (A) is correct. The larger the number of lines per unit length of a diffraction grating, the more opportunities arise for destructive interference to sharpen the maxima—and the more widely spaced are the maxima, thus allowing for more precise measurement. Whether monochromatic or polychromatic light is used affects only the number of fringe patterns that appear, not the precision with which they can be measured.
61. (C) is correct. From the lens equation,  $1/f = 1/d_o + 1/d_i$ , which here is  $1/F = 1/D + 1/d_i$ . This includes the correct sign conventions for a converging (double convex) lens. Solving gives  $1/d_i = 1/F - 1/D = D - F/FD$ , so  $d_i = FD/(D - F)$ .
62. (B) is correct. By geometry,  $\theta_2 = \theta_3$ . Next, from setting up Snell's law for the two situations,  $\sin \theta_1/\sin \theta_2 = n_2/n_1$  and  $\sin \theta_3/\sin \theta_4 = n_1/n_2$ , so  $\sin \theta_1/\sin \theta_2 = \sin \theta_3/\sin \theta_4$ , so  $\theta_1 = \theta_4$ , so the refracted ray will be 're-refracted' back to its original angle from the normal.
63. (D) is correct. Total internal reflection occurs at the critical angle when  $\sin \theta_c = n_2/n_1$ , as derived from Snell's law where the refracted ray is at a  $90^\circ$ ,  $\sin \theta = n_2/n_1 = 1.3/1.5 = 0.87$ , so  $\theta = 60^\circ$ .
64. (B) is correct. From the mirror equation,  $1/f = 1/d_o + 1/d_i$   
 $= 1/2D + 1/D = 3/2D$ , so  $f = 2D/3$ , so the radius of curvature  $r = 2f = 4D/3$ .
65. (B) is correct. Planck's quantum hypothesis suggests that each energy state is a whole number multiple of the minimum energy state,  $E_{\min} = hf$ , called a quantum of energy.
66. (A) is correct. Rewriting the relation between energy and frequency as  $f = E/h$  and substituting for  $f = c/\lambda$ ,  $\lambda = hc/E$   
 $= (6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.0 \times 10^8 \text{ m/s})/10^{-20} = 2 \times 10^{-5} \text{ m}$ .
67. (A) is correct. By accounting for the total number of nucleons, there are  $1 + A$  on the left side of the fission equation, and  $C + E + 4$  on the right. Since  $1 + A = C + E + 4$ ,  $C = (1 + A) - (E + 4)$ .
68. (D) is correct. Transmutation is defined as having occurred when the daughter nucleus differs from the parent nucleus such that a different element has been formed. In alpha decay, an alpha particle is emitted, and in beta decay a nonorbital electron is emitted, having fundamentally changed the element.
69. (A) is correct. In the Bohr model, the energies of orbits are given by  $E_n = -13.6 \text{ eV}/n^2$ , where excited states are given by the natural numbers of  $n > 1$ . The ground state,  $n = 1$ , has an orbital energy of  $-13.6 \text{ eV}$ . (B)–(E) represent energies for successive orbits, that is, consecutive values of  $n$ , from 2 to 5.

GO ON TO THE  
NEXT PAGE 

- 70. (E) is correct. The values for magnetic quantum number include all integers and their additive inverses from 0 to  $X - 1$ , that is,  $-(X - 1)$ ,  $-(X - 2)$ ,  $\dots$ ,  $-1$ ,  $0$ ,  $1$ ,  $\dots$ ,  $(X - 2)$ ,  $(X - 1)$ . Of the answers listed, only 0, (answer E) fits this criteria, since  $X > 0$ .

### Free-Response Questions

1. (a)



Because the sign is held (in static equilibrium),  $\Sigma F_x = \Sigma F_y = 0$ . Resolved into components,  $\Sigma F_x = T_2 - T_1 \cos \theta = 0$  and  $\Sigma F_y = T_1 \sin \theta - mg = 0$ .

