CHAPTER 10

Fluids

Density and Specific Gravity

- Matter is most commonly arranged in solid, liquid, or gas form. **Solids** have an unchanging shape and volume; solids are incompressible—that is, their volume cannot be changed without the application of substantial force. A **liquid** assumes the form of its container, is generally incompressible, and has a fixed volume. A **gas** assumes both the shape and volume of its container. Liquids and gases are collectively treated as **fluids.**
- Density and specific gravity are characteristics of a substance. **Density** is the ratio of its mass to its volume, which is $\rho = m/V$, whose units are kg/m^3. In a pure substance, density is uniform throughout. Temperature and pressure can affect the density of a material.
- **Specific gravity** is the ratio of its density to that of water at 4.0°C. As a ratio of values with like units, specific gravity has no units.

Pressure in Fluids

Pressure is the ratio of force to surface area—that is, $P = F/A$—whose units are **pascals,** abbreviated Pa, equivalent to N/m^2. Pressure acts perpendicularly on a surface. For fluids, pressure is exerted equally in all directions. As such, pressure must be uniform for a fluid to remain at rest.

- For a liquid of uniform density, the pressure is proportional to the depth at which it is measured, such that $P = \rho gh$. This pressure is uniform everywhere along the plane of depth perpendicular to the force of gravity.
- For gases with only slight variation in density due to depth, change in pressure is proportional to the change in depth at which it is measured, $\Delta P = \rho g \Delta h$. This is independent of the cross-sectional area of the fluid's container.

Atmospheric Pressure and Gauge Pressure

- An **atmosphere,** abbreviated atm, is a unit of pressure equivalent to 1.013×10^5 N/m^2, the average atmospheric pressure at sea level.
- A **bar** is a unit of pressure equivalent to 1.00×10^5 N/m^2.
- **Gauge pressure** represents pressure above atmospheric pressure. The sum of gauge pressure and atmospheric pressure is absolute pressure—that is, $P = P_A + P_G$.

Pascal's Principle

The internal pressure of a confined fluid increases uniformly by the amount of pressure applied to it. Hydraulic lifts operate because the pressure applied to an input piston is equal to the pressure on an output piston initially at the same height. Therefore, the ratio of force on the output piston to the force on the input piston is equal to the ratio of area of the output piston to the area of the input piston. The ratio of force out to force in is called the **mechanical advantage.** In equation form, $F_{out}/F_{in} = A_{out}/A_{in}$ = **mechanical advantage.**

Measurement of Pressure; Gauges and the Barometer

In **manometers,** horseshoe-shaped tubes housing a liquid of uniform density, the change in height of the liquid in a column open to a known atmospheric pressure can be used to measure the pressure of a system at the other end. That is, $P = Po + \rho gh$.

- When the fluid is mercury, pressure is often referred to by the height of the mercury in the column. A unit of pressure is **torrs,** or millimeters of mercury, abbreviated mm-Hg.
- A **barometer** is an adapted manometer with one end closed so that its pressure at that end is zero, enabling measurement of atmospheric pressure with the open end. The mercury in a barometer of sufficient height is 76 cm at standard atmospheric pressure.

Buoyancy and Archimedes' Principle

The pressure on the undersurface of an object immersed in a fluid increases with depth.

- **Archimedes' principle** states that the upward **buoyant force** on a submerged object fluid is equal to the weight of the fluid it displaces, $F_B = \rho_F gV$.
- An object will float in a fluid of lower density. The ratio of the object's volume that is submerged to the volume of the entire object is equal to the ratio of the object's density to the density of the fluid. That is, $V_{sub}/V_O = \rho_o/\rho_F$.

Fluids in Motion; Flow Rate and the Equation of Continuity

There are two types of flow for fluids in motion.

