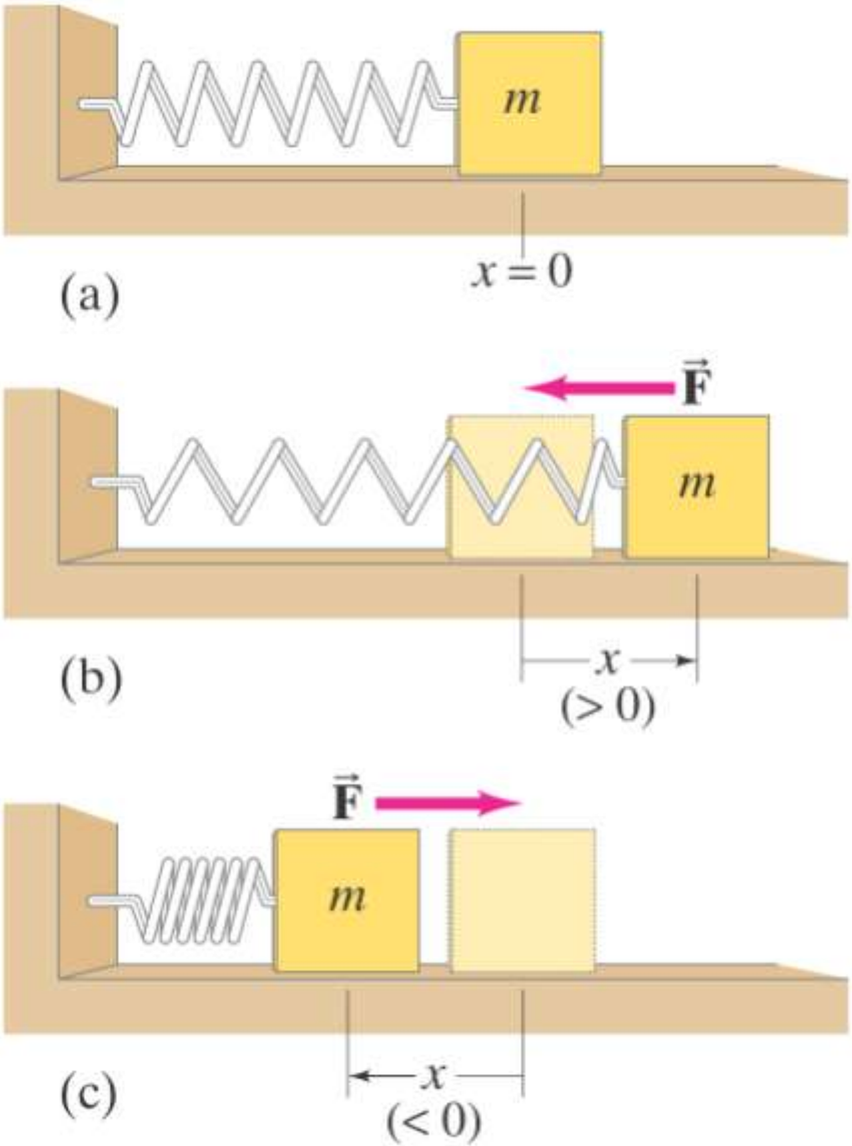
Chapter 11

Vibrations and Waves

11-1 Simple Harmonic Motion



If an object vibrates or oscillates back and forth over the same path, each cycle taking the same amount of time, the motion is called periodic. The mass and spring system is a useful model for a periodic system.

11-1 Simple Harmonic Motion

We assume that the surface is frictionless. There is a point where the spring is neither stretched nor compressed; this is the equilibrium position. We measure displacement from that point (*x* = 0 on the previous figure).

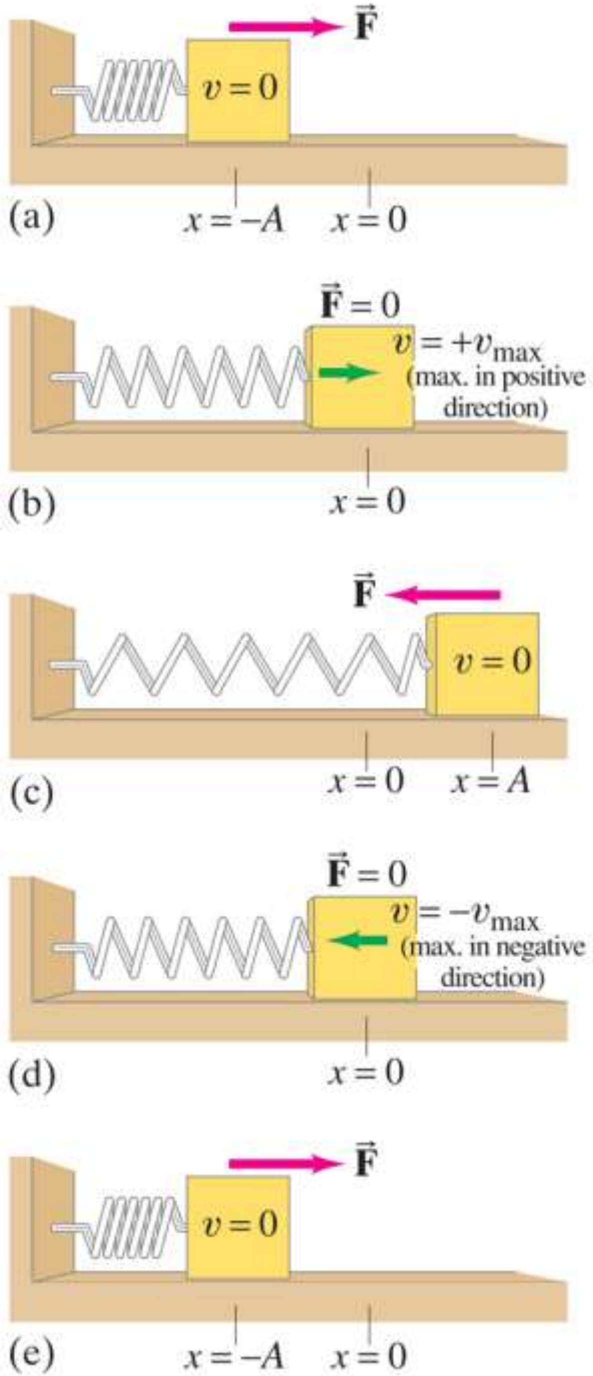
The force exerted by the spring depends on the displacement:

$$F = -kx \qquad (11\text{-}1)$$

11-1 Simple Harmonic Motion

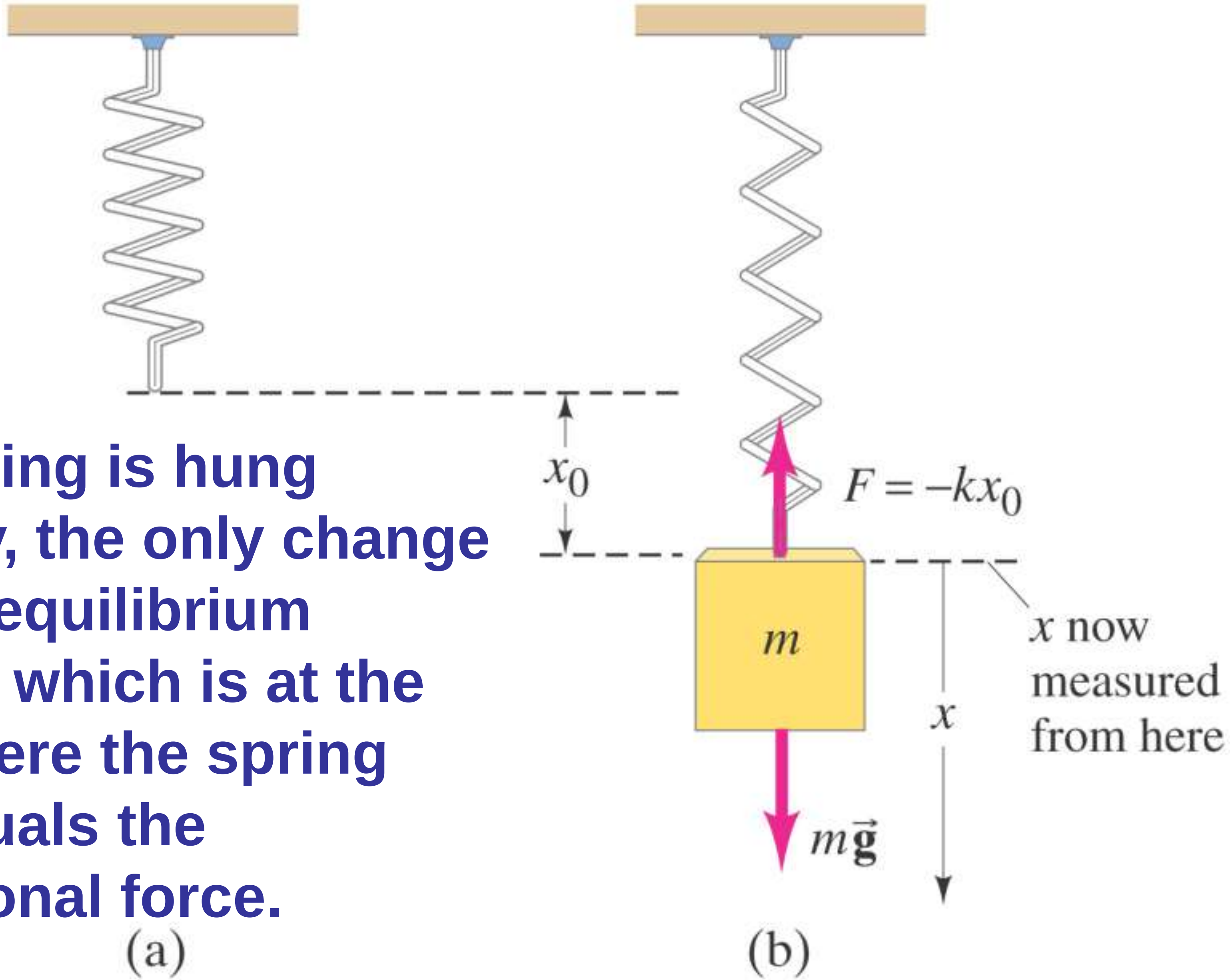
- The minus sign on the force indicates that it is a restoring force – it is directed to restore the mass to its equilibrium position.
- *k* is the spring constant
- The force is not constant, so the acceleration is not constant either

11-1 Simple Harmonic Motion



- Displacement is measured from the equilibrium point
- Amplitude is the maximum displacement
- A cycle is a full to-and-fro motion; this figure shows half a cycle
- Period is the time required to complete one cycle
- Frequency is the number of cycles completed per second

11-1 Simple Harmonic Motion



If the spring is hung vertically, the only change is in the equilibrium position, which is at the point where the spring force equals the gravitational force.

11-1 Simple Harmonic Motion

Any vibrating system where the restoring force is proportional to the negative of the displacement is in simple harmonic motion (SHM), and is often called a simple harmonic oscillator.

11-2 Energy in the Simple Harmonic Oscillator

We already know that the potential energy of a spring is given by:

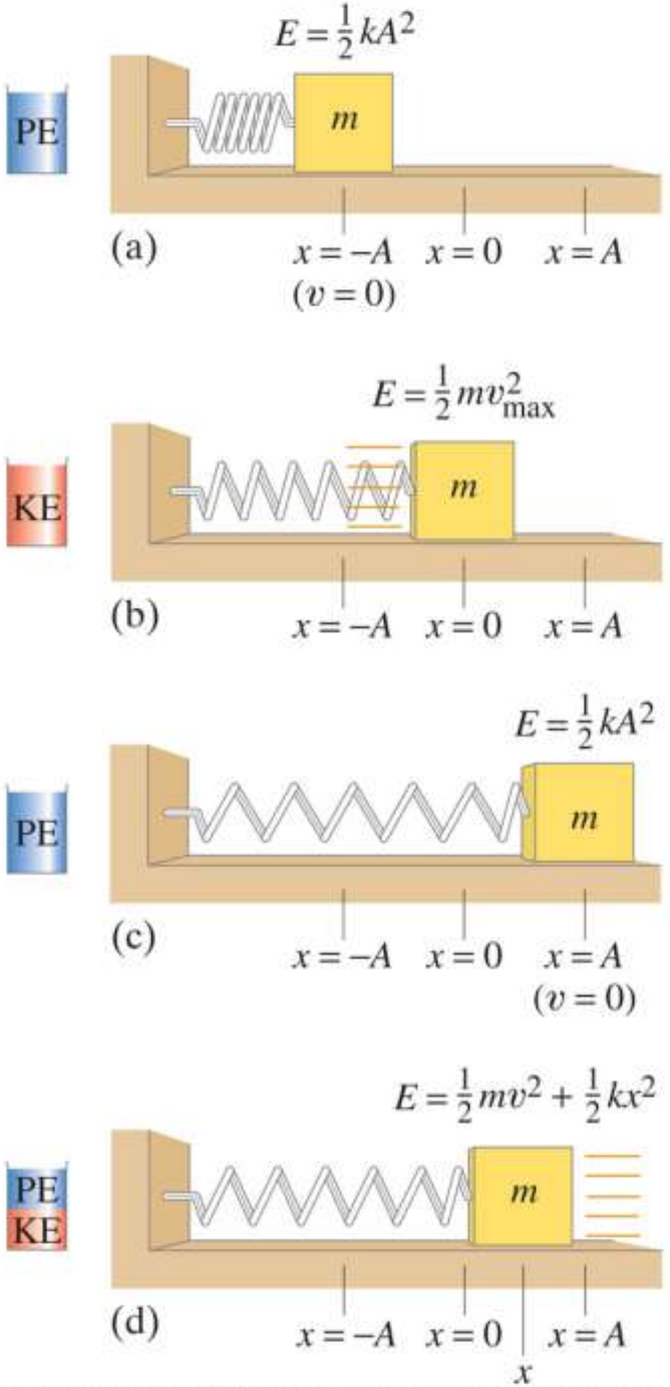
$$\text{PE} = \frac{1}{2}kx^2$$

The total mechanical energy is then:

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \qquad \textbf{(11-3)}$$

The total mechanical energy will be conserved, as we are assuming the system is frictionless.

11-2 Energy in the Simple Harmonic Oscillator



If the mass is at the limits of its motion, the energy is all potential.

If the mass is at the equilibrium point, the energy is all kinetic.

We know what the potential energy is at the turning points:

$$E = \frac{1}{2}kA^2 \qquad \textbf{(11-4a)}$$

11-2 Energy in the Simple Harmonic Oscillator

The total energy is, therefore $\frac{1}{2}kA^2$

And we can write:

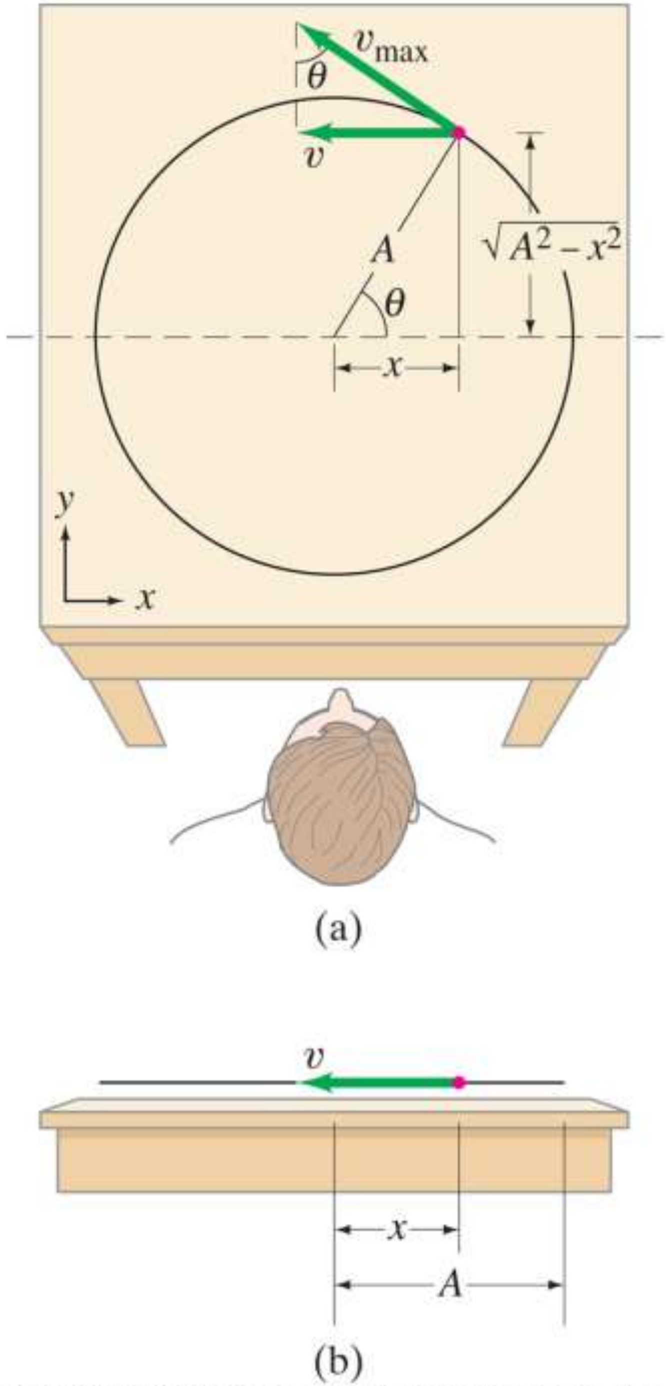
$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \qquad \textbf{(11-4c)}$$

This can be solved for the velocity as a function of position:

$$v = \pm v_{\max}\sqrt{1 - \frac{x^2}{A^2}} \qquad \textbf{(11-5)}$$

where $v_{\max}^2 = (k/m)A^2$

11-3 The Period and Sinusoidal Nature of SHM



If we look at the projection onto the *x* axis of an object moving in a circle of radius *A* at a constant speed v_{max}, we find that the *x* component of its velocity varies as:

$$v = v_{\max}\sqrt{1 - \frac{x^2}{A^2}}$$

This is identical to SHM.

11-3 The Period and Sinusoidal Nature of SHM

Therefore, we can use the period and frequency of a particle moving in a circle to find the period and frequency:

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad \textbf{(11-7a)}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad \textbf{(11-7b)}$$