- (b) Given  $\theta = 30^\circ$  and  $m = 5 \text{ kg}$ , from  $\Sigma F_y = T_1 \sin \theta - mg = 0$ ,  
 $T_1 \sin \theta = mg = (5 \text{ kg})(9.8 \text{ m/s}^2)$ ,  
 so  $T_1 = (5 \text{ kg})(9.8 \text{ m/s}^2) / \sin 30^\circ = 98 \text{ N}$ .  
 Applying this to  $\Sigma F_x = T_2 - T_1 \cos \theta = 0$ ,  
 $T_2 = T_1 \cos \theta = (98 \text{ N})(\cos 30^\circ) = 85 \text{ N}$ .
- (c) The period of a simple pendulum is independent of mass, relating only to the length of the string that supports it. Since the period is given by  $T = 2\pi\sqrt{l/g}$ ,  $l = (T/2\pi)^2 g = (1.8 \text{ s}/2\pi)^2 (9.8 \text{ m/s}^2) = 0.80 \text{ m}$ .

*This response accurately depicts the forces acting on the sign in part a, resolving  $T_1$  into its components, and it applies the quantitative implications of static equilibrium for use in part b. The information given allows for the application of the simple pendulum equation in the response to part c, and the student notes that such motion is independent of mass.*

2. (a) Converting first to SI units,  $(781 \text{ km/h})(1000 \text{ m}/1 \text{ km})(1 \text{ h}/3600 \text{ s}) = 217 \text{ m/s}$ . Breaking into the initial velocity into components:  $V_x = V \cos \theta = (217 \text{ m/s}) \cos 50^\circ = 139 \text{ m/s}$  and  $V_y = V \sin \theta = (217 \text{ m/s}) \sin 50^\circ = 166 \text{ m/s}$ .
- (b) The maximum height will occur when  $V_y$  velocity is 0.  
 Using  $v_y^2 = v_{y0}^2 - 2gy = (0 \text{ m/s})^2 = (166 \text{ m/s})^2 - (19.6 \text{ m/s}^2)y$ ,  
 so  $y = 1400 \text{ m}$  to the appropriate significant figures.
- (c) With the starting height at 765 m and finishing height 0 m, the projectile will be in the air for  $t$  seconds for  $y = y_0 + v_{y0}t - 1/2(gt^2)$ , so  $0 = 765 + (166 \text{ m/s})t - (4.9 \text{ m/s}^2)t^2$ . By the quadratic equation, the positive value is  $t = 38 \text{ s}$ , so the projectile travels  $x$  meters, where  $x = x_0 + v_{x0}t = 0 + (139 \text{ m/s})(38 \text{ s}) = 5300 \text{ m}$ .
- (d) Such a velocity would have an  $x$  component of  $V_x = 217 \text{ m/s} \cos \theta$  and a  $y$  component of  $V_y = 217 \text{ m/s} \sin \theta$ . The equations  $x = x_0 + v_{x0}t$  and  $y = y_0 + v_{y0}t - 1/2(gt^2)$  can be used, where  $2500 \text{ m} = 0 \text{ m} + (217 \text{ m/s} \cos \theta)t$ , so  $t = 2500 \text{ m}/217 \cos \theta$ , which can substituted into the latter equation, so

$$765 \text{ m} = 765 \text{ m} + (217 \sin \theta \text{ m/s})(2500 \text{ m}/217 \cos \theta) - 1/2(9.8 \text{ m/s}^2)(2500 \text{ m}/217 \cos \theta)^2.$$

From this,  $\sin \theta \cos \theta = (4.9 \text{ m/s}^2)(2500 \text{ m})/(217 \text{ m/s})^2$ .  
 Using the trigonometric identity  $2 \sin \theta \cos \theta = \sin 2\theta$ ,  
 $\sin 2\theta = 2(4.9 \text{ m/s}^2)(2500 \text{ m})/(217 \text{ m/s})^2$ , and  $\theta = 15.7^\circ$ .

The response correctly changes the given units to SI, and it breaks the velocity vector into its components for part a. Projectile motion equations are then applied to find maximum height and distance in the responses to parts b and c, using the quadratic equation for the latter, although the "table" function of an approved calculator could also be used. These same methods are applied in the response to part d, this time solving for the angle, which requires use of a trigonometric identity for a solution.

