CHAPTER 11

Vibrations and Waves

CHAPTER 12

Sound

Vibrations produce waves that travel through a medium, functioning as both the source and detector of waves. This occurs on both the microscopic and macroscopic scales.

Simple Harmonic Motion

Cyclic vibrations or oscillations are described as **periodic.** This describes a range of repeating motion, including an object attached to a spring.

- In an idealized form, an object attached to a spring oscillates horizontally, while the spring itself is considered massless and friction is ignored. The **natural length** of the spring is defined as its length where the force on the object is zero. The object attached to the spring at its natural length is said to be at its **equilibrium position.**
- When the object is moved, thereby altering the spring's length, the spring acts to return the object to its equilibrium position. The force on the object is proportional to the product of its **displacement,** or the distance from equilibrium position, x, and the spring constant, k, such that $F = -kx$. The sign is negative because it acts in the opposite direction of displacement. This equation is accurate in the range of displacements that do not deform the spring.
- The displacement causes the force to push the object back toward equilibrium, but under these idealized conditions it overshoots and is displaced on the other side of the equilibrium position to the same distance away.
- If the initial displacement is A, called the **amplitude,** the object's position oscillates repeatedly between A and $-A$. Each complete oscillation is called a **cycle,** and the time each cycle takes, T, is called **period.** The **frequency,** or cycles per second, is given by $f = 1/T$. The unit of frequency is given by **hertz,** equivalent to cycles per second.
- The equilibrium position would be different for the same spring hung vertically due to stretching from the force of gravity. Subsequent displacement would cause cyclic motion for the vertical spring as well.

- These scenarios, in which $F = -kx$, describe **simple harmonic motion,** and such systems are referred to as **simple harmonic oscillators.**

Energy in the Simple Harmonic Oscillator

A spring not at its equilibrium position stores potential energy, where $PE = 1/2(kx^2)$, and the total mechanical energy of the system is given by $E = 1/2(mv^2) + 1/2(kx^2)$.

- Under idealized circumstances, the total mechanical energy of an oscillating system is conserved.
- When the object is at the distance equal to the amplitude, the velocity is zero just before reversing direction, and all energy is stored as potential energy. When the oscillating object is at equilibrium distance, the velocity is maximized, and all energy at that instant is in the form of kinetic energy. Thus $1/2(mv^2) + 1/2(kx^2) = 1/2(mv_{max}^2) = 1/2(kA^2)$.
- The total mechanical energy of an object undergoing simple harmonic motion is proportional to the square of the amplitude. This relation can provide the velocity at any given time, $v = \pm v_{max}\sqrt{(1 - x^2/A^2)}$.

The Period and Sinusoidal Nature of SHM

The period of an object undergoing simple harmonic motion is independent of the initial displacement.

- Projecting circular motion to one dimension along the plane of the circular pathway provides a precise analog to evaluate simple harmonic motion. This enables a calculation of maximum velocity in terms of amplitude and frequency, $v_0 = 2\pi Af$, and of period in terms of mass and the spring constant, $T = 2\pi\sqrt{(m/k)}$.
- Similar analysis provides position as a function of amplitude and frequency, $x = A \cos 2\pi ft = A \cos (2\pi t/T)$.
- Because these are trigonometric functions and are graphed accordingly, the position as a function of the time path of simple harmonic motion is described as **sinusoidal.**
- Velocity is a function of maximum velocity and frequency, $v = -v_0 \sin 2\pi ft = -v_0 \sin (2\pi t/T)$, and, similarly for acceleration, $a = -a_0 \cos 2\pi ft = -a_0 \cos (2\pi t/T)$, where maximum acceleration is given by $a_0 = kA/m$.

The Simple Pendulum

For a **simple pendulum,** an object at the end of a taut, massless string traces out an arc of a circular path in a cyclic motion.

- Because friction and air resistance are ignored in this idealization, the amplitude is the same on both sides of the object's vertical hanging position.
- The restoring force is approximately proportional to the angular displacement from the hanging position and in the opposite direction, such that $F = -mg \sin \theta$. For small angular displacements, this can be viewed as an approximation of simple harmonic motion. The period in such cases is equal to $T = 2\pi\sqrt{(L/g)}$.