11-3 The Period and Sinusoidal Nature of SHM

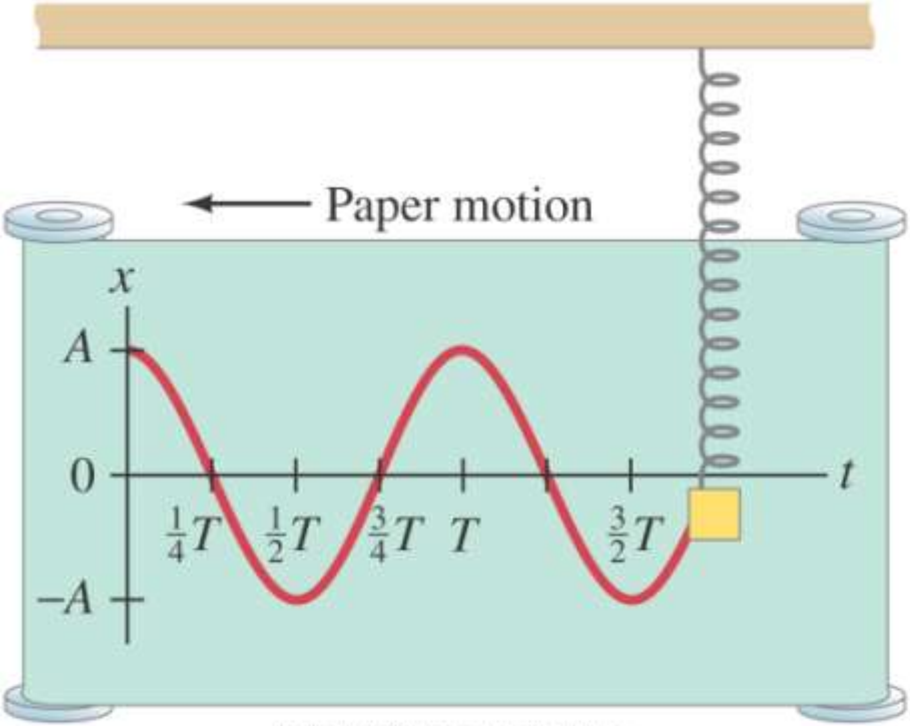
We can similarly find the position as a function of time:

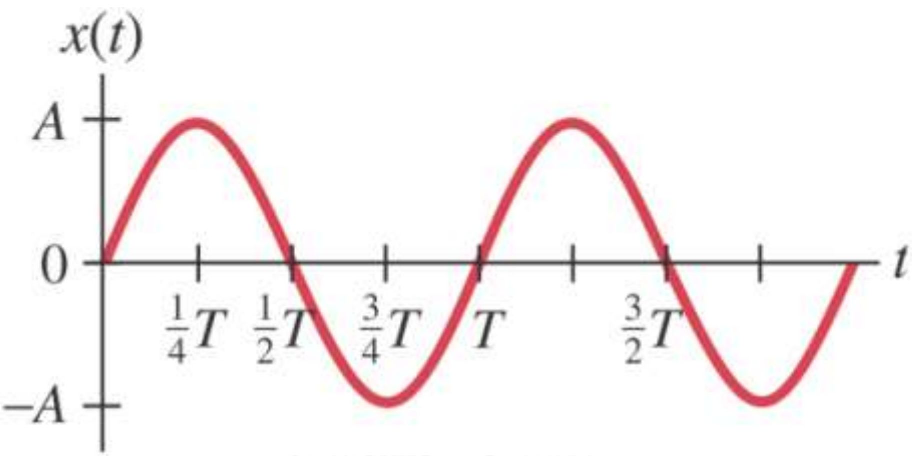
$$x = A\cos\omega t \qquad \text{(11-8a)}$$

$$= A\cos(2\pi f t) \qquad \text{(11-8b)}$$

$$= A\cos(2\pi t/T) \qquad \text{(11-8c)}$$

11-3 The Period and Sinusoidal Nature of SHM





The top curve is a graph of the previous equation.

The bottom curve is the same, but shifted ¼ period so that it is a sine function rather than a cosine.

11-3 The Period and Sinusoidal Nature of SHM

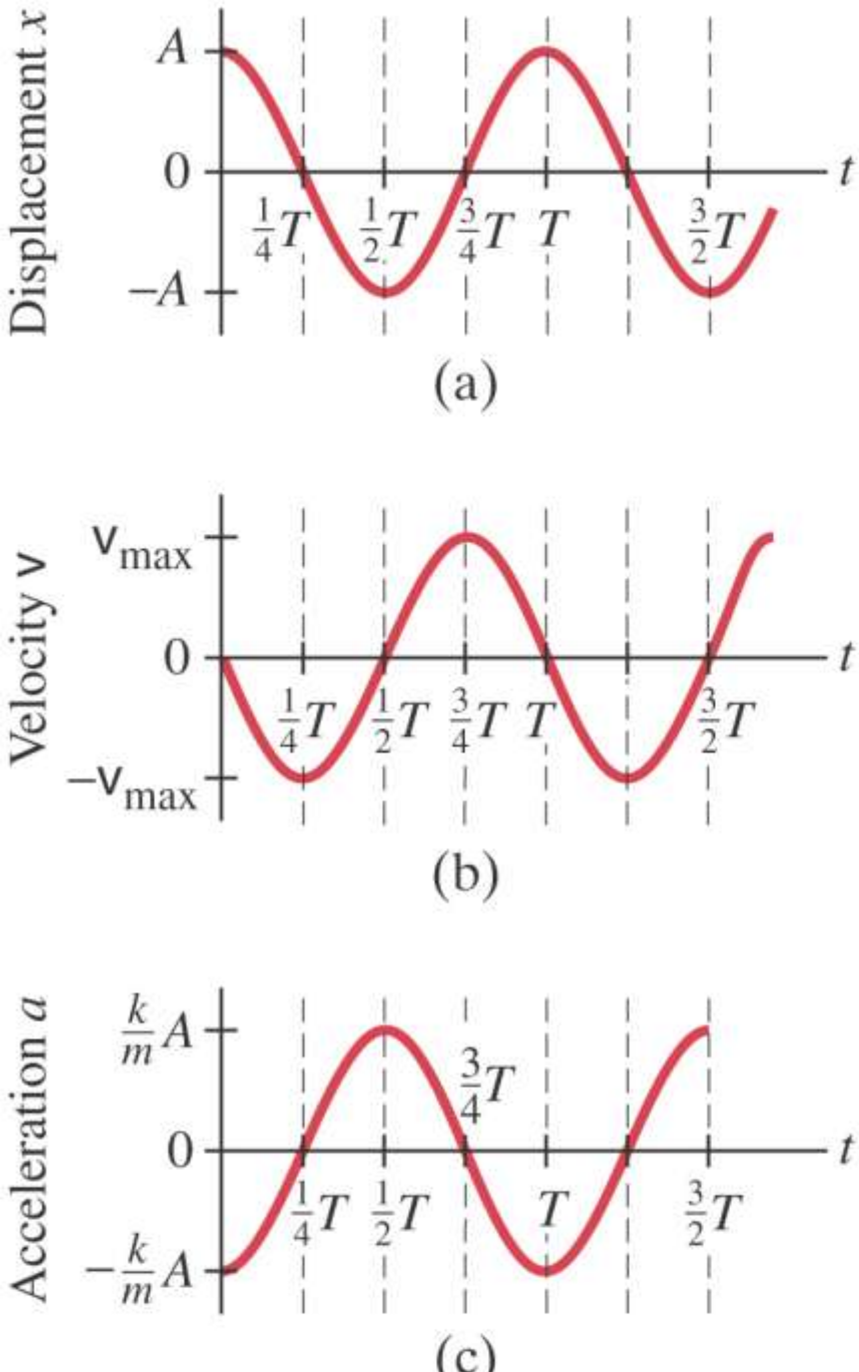
The velocity and acceleration can be calculated as functions of time; the results are below, and are plotted at left.