- In **streamline flow,** also known as **laminar flow,** adjacent layers of fluid appear to slide over one another. For laminar flow in a tube, velocity varies with the cross-sectional area of the tube, such that $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$. This is called **the equation of continuity.** This implies $A_1 v_1 = A_2 v_2$ if the fluid is incompressible.
- In **turbulent flow,** the intersection and interaction of fluid layers create small circular currents called eddies, and these eddies absorb energy. **Viscosity** is the internal friction of a fluid present to varying degrees in both types of flow.

Bernoulli's Equation

- **Bernoulli's principle** states that velocity and pressure are inversely proportional for fluids—that is, the higher a fluid's velocity, the lower its pressure, as well as the converse.
- **Bernoulli's equation** applies to an incompressible fluid with laminar flow and negligible viscosity. The relationship between two portions of a pipe of differing height and cross-sectional area is as follows:
  $P_1 + 1/2(\rho v_1^2) + \rho g y_1 = P_2 + 1/2(\rho v_2^2) + \rho g y_2$
  where y_1 and y_2 are the heights of each segment of pipe.

For Additional Review

Follow the derivation of Bernoulli's equation from the work-energy principle, and consider how it serves to model the conservation of energy.

Multiple-Choice Questions

1. If the density of granite is 2.7×10^3 kg/m^3, what is the largest possible height for solid cylindrical pillar of radius 25 cm with a mass of 750 kg?
   (A) 0.35 m
   (B) 0.71 m
   (C) 1.4 m
   (D) 3.1 m
   (E) 7.4 m

2. What is the pressure on the ground of a 13 kg concrete cube of side length 20 cm?
   (A) 1100 Pa
   (B) 1600 Pa
   (C) 1800 Pa
   (D) 2400 Pa
   (E) 3200 Pa

3. What is the apparent weight of an 8 kg object of volume 0.005 m^3 submerged in a bathtub whose water has a density of 1.00×10^3 kg/m^3?
   (A) 10 N
   (B) 17 N
   (C) 29 N
   (D) 49 N
   (E) 78 N

4. What is the minimum mass needed for a spherical object with a 2 m radius to sink in an ocean whose water has a density of 1.1×10^3 kg/m^3?
   (A) 9.7×10^2 kg
   (B) 2.7×10^3 kg
   (C) 7.4×10^3 kg
   (D) 1.8×10^4 kg
   (E) 3.7×10^4 kg

5. How much of a 10 cm-thick block of ice ($\rho = 0.92 \times 10^3$ kg/m^3) will be submerged in an ocean ($\rho = 1.03 \times 10^3$ kg/m^3), assuming that there is no thermal exchange?
   (A) 1.1 cm
   (B) 3.1 cm
   (C) 5.7 cm
   (D) 8.9 cm
   (E) 10 cm

6. The main pipe that carries water into a home has a radius R at a velocity v. What must be the radius for each of five identical faucets (fed by the pipe) to have the same velocity of water?
   (A) 0.45 R
   (B) 0.9 R
   (C) R
   (D) 2 R
   (E) 2.2 R

7. The buoyant force on an object is equal to the weight of water displaced by a submerged object. This is a principle attributed to
   (A) Pascal
   (B) Bernoulli
   (C) Archimedes
   (D) Galileo
   (E) Torricelli

8. Which of the following are NOT units of pressure?
   I. Atmospheres
   II. Torrs
   III. Kilopascals
   (A) II
   (B) III
   (C) II and III
   (D) I, II, and III
   (E) None of the above