3. (a) The flow speed in the outlet pipe is given by the equation of continuity,  
 $v_2 = v_1 A_1 / A_2 = (0.88 \text{ m/s})(0.0016\pi) / (0.0001\pi) = 14.1 \text{ m/s}$ .

- (b) The results of part (a) can be applied to Bernoulli's equation:

$$P_1 + 1/2(\rho v_1^2) + \rho g y_1 = P_2 + 1/2(\rho v_2^2) + \rho g y_2$$

$$3.2 \times 10^5 \text{ N/m}^2 + 1/2(0.55 \times 10^3 \text{ kg/m}^3)(0.88 \text{ m/s})^2 + (0.55 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0 \text{ m}) = P_2 + 1/2(0.55 \times 10^3 \text{ kg/m}^3)(14.1 \text{ m/s})^2 + (0.55 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(25 \text{ m})$$

$$P_2 = 1.3 \times 10^5 \text{ N/m}^2.$$

- (c) The result from the equation of continuity in part (a) is the same, as it is not dependent on height, and it can be applied to Bernoulli's equation:

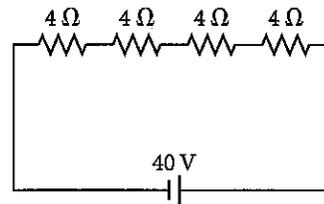
$$P_1 + 1/2(\rho v_1^2) + \rho g y_1 = P_2 + 1/2(\rho v_2^2) + \rho g y_2$$

$$3.2 \times 10^5 \text{ N/m}^2 + 1/2(0.55 \times 10^3 \text{ kg/m}^3)(0.88 \text{ m/s})^2 + (0.55 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0 \text{ m}) = P_2 + 1/2(0.55 \times 10^3 \text{ kg/m}^3)(14.1 \text{ m/s})^2 + (0.55 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0 \text{ m})$$

$$P_2 = 2.7 \times 10^5 \text{ N/m}^2.$$

This response correctly applies the equation of continuity for parts a and c and Bernoulli's equation for parts b and c. The given radii are used to find areas for the equation of continuity, and terms in atmospheres are correctly converted to  $\text{N/m}^2$  as needed so that Bernoulli's equation can be used.

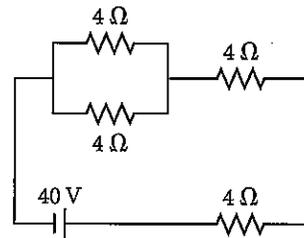
4. (a)



$$R_{\text{eq}} = 16 \Omega, I = 2.5 \text{ A}$$

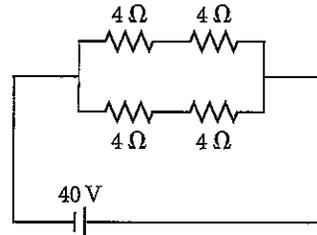


(b)



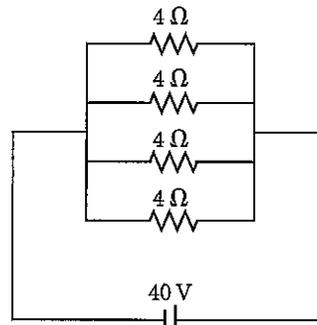
$$R_{\text{eq}} = 10 \Omega, I = 4 \text{ A}$$

(c)



$$R_{\text{eq}} = 4 \Omega, I = 10 \text{ A}$$

(d)



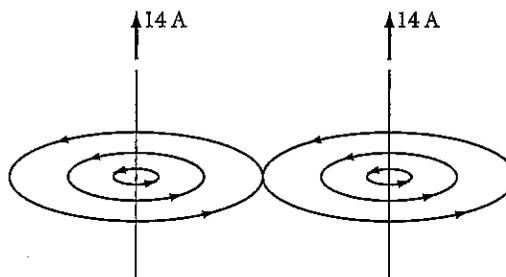
$$R_{\text{eq}} = 1 \Omega, I = 40 \text{ A}$$