$$v = -v_{max} \sin \omega t \qquad \textbf{(11-9)}$$

$$v_{max} = A\sqrt{\frac{k}{m}}$$

$$a = -a_{max} \cos(2\pi t / T) \qquad \textbf{(11-10)}$$

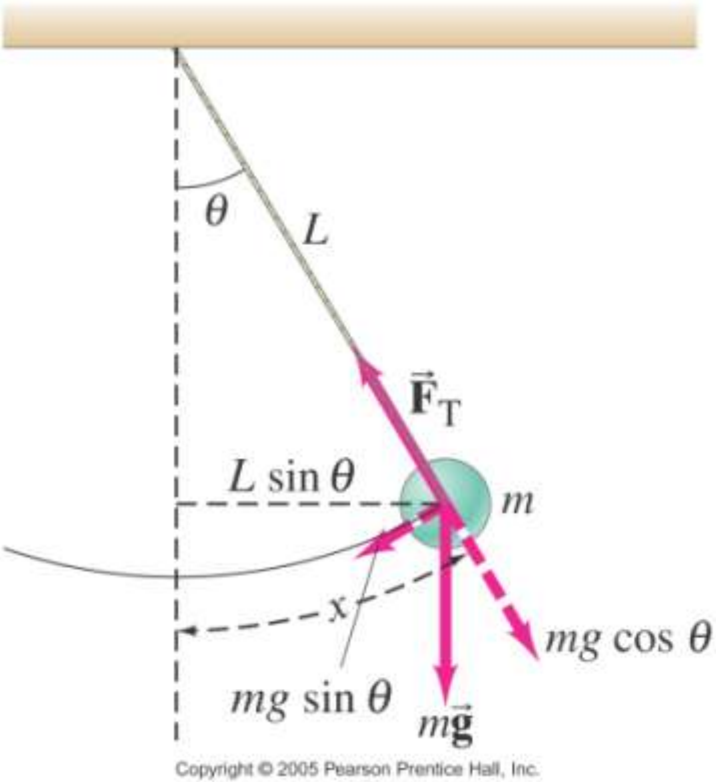
$$a_{max} = kA/m$$



11-4 The Simple Pendulum

A simple pendulum consists of a mass at the end of a lightweight cord. We assume that the cord does not stretch, and that its mass is negligible.

11-4 The Simple Pendulum



In order to be in SHM, the restoring force must be proportional to the negative of the displacement. Here we have: $F = -mg\sin\theta$

which is proportional to sin *θ* and not to *θ* itself.

However, if the angle is small, sin *θ* ≈ *θ*.

TABLE 11–1
Sin *θ* at Small Angles

θ (degrees)	*θ* (radians)	sin *θ*	% Difference
0	0	0	0
1°	0.01745	0.01745	0.005%
5°	0.08727	0.08716	0.1%
10°	0.17453	0.17365	0.5%
15°	0.26180	0.25882	1.1%
20°	0.34907	0.34202	2.0%
30°	0.52360	0.50000	4.7%

11-4 The Simple Pendulum

Therefore, for small angles, we have:

$$F \approx -\frac{mg}{L}x$$

where $x = L\theta$

The period and frequency are:

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \text{(11-11a)}$$

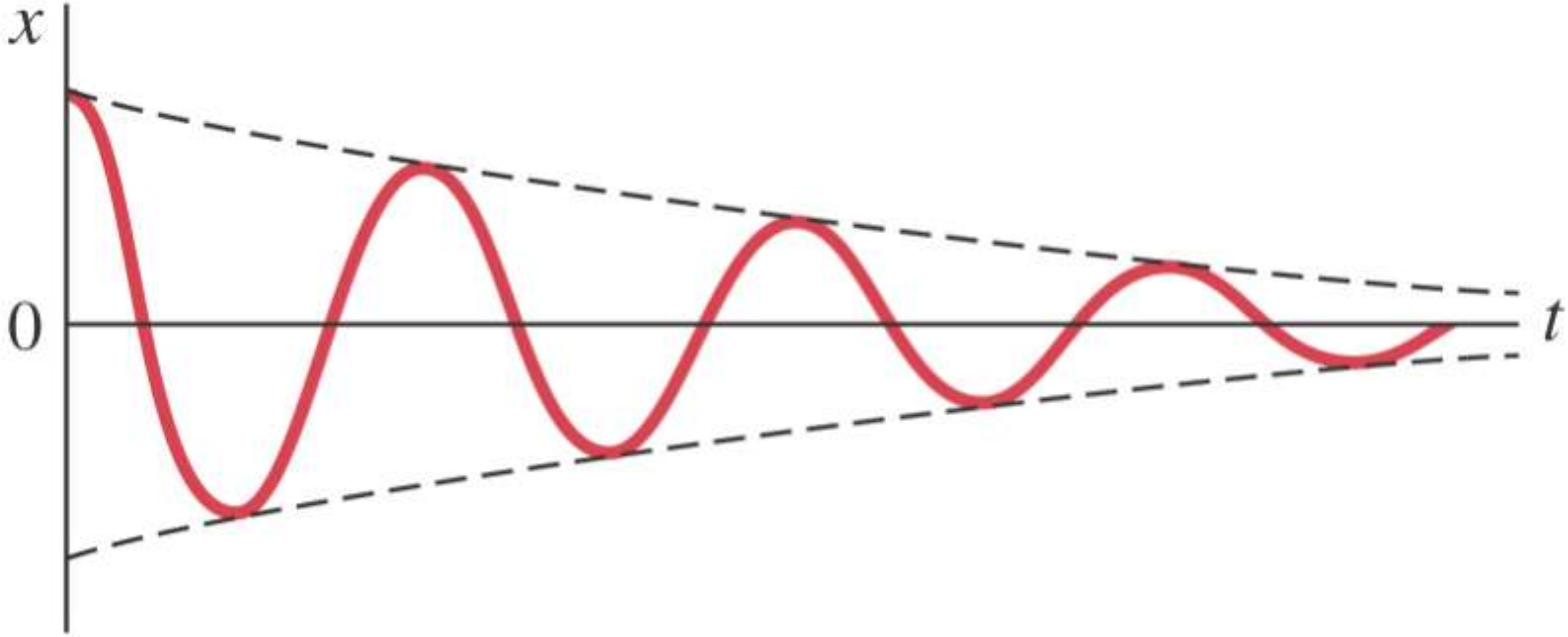
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad \text{(11-11b)}$$