Damped Harmonic Motion

These idealizations deviate from real pendulum and oscillating spring motion, as they do not account for friction or air resistance.

- In reality, the amplitude in either case decreases as a function of time until they return to equilibrium. This is called **damped harmonic motion.**
- Extremes of damped motion are described as **overdamped,** in which the time necessary for the system to come to rest is large due to the high level of damping, and **underdamped,** in which the system comes to rest after a number of oscillations.
- **Critical damping** occurs as an intermediate form of damping, and the system comes to rest in the shortest time.

Forced Vibrations; Resonance

A vibrating system that has an external force interacting with it produces a **forced vibration.**

- The system then adopts the frequency of the external force rather than the **natural frequency** of the system.
- The resulting amplitude is a function of the natural frequency and the frequency of the forced vibration, and it is maximized when these frequencies are equal.
- This phenomenon is known as **resonance,** and the natural frequency is referred to as **resonant frequency.**

Wave Motion

In the motion of transverse **mechanical waves,** the particles of the medium travel perpendicular to the wave, oscillating regularly above and below their equilibrium points.

- The wave is not matter itself, but rather an oscillating motion traveling through a medium transporting energy across a distance.
- A **pulse** or any disturbance imparted to a string can create a vibration that produces a mechanical wave.
- Each adjacent particle moves up and down carrying the wave along a string's length.
- A continuous oscillating disturbance creates a **continuous** or **periodic wave.**
- In a perfectly elastic medium, a simple harmonic vibration creates a sinusoidal path along the string's length, and every particle has a sinusoidal motion as a function of time.
- A particle's maximum displacement from equilibrium is the wave's **amplitude,** and the distance between two consecutive matching points on the wave is referred to as the **wavelength,** λ. The **frequency** refers to the number of cycles per second, and it is the inverse of the period. The **wave velocity,** v, is the velocity at which any part of the wavelength travels perpendicular to the motion of the individual particles of the medium, where $v = \lambda f$.
- The wave velocity for a string is a function of the string tension, its mass and length, such that $v = \sqrt{[F_T/(m/L)]}$.

Types of Waves: Transverse and Longitudinal

In a **transverse** wave, the particles of a medium move perpendicular to the wave velocity. In a **longitudinal wave,** the particles of a medium move parallel to the wave velocity.

- Imparting a pulse in a spring in the direction parallel to its length can create a longitudinal wave as compressions and expansions travel its length.
- **Sound** is a longitudinal wave that travels through air or some other medium.
- The previously defined terms describing waves are analogous for transverse and longitudinal waves.

Energy Transported by Waves

For a uniform medium, the energy in waves travels as a series of increasingly larger concentric spheres.

- The energy carried by waves is quantified by **intensity,** which is energy per second per unit area perpendicular to the direction of energy flow, $I =$ power/surface area $= P/(4\pi r^2)$.
- Both energy and intensity are proportional to the square of the wave's amplitude. The ratio of the intensities at two distances from the source of a spherical wave is given by $I_2/I_1 = r_1^2/r_2^2$.
- The corresponding relation in terms of amplitude is given by $A_2/A_1 = r_1/r_2$.
- For an idealized one-dimensional wave, intensity and amplitude do not decrease with distance.

Reflection and Interference of Waves

Some or all of a wave is reflected when it hits a barrier.

- A transverse wave pulse in a string that reaches a fixed end will return on the opposite side of the string. Energy is lost as heat, and some is transferred to the wall.
- A transverse wave pulse in a string that reaches a free end will return on the same side of the string.
- When a string's pulse reaches a heavier section of string, some of the pulse is reflected as if hitting a fixed end, and some of the pulse is transmitted along the heavier end, although its amplitude is less due to the reduction in energy.

For a multidimensional **plane wave,** there is a linear crest called a **wave front.**

- Wave fronts are perpendicular to **rays,** lines denoting a plane wave's direction of motion at a given point.
- When a wave front strikes a barrier at an angle, the angle of incidence is described as the angle between the incident ray and a line perpendicular to the barrier. The wave is reflected at equal angle, called the **angle of reflection,** from the other side of the perpendicular line.