- (e) These values (voltage,  $V$ ; current,  $I$ ; and resistance,  $R$ ) are related by Ohm's law, in which  $V = IR$ .
- (f) Their respective power can be determined from the relation  $P = IV$ . Thus:  
For part (a)  $P = IV = (2.5 \text{ A})(40 \text{ V}) = 100 \text{ W}$   
For part (b)  $P = IV = (4 \text{ A})(40 \text{ V}) = 160 \text{ W}$   
For part (c)  $P = IV = (10 \text{ A})(40 \text{ V}) = 400 \text{ W}$   
For part (d)  $P = IV = (40 \text{ A})(40 \text{ V}) = 1600 \text{ W}$

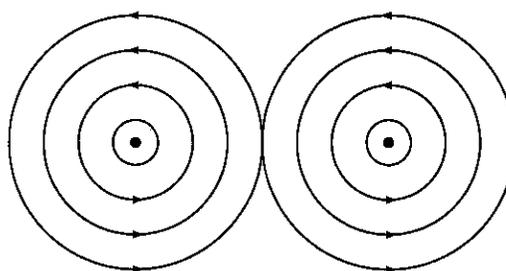
*This response demonstrates an understanding of equivalent resistance for parallel and series resistors. While there are several possible accurate responses, the arrangements here fit the necessary criteria, and they demonstrate a full understanding of Ohm's law. The relation of current and voltage to power given by  $P = IV$  is then applied for the situations depicted in parts a through d.*

5. (a) As the force on a current-carrying wire is given by  $F = I\ell B \sin \theta$ , current is given by  $I = F/\ell B \sin \theta$ . Thus,  $I = F/\ell B \sin \theta = 26 \text{ N}/(0.24 \text{ m})(12 \text{ T})(\sin 40^\circ) = 14 \text{ A}$ .
- (b) The maximum force on the wire given these constraints can be produced when the wire is perpendicular, that is  $\theta = 90^\circ$ , and  $F = I\ell B \sin \theta = I\ell B = (14 \text{ A})(0.24 \text{ m})(12 \text{ T}) = 40 \text{ N}$ .

- (c) The magnetic field due to a straight wire is given by  $B = \mu_0 I / 2\pi r = (4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(14 \text{ A}) / 2\pi(0.30 \text{ m}) = 9.3 \times 10^{-6} \text{ T}$ .
- (d) The force between two parallel wires is given by  $F = \mu_0 I_1 I_2 l / 2\pi L = (4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(14 \text{ A})(14 \text{ A})(0.24 \text{ m}) / 2\pi(0.02 \text{ m}) = 4.7 \times 10^{-4} \text{ N}$ .
- (e) The force is attractive because their currents are in the same direction.



Above and side, foreshortened view



Top view

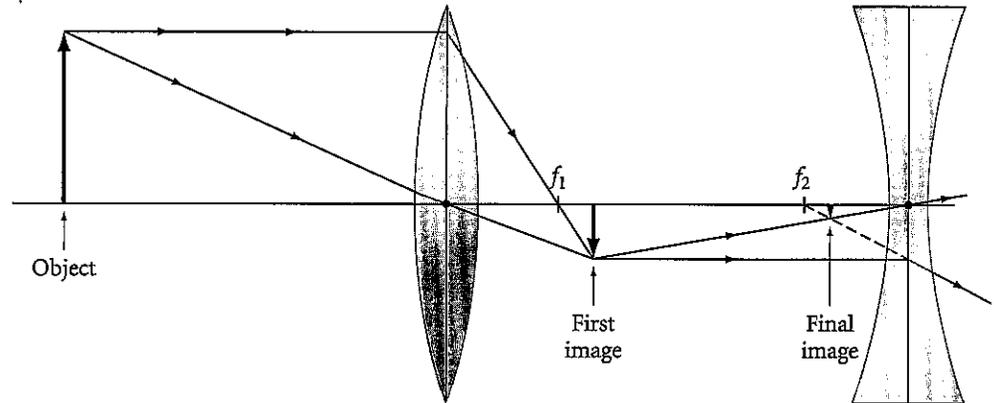
*This response correctly applies the relation  $F = IlB \sin \theta$  to the given information for part a. Next, trigonometry indicates the maximum force will occur when  $\theta$  is  $90^\circ$ , such that  $F_{\max} = IlB$ , for part b. The equations  $B = \mu_0 I / 2\pi r$  and  $F = \mu_0 I_1 I_2 l / 2\pi L$  are correctly applied to parts c and d respectively. The depiction in the response to part e is accurate, although it could also be drawn other correct and acceptable ways.*