11-4 The Simple Pendulum

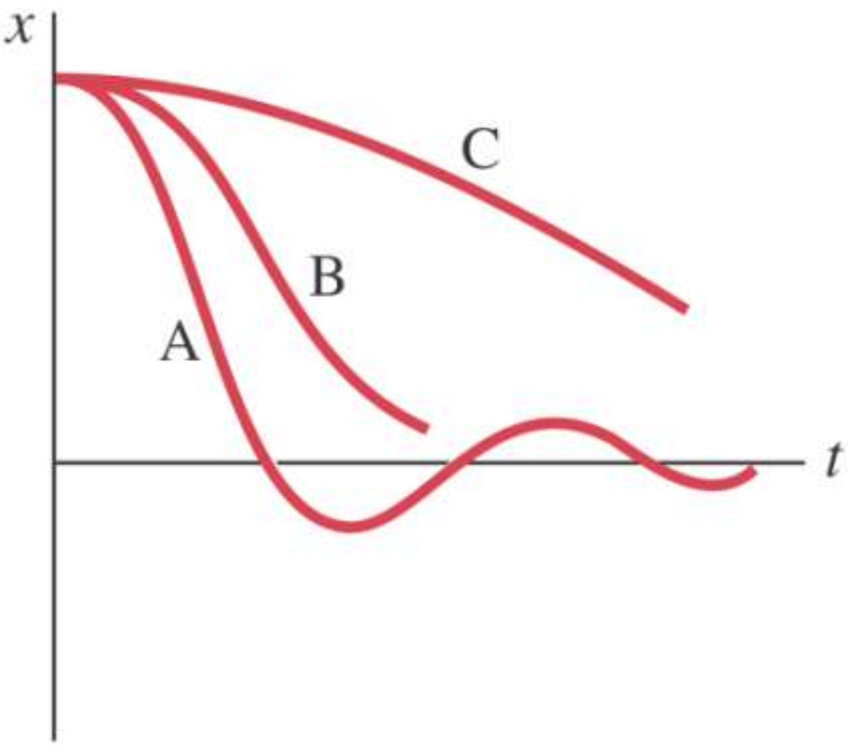
So, as long as the cord can be considered massless and the amplitude is small, the period does not depend on the mass.

11-5 Damped Harmonic Motion

Damped harmonic motion is harmonic motion with a frictional or drag force. If the damping is small, we can treat it as an "envelope" that modifies the undamped oscillation.



11-5 Damped Harmonic Motion



However, if the damping is large, it no longer resembles SHM at all.

A: underdamping: there are a few small oscillations before the oscillator comes to rest.

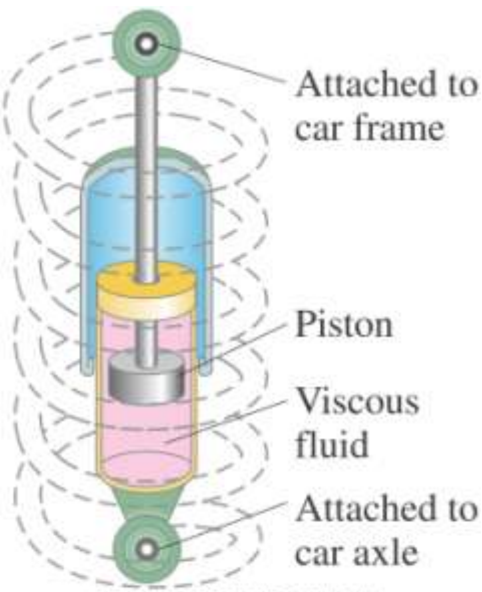
B: critical damping: this is the fastest way to get to equilibrium.

C: overdamping: the system is slowed so much that it takes a long time to get to equilibrium.

11-5 Damped Harmonic Motion

There are systems where damping is unwanted, such as clocks and watches.

Then there are systems in which it is wanted, and often needs to be as close to critical damping as possible, such as automobile shock absorbers and earthquake protection for buildings.

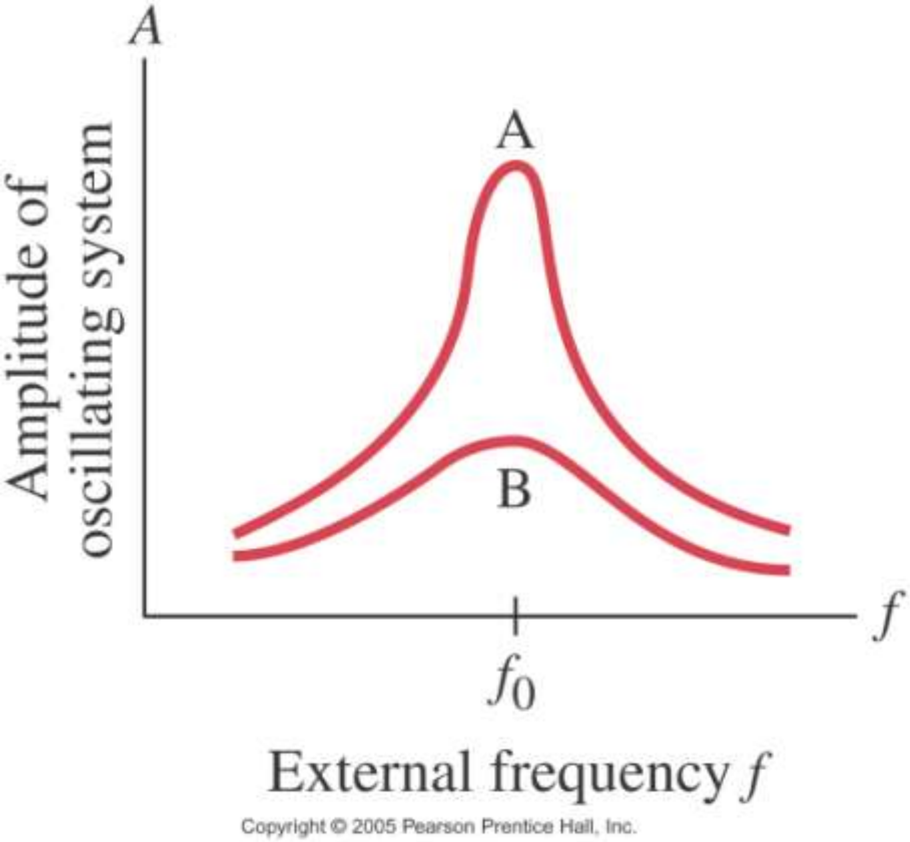


11-6 Forced Vibrations; Resonance

Forced vibrations occur when there is a periodic driving force. This force may or may not have the same period as the natural frequency of the system.