Interference describes the interaction of waves. The waves' resulting displacement is the algebraic sum of the waves according to the **principle of superposition.**

- Two interacting pulses of equal amplitude on the same side of a string experience **constructive interference,** and their joint amplitude at the moment of interaction is double the amplitude.
- Two interacting pulses of equal amplitude on opposite sides of a string experience **destructive interference,** and their joint amplitude at the moment of interaction is zero. For the interaction of continuously oscillating waves, interference may occur at several locations.
- When all interference is constructive, the waves are said to be **in phase.** Otherwise, the waves are **out of phase,** which can mean interference is partially or completely destructive.

Standing Waves; Resonance

When two ends of a cord are fixed, certain vibrations can produce a wave that appears not to move. This is called a **standing wave.**

- The **nodes** of a cord are the locations where it appears motionless as a consequence of destructive interference, whereas the **antinodes** refer to where the cord has the greatest displacement as a consequence of constructive interference.
- At the **fundamental frequency,** a standing wave has a single antinode between the two nodes at its ends.
- Integer multiples of the fundamental frequency, called **natural** or **resonant frequencies,** also produce standing waves. The wavelengths for such frequencies are a function of the string length, $L = n\lambda_n/2$, for $n = 1, 2, 3. \ldots$
- The fundamental frequency is called the **first harmonic,** and all subsequent harmonics are given a prefix indicated by the applicable value of n where $f_n = nf_1$. The term **overtones** refers to these subsequent harmonics, and is given a prefix indicated by $n - 1$ for the applicable value of n where $f_n = nf_1$. Thus, the second harmonic is the first overtone, and so on.
- The energy of a standing wave is zero at the nodes; therefore, energy is not transmitted down a string's length.

Refraction and Diffraction

Refraction describes the change of a wave's direction as it enters a different medium, whereas **diffraction** describes the bending of a wave front after it meets an obstruction.

- Refraction is a function of the incident angle and the wave velocity in each medium. The law of refraction states that $\sin \theta_2/\sin \theta_1 = v_2/v_1$.
- The approximate angle of diffraction is a function of the wavelength of the wave front, λ, and the width of the obstruction, L, such that θ (radians) $\approx \lambda/L$.

General Terms for Sound

The longitudinal waves created by a vibration are received by the human ear and perceived by the human brain as **sound.** A medium is necessary for sound waves to propagate.

- The speed of sound waves depends on the medium through which they travel, as well as the temperature and pressure of the medium. Generally, the term **speed of sound** refers to its speed in air, which is 331 m/s at 0°C and 1 atm. The speed is greater in liquids than in gases and greatest in solids.
- **Pitch** refers to the frequency of sound. The **audible range** of pitch is 20 Hz to 20,000 Hz. Frequencies above the audible range are referred to as **ultrasonic,** whereas those below it are called **infrasonic.**
- Longitudinal waves can be viewed equivalently as **pressure waves,** as their expansions and contractions produce quantifiable pressure changes.
- The perceived loudness of sound is related to wave **intensity.** As a logarithmic function, the intensity of a sound, β, is given by $\beta = 10 \log (I/I_0)$ whose units are **decibels,** where I_0 represents a reference level, typically 1.0×10^{-12} W/m^2.
- The **quality** of a sound (known as timbre or tone color in music) refers to its audible distinctiveness, and the quality depends on the overtones that accompany a fundamental frequency.
- A large number of frequencies and overtones can render individual pitches indistinguishable, and so would be perceived as noise.

The Ear and Its Response; Loudness

The human ear uses vibrations to transform sound waves to electrical impulses through a multi-step process. Sound waves travel through the ear canal and vibrate the eardrum and the adjacent tiny bones of the middle ear. These, in turn, vibrate the oval window of the inner ear and carry the vibrations to the liquid-filled cochlea, where vibrations are converted into electrical impulses to be interpreted by the human nervous system.

Sources of Sound: Vibrating Strings and Air Columns

Vibrating strings function as standing waves whose pitch corresponds to their fundamental frequency, given by $f = v/2L$. This equation is the same for standing waves in a long narrow tube with both ends open, which produces sound waves such that the pressure is analogous to the displacement of a vibrating string. This is only slightly different for a tube with one end closed, as the pressure at the closed end is an antinode, and the fundamental frequency is given by $f_1 = v/4L$.