6. (a) For combinations of lenses, the image height and distance produced by the first lens determine the object height and distance, respectively, for the second lens. The equation  $1/d_o + 1/d_i = 1/f$  where the appropriate conventions are used. For the first lens of focal length 5 cm, the object of height  $H$  has an object distance of 16 cm so  $1/d_i = 1/f - 1/d_o = 1/5 \text{ cm} - 1/16 \text{ cm}$  so  $d_i = 7.3 \text{ cm}$ . This is on the other side of the convex lens from the object. Since  $h_i/h_o = -d_i/d_o$ ,  $h_i/H \text{ m} = -7.3 \text{ cm}/16 \text{ cm}$  so  $h_i = -0.46 H$ . By the sign, this first image is inverted. This image is  $16 \text{ cm} - 7.3 \text{ cm} = 8.7 \text{ cm}$  from the second lens and is treated as the

GO ON TO THE  
NEXT PAGE

object. Then,  $1/d_o - 1/d_i = -1/f$ . So  $1/d_o + 1/f = 1/d_p$ , thus  $1/8.7 \text{ cm} + 1/4 \text{ cm} = 2.7 \text{ cm}$  on the same side of the convex lens as the image created by the first lens. The height is  $h_i = h_o(-d_i/d_o) = 0.46 H(-2.7 \text{ cm}/8.7 \text{ cm}) = -0.14 H$  and remains inverted.

(b)



(c) The rays of geometric optics can be thought of as a practical representation of very narrow light waves. Rays, like the waves they represent, generally travel in straight lines. Moreover, geometric optics accurately models observations of reflection and refraction.

*The lens equation is applied twice consecutively, following the method for finding images resulting from combinations of lenses. Appropriate sign conventions are used. The response to part b shows two distinct rays of light from the top of the object and their paths to the top of the image. Other rays are acceptable, of course. The justification for geometric optics in the response to part c gives several correct reasons.*

7. (a) The two seemingly contradictory theories of the nature of light are the wave theory, which describes light as electromagnetic waves, and the particle theory, which describes light as streams of photons. Each theory has several experimental observations to support it.
- (b) Diffraction describes the behavior of light as it interacts with an obstacle. The wave theory predicts that the wave fronts would bend around the obstacle on the side opposite from the source, whereas the particle theory would suggest that there is no bending and would be similar to ray theory predictions. In experiments, such bending is seen, so the wave theory of light best describes diffraction.
- (c) The photoelectric effect describes the emission of electrons from a metal surface exposed to light. Both theories predict that the number of electrons emitted is proportional to the light's intensity. However, the wave theory predicts that the maximum kinetic energy of the electrons will also be proportional to the intensity, whereas the particle theory predicts that maximum kinetic energy is independent of intensity. In experiments, the latter is observed, so the particle theory of light best describes the photoelectric effect.

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- (d) Interference describes the behavior of light as it interacts with slits in an otherwise opaque screen. The wave theory predicts that monochromatic light passing through the slits onto another screen will produce more lines than slits on this second screen, indicating areas of constructive interference where the wave fronts are in phase. The particle theory would suggest that light passing through the slits onto another screen will produce as many lines as there are slits on the screen in line with ray theory predictions. In experiments, a greater number of lines than slits is seen, indicating that the wave theory of light best describes interference.
- (e) These contradictory theories are conjoined in the wave-particle duality of light, which states that all phenomenon regarding light can be described by the wave theory or the particle theory, though not by both of these theories simultaneously.

*This response accurately describes a general overview of the two competing theories and their hypothetical implications for three different phenomena. It also describes which observations were actually made under experimental conditions. Finally, the counterintuitive notion of wave-particle duality is identified and broadly outlined.*

GO ON TO THE  
NEXT PAGE 