If the frequency is the same as the natural frequency, the amplitude becomes quite large. This is called resonance.

11-6 Forced Vibrations; Resonance



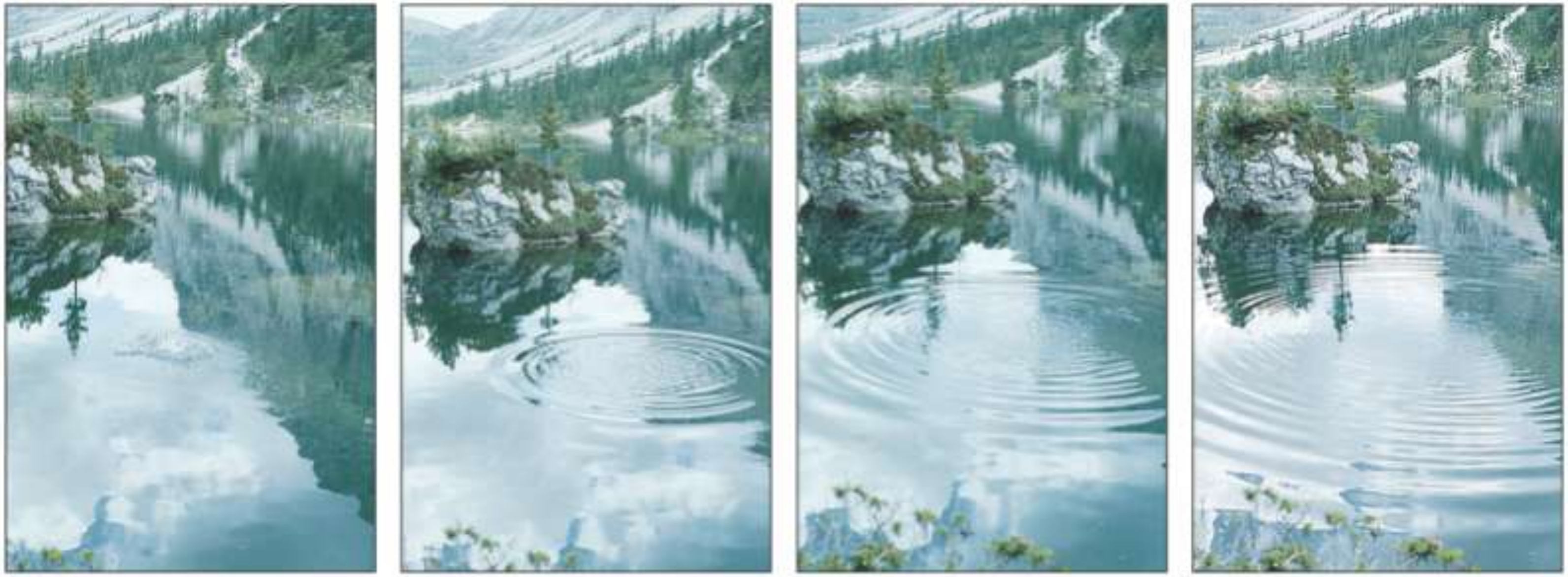
The sharpness of the resonant peak depends on the damping. If the damping is small (A), it can be quite sharp; if the damping is larger (B), it is less sharp.

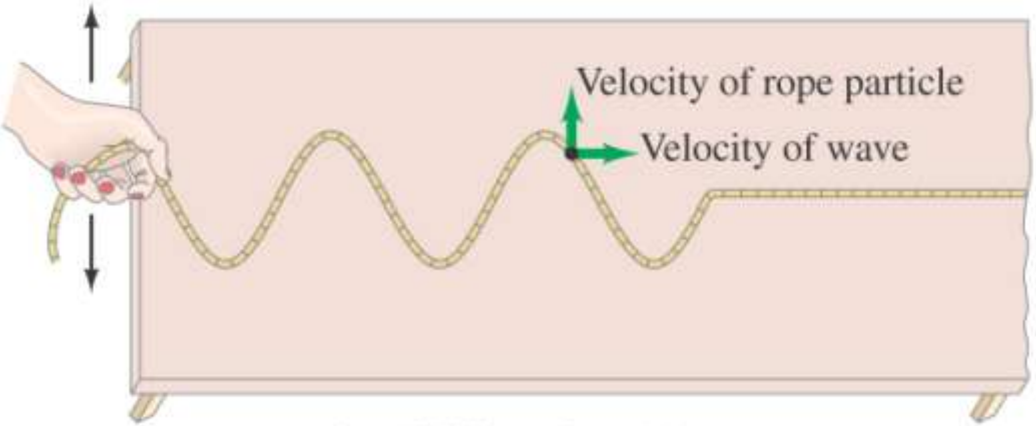
Like damping, resonance can be wanted or unwanted. Musical instruments and TV/radio receivers depend on it.

11-6 Forced Vibrations; Resonance

[Time for a Gizmo!]