Interference of Sound Waves; Beats

The expansions and contractions of sound waves from different sources interfere with each other in a manner such that if two sources emit the same frequency in phase, constructive interference will occur when their compressions and expansions align. Otherwise, destructive interference will occur to varying degrees (dependent on the misalignment of wavelength).

- Interference of two sources of similar frequency produce audible recurring intensity changes called **beats.**
- The **beat frequency** is given by the frequency difference of the waves.

Doppler Effect

The **Doppler effect** describes frequency perceived by an observer as a function of the velocity of the source that is emitting sound waves.

- The new frequency the observer perceives is given by $f' = f/(1 - [v_s/v])$ when a source is moving toward a stationary observer, and $f' = f/(1 + [v_s/v])$ when a source is moving away from a stationary observer. For these equations, v_s is the velocity of the source, and v is the velocity of the sound waves in air.
- Alternately, when the source is at rest and the observer is moving toward it at a constant velocity, v_o, the perceived frequency is given by $f' = (1 + [v_o/v])f$, and by $f' = (1 - [v_o/v])f$ when the observer is moving away from a stationary source.
- The Doppler effect applies to all waves—including light waves, which shift to a lower frequency when their source is moving away. This is called a **red shift.**

Shock Waves and the Sonic Boom

Supersonic speed describes velocities faster than the speed of sound, and it is quantified in **Mach number,** given by the ratio of an object's velocity to that of sound in the same medium.

- **Shock waves** occur when a sound wave source travels faster than sound and creates a powerful constructive interference by its coinciding wave fronts. The angle of the cone, θ, that is created at the front of an object traveling at supersonic speeds is related to the velocity of the object, v_{obj}, and the sound in the medium, v_{snd}, by $\sin\theta = v_{snd}/v_{obj}$.

For Additional Review

With regard to its energy, consider which properties of waves and which properties of particles apply to light.

Multiple-Choice Questions

1. If the length of a pendulum with period 10 seconds were halved, what would be its new period?
   (A) 3.2 s
   (B) 5.0 s
   (C) 7.1 s
   (D) 10 s
   (E) 13 s

2. How many oscillations per second does a 2.0 kg mass have on a spring ($k = 4.44$ N/m) undergoing simple harmonic motion horizontally?
   (A) 0.24 Hz
   (B) 0.67 Hz
   (C) 1.5 Hz
   (D) 3.1 Hz
   (E) 4.2 Hz

3. What is the maximum velocity for a 4.0 kg mass attached to a horizontally placed spring ($k = 22$ N/m) if its amplitude is 0.1 m?
   (A) 0.23 m/s
   (B) 0.45 m/s
   (C) 0.98 m/s
   (D) 1.3 m/s
   (E) 1.8 m/s

4. How far from equilibrium will a 45-gram mass—with an amplitude of 0.35 m for a spring ($k = 21$ N/m) undergoing simple harmonic motion—have a speed that is half its maximum velocity?
   (A) 0.30 m
   (B) 0.45 m
   (C) 0.70 m
   (D) 0.92 m
   (E) 3.5 m

5. A massless vertical spring is displaced 0.15 m when a 3.25 kg mass is attached. When used horizontally, what is the total energy if the spring has an amplitude of 0.25 m?
   (A) 0.65 J
   (B) 1.0 J
   (C) 3.4 J
   (D) 6.6 J
   (E) 9.8 J

6. When a wave hits a medium of differing density at 35°, its angle of refraction is 6°. If its final velocity is 5.25 m/s, the initial velocity was
   (A) 1 m/s
   (B) 5.25 m/s
   (C) 11 m/s
   (D) 29 m/s
   (E) 31 m/s

7. Which of the following could be the fundamental frequency for a vibration that has an overtone frequency of 990 Hz?
   (A) 330 Hz
   (B) 660 Hz
   (C) 1485 Hz
   (D) 1980 Hz
   (E) 1990 Hz