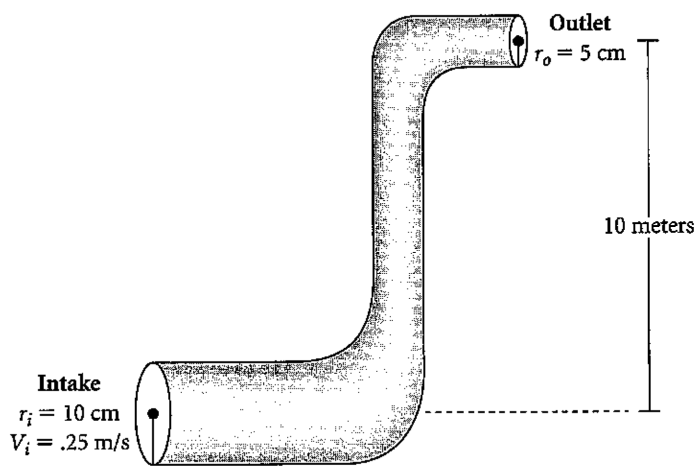
9. What is the specific gravity of water at 4.0 C?
   (A) 1.0×10^0
   (B) 1.0×10^1
   (C) 1.0×10^2
   (D) 1.0×10^3
   (E) 1.0×10^4

10. If the gauge pressure of a device reads 2.026×10^5 N/m^2, the absolute pressure it is measuring is
   (A) 1.013×10^5 N/m^2
   (B) 2.052×10^5 N/m^2
   (C) 2.026×10^5 N/m^2
   (D) 3.039×10^5 N/m^2
   (E) 6.078×10^5 N/m^2

Free-Response Questions

1. A specially constructed barometer is filled with ethyl alcohol instead of mercury.
   (a) What would be the height of a column of liquid, whose density is 0.79×10^3, at standard atmospheric pressure?
   (b) Why do you think mercury instead of ethyl alcohol is used in barometers?
   (c) What would be the atmospheric pressure on a distant planet if this device indicated a column of 76 cm?

2. (a) What is the pressure in a gasoline pipe with a radius of 5 cm 10 meters above its intake pipe, which has a radius of 10 cm and flows at a speed of 0.25 m/s at 1.0 atm? (Gasoline has a density of 0.68×10^3 kg/m^3.)



(b) Assuming other factors are constant, what happens to the flow speed in the output pipe as the radius of the intake pipe changes, and how does this affect the pressure?

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. (C) is correct. The volume of a cylinder is $V = \pi r^2 h$. Since $\rho = m/V$, $\rho = m/\pi r^2 h$, then $h = m/\pi r^2 \rho = 750\ \text{kg}/\pi(0.25\ \text{m})^2 2.7 \times 10^3\ \text{kg/m}^3 = 1.4\ \text{m}$.

2. (E) is correct. The surface area of the cube touching the ground is $(0.2\ \text{m})^2 = 0.04\ \text{m}^2$. Since $P = F/A$, $P = mg/A = (13\ \text{kg})(9.8\ \text{m/s}^2)/(0.04\ \text{m}^2) = 3200\ \text{Pa}$.

3. (C) is correct. The weight of the object is $mg = (8\ \text{kg})(9.8\ \text{m/s}^2) = 78\ \text{N}$ downward. The buoyant force is equal to the weight of water displaced by the object, such that $F_B = mg = \rho Vg(1.00 \times 10^3\ \text{kg/m}^3)(0.005\ \text{m}^3)(9.8\ \text{m/s}^2) = 49\ \text{N}$. It acts upward, so the apparent weight is $78\ \text{N} - 49\ \text{N} = 29\ \text{N}$.

4. (E) is correct. For the object to sink, the buoyant force must be less than the weight of the object. The weight of displaced seawater is $F = mg = \rho_F gV = (1.1 \times 10^3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(4\pi(2\ \text{m})^3/3) = 3.6 \times 10^5\ \text{N}$. The weight of the object is $(x\ \text{kg})(9.8\ \text{m/s}^2) > 3.6 \times 10^5\ \text{N}$. The mass would have to be greater than 3.7×10^4 kg. Otherwise the buoyant force would be greater than the weight of the diving bell.

5. (D) is correct. The ratio of an object submerged is given by the ratio of densities. $V_{\text{submerged}}/V_O = \rho_o/\rho_F = (0.92 \times 10^3\ \text{kg/m}^3/1.03 \times 10^3\ \text{kg/m}^3) = 0.89$. So 8.9 cm will be submerged.