8. The intensity of a spherical wave is 21 W/m^2 at a distance of 16 meters away from a constantly emitting source. What will it be from 3 meters away?
   (A) 1000 W/m^2
   (B) 600 W/m^2
   (C) 112 W/m^2
   (D) 3.9 W/m^2
   (E) 0.74 W/m^2

9. Which of the following values would NOT be sufficient to determine the force of tension in a string vibrating at its fundamental frequency?
   (A) Wavelength, frequency, and mass
   (B) Mass, wave velocity, and string length
   (C) Wave velocity, mass, and wavelength
   (D) String length, mass, and frequency
   (E) Wave velocity, string length, and frequency

10. An object attached to a spring undergoing simple harmonic motion reaches its maximum velocity of 4 m/s. What is its amplitude?
   (A) 0.02 m
   (B) 0.12 m
   (C) 0.25 m
   (D) 0.30 m
   (E) Cannot determine amplitude with information given

Free-Response Questions

1. An object attached to a spring ($k = 30$ N/m) has a velocity of 2.5 m/s when it is 0.55 m from equilibrium, and it has a period of 2.45 seconds.
   (a) What is the mass of the object?
   (b) What is the total energy of the system?
   (c) What is the amplitude?
   (d) What is the maximum speed?
   (e) At what displacement is the speed maximized?

2. A horizontal string of length 1.5 meters vibrates with a wave velocity of 1320 m/s at its fundamental frequency.
   (a) What is the fundamental frequency?
   (b) What is the frequency of the fourth overtone, and how many nodes and antinodes will it have?
   (c) In terms of interference, to what do the nodes and antinodes correspond?
   (d) How many grams must the string be to have a tension of 10^5 N?

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. **(C) is correct.** As stated, the relation between the initial length L_i and the final length L_f is given by $L_i = 2L_f$. The relation between a pendulum's period and length is given by $T = 2\pi\sqrt{(l/g)}$. So, 10 s $= 2\pi\sqrt{(L_i/g)}$. $T_2 = 2\pi\sqrt{(L_f/g)} = 2\pi\sqrt{(L_i/2g)} = 7.1$ s.
2. **(A) is correct.** The period of simple harmonic motion is given by $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{(2.0 \text{ kg})/(4.44 \text{ N/m})} = 4.22$ s. Frequency, or oscillations per second, is $1/T = 1/(4.22 \text{ s}) = 0.24$ Hz.
3. **(A) is correct.** Using the relation $1/2(mv_1^2) + 1/2kx_1^2 = 1/2(mv_2^2) + 1/2kx_2^2$, the velocity is zero at maximum displacement, which is equal to the amplitude. Similarly, the velocity will be greatest when the spring is not stretched or compressed—that is, $x_2 = 0$. $1/2(4 \text{ kg})(0 \text{ m})^2 + 1/2(22 \text{ N/m})(0.1 \text{ m})^2 = 1/2(4 \text{ kg})(v_2^2) + 1/2(22 \text{ N/m})(0 \text{ m})^2$ and $v_2 = .23$ m/s.
4. **(A) is correct.** The following relationships $1/2(mv_{max}^2) = 1/2(kA^2) = 1/2m(v_{max}/2)^2 + 1/2(kx^2)$ can be used to find the value of x sought. $1/2(0.045 \text{ kg})v_{max}^2 = 1/2(21 \text{ N/m})(0.35 \text{ m})^2$, so $v_{max} = 7.6$ m/s.
   $1.3 \text{ J} = 1/2m(v_{max}/2)^2 + 1/2(kx^2) = 1/2(0.045 \text{ kg})(7.6/2 \text{ m/s})^2 + 1/2(21 \text{ N/m})(x)^2$
   so $x = 0.30$ m.
5. **(D) is correct.** The first of two steps is to determine the spring constant. From the information given, $F = kd = mg$,
   so $k = mg/d = (3.25 \text{ kg})(9.8 \text{ m/s}^2)/(0.15 \text{ m}) = 212$ N/m.
   Total energy is given by $1/2(kA^2) = 1/2(212 \text{ N/m})(0.25 \text{ m})^2 = 6.6$ J.
6. **(D) is correct.** The law of refraction states that the relation between angle and velocity is given by $\sin\theta_2/\sin\theta_1 = v_2/v_1$,
   $v_1 = v_2 \sin\theta_1/\sin\theta_2 = (5.25 \text{ m/s})(\sin 35°)/(\sin 6°) = 29$ m/s.
7. **(A) is correct.** All overtones are integer multiples of the fundamental frequency, $f_n = nf_1$. As such, an overtone is a value into which the fundamental frequency divides evenly, 990 Hz$/n = f_1$. Of the values listed, 990 Hz/3 = 330 Hz is the only correct answer of the five presented.
8. **(B) is correct.** $I_2/I_1 = r_1^2/r_2^2$ $I_2 = I_1(r_1^2/r_2^2) = 21 \text{ W/m}^2((16 \text{ m})^2/(3 \text{ m})^2) = 598 \text{ W/m}^2$, or 600 W/m^2 to significant figures.
9. **(E) is correct.** The force of tension in a string is generally given by $F_T = mv^2/L$. Since $v = \lambda f$ by definition and $\lambda = 2L$ because it is vibrating at its fundamental frequency, $F_T = mv^2/L = 2mv^2/\lambda = m(\lambda f)^2/L = m(2Lf)^2/L = 4mLf^2 = 2m\lambda f^2$.
10. **(E) is correct.** Not enough information is provided to determine amplitude. The maximum displacement could have been determined if the mass of the object and the spring constant value were provided.

Free-Response Questions

1. (a) Period is a function of mass, such that $T = 2\pi\sqrt{(m/k)}$. This can be rearranged so that mass is a function of period and the spring constant, $m = (T/2\pi)^2k = (2.45 \text{ s}/2\pi)^2(30.0 \text{ N/m}) = 4.56$ kg.

(b) The total mechanical energy of the system is given by $E = 1/2(mv^2) + 1/2(kx^2)$ where v is the velocity at a displacement x, which is given, so $E = 1/2(4.56 \text{ kg})(2.50 \text{ m/s})^2 = 1/2(30.0 \text{ N/m})(.550 \text{ m})^2 = 18.8 \text{ J}$.

(c) Because the total energy of the system is given by $E = 1/2(mv^2) + 1/2(kx^2) = 1/2(mv^2_{max}) = 1/2(kA^2)$, $E = 18.8 \text{ J} = 1/2(kA^2) = 1/2(30 \text{ N/m})(A)^2$, the resulting amplitude is $A = \sqrt{2E/k} = 1.12$ m.

(d) $18.8 \text{ J} = 1/2(mv^2_{max}) = 1/2(4.56 \text{ kg})(v_{max})^2$, so the maximum velocity $v_{max} = 2.87$ m/s.

(e) The maximum velocity occurs when the spring has no potential energy stored in it, which is at the instant the object is at equilibrium, $x = 0$.

This response correctly identifies and utilizes the total mechanical energy of an oscillating system undergoing simple harmonic motion, as well as the relation of period to mass developed from the one-dimensional analog of circular motion. For the former, the relation between amplitude and maximum velocity is used for the responses to parts c and d. Furthermore, the relationship between displacement from equilibrium and velocity is analyzed correctly for the response to part e.

2. (a) The relation between wavelength and string length is given by $\lambda = 2L = 2(1.5 \text{ m}) = 3$ m, which can be used to determine velocity, $v = \lambda f$, so $1320 \text{ m/s} = (3 \text{ m})f$. Therefore, 440 Hz is the fundamental frequency.

   (b) The fourth overtone, or fifth harmonic, is given by $5f_1 = 5(440 \text{ Hz}) = 2200$ Hz and would have five antinodes and six nodes.

   (c) The nodes correspond to points of destructive interference, whereas the antinodes correspond to points of constructive interference.

   (d) The tension of a string is given by $F_T = mv^2/L$, so $LF_T/v^2 = m$, so $m = 0.086 \text{ kg} = 86$ grams.

This response uses the given quantities to solve for frequency in part a by first solving for wavelength from string length. For the response to part b, the relationship between harmonic number and overtone number is correctly identified, as well as the corresponding number of nodes and antinodes and their definitions in the response to part c. Finally, the response to part d rearranges the equation for the force of tension to solve for mass.