6. (A) is correct. The equation of continuity for an incompressible fluid is given by $A_1 v_1 = A_2 v_2$, in this case, $\pi R^2 v = 5\pi R_f^2 v$. The ratio of pi over pi is 1, so $R_f = \sqrt{(R^2/5)} = 0.45\ R$.

7. (C) is correct. This is Archimedes' principle, the earliest of this section's principles.

8. (E) is correct. All three are units of pressure equivalent to a value in N/m^2. One atmosphere is defined to be $1.013 \times 10^5\ \text{N/m}^2$. One torr is defined to be $133\ \text{N/m}^2$. One kilopascal is defined to be $1000\ \text{N/m}^2$.

9. (A) is correct. Specific gravity refers to the ratio of the density of a substance to the density of water at 4.0° C. As such, the specific gravity of water is the ratio of the density of water to itself, which is equal to 1. In scientific notation, this would be written as 1.0×10^0.

10. (D) is correct. Gauge pressure measures the pressure above atmospheric pressure, and it is related to absolute pressure by the equation $P = P_A + P_G = 1.013 \times 10^5\ \text{N/m}^2 + 2.026 \times 10^5\ \text{N/m}^2 = 3.039 \times 10^5\ \text{N/m}^2$.

Free-Response Questions

1. (a) Barometers are manometers whose function is to measure atmospheric pressure. The height of fluid in the column with a closed end (where the pressure is zero) represents this atmospheric pressure. $P = \rho gh = 1.013 \times 10^5\ \text{N/m}^2 = (0.79 \times 10^3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(h)$, so the height of fluid is 13 meters.

(b) The atmospheric pressure of the Earth creates a column of mercury that is 76 cm high, which is far more practical than 13 meters.

(c) Again, the same equation applies. $P = (0.79 \times 10^3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(h) = (0.79 \times 10^3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.76\ \text{m}) = 5.9 \times 10^3\ \text{N/m}^2$. On this distant planet, the lower pressure is such that ethyl alcohol would be a practical fluid for measuring atmospheric pressure.

This response demonstrates an understanding of practical application of pressure relationships. In the response to part a, the density of this alternate liquid is applied to the pressure equation, $P = \rho gh$. The response to part b identifies why such a setup would be impractical on the Earth, but the response to part c shows a hypothetical extraterrestrial use.

2. (a) To find the flow speed in the other end of the gasoline pipe, $v_2 = v_1A_1/A_2 = (0.25\ \text{m/s})(0.01\pi)/(0.0025\pi) = 1\ \text{m/s}$.
Using Bernoulli's equation: $P_1 + 1/2(\rho v_1^2) + \rho g y_1 = P_2 + 1/2(\rho v_2^2) + \rho g y_2$,
$1 \times 10^5\ \text{N/m}^2 + 1/2(0.68 \times 10^3\ \text{kg/m}^3)(0.25\ \text{m/s})^2$
$+ (0.68 \times 10^3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0)$
$= P_2 + 1/2(0.68 \times 10^3\ \text{kg/m}^3)(1\ \text{m/s})^2$
$+ (0.68 \times 10^3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(10\ \text{m})$,
$1.0021 \times 10^5\ \text{N/m}^2 = P_2 + (0.34 \times 10^3\ \text{N/m}^2) + 6.7 \times 10^4\ \text{N/m}^2$,
$P_2 = 3.2 \times 10^4\ \text{N/m}^2$.

(b) If all other factors are constant, the flow speed for the output pipe and the square of the intake radius are directly proportional. As flow speed increases, the pressure in the output must decrease, according to Bernoulli's equation.

This response successfully employs the equation of continuity and Bernoulli's equation in the response to part a. The response to part b demonstrates an understanding of the mathematical relationship represented by the variables in these equations.