11-7 Wave Motion

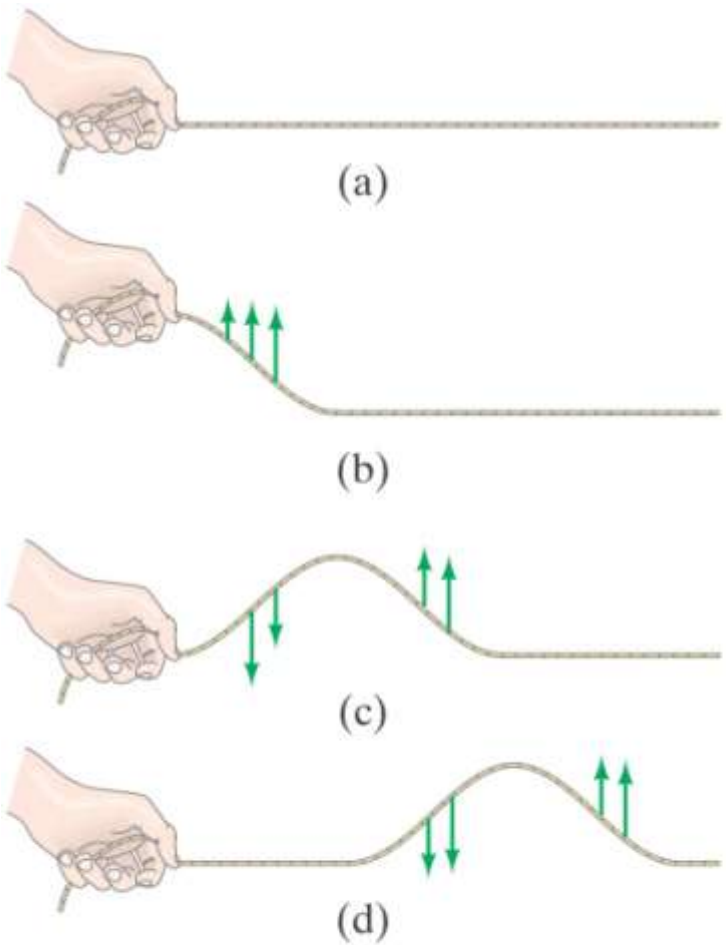




A wave travels along its medium, but the individual particles just move up and down.

11-7 Wave Motion

All types of traveling waves transport energy.



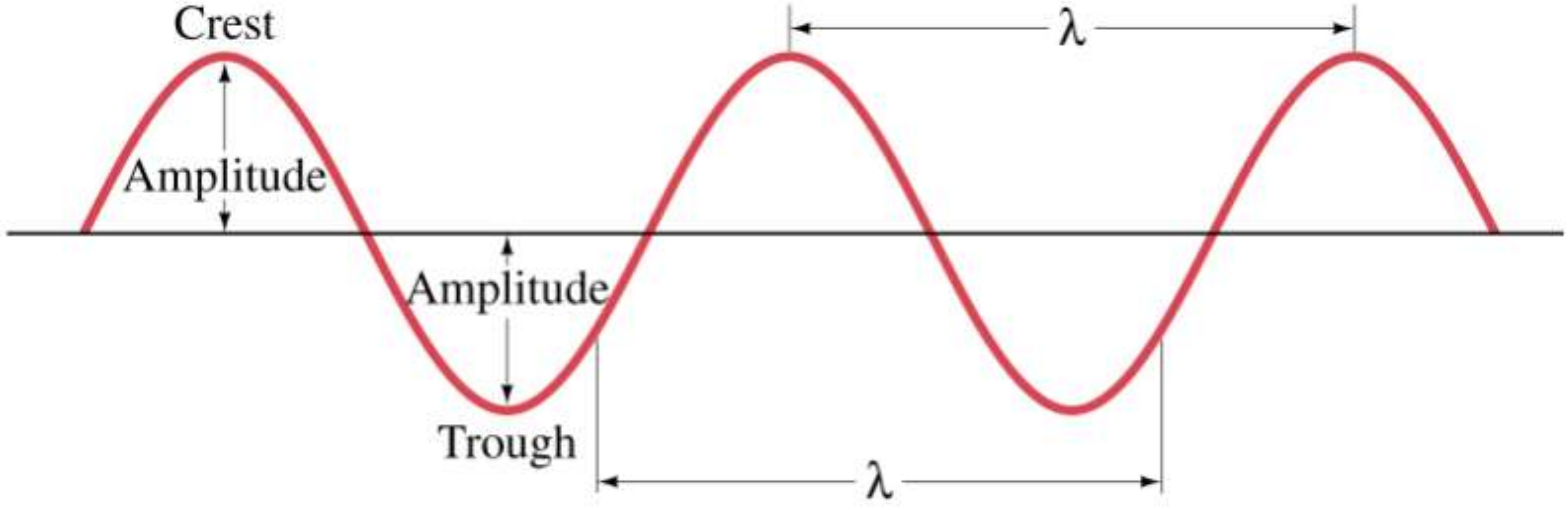
Study of a single wave pulse shows that it is begun with a vibration and transmitted through internal forces in the medium.

Continuous waves start with vibrations too. If the vibration is SHM, then the wave will be sinusoidal.

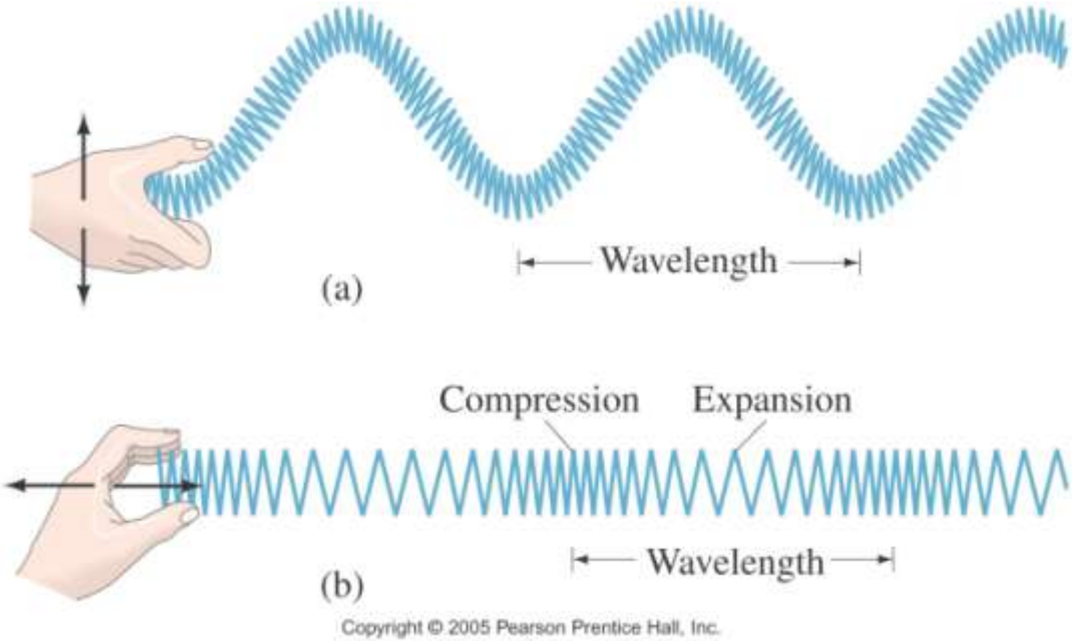
11-7 Wave Motion

Wave characteristics:

- **Amplitude, *A***
- **Wavelength, *λ***
- **Frequency *f* and period *T***
- **Wave velocity** $v = \lambda f$ **(11-12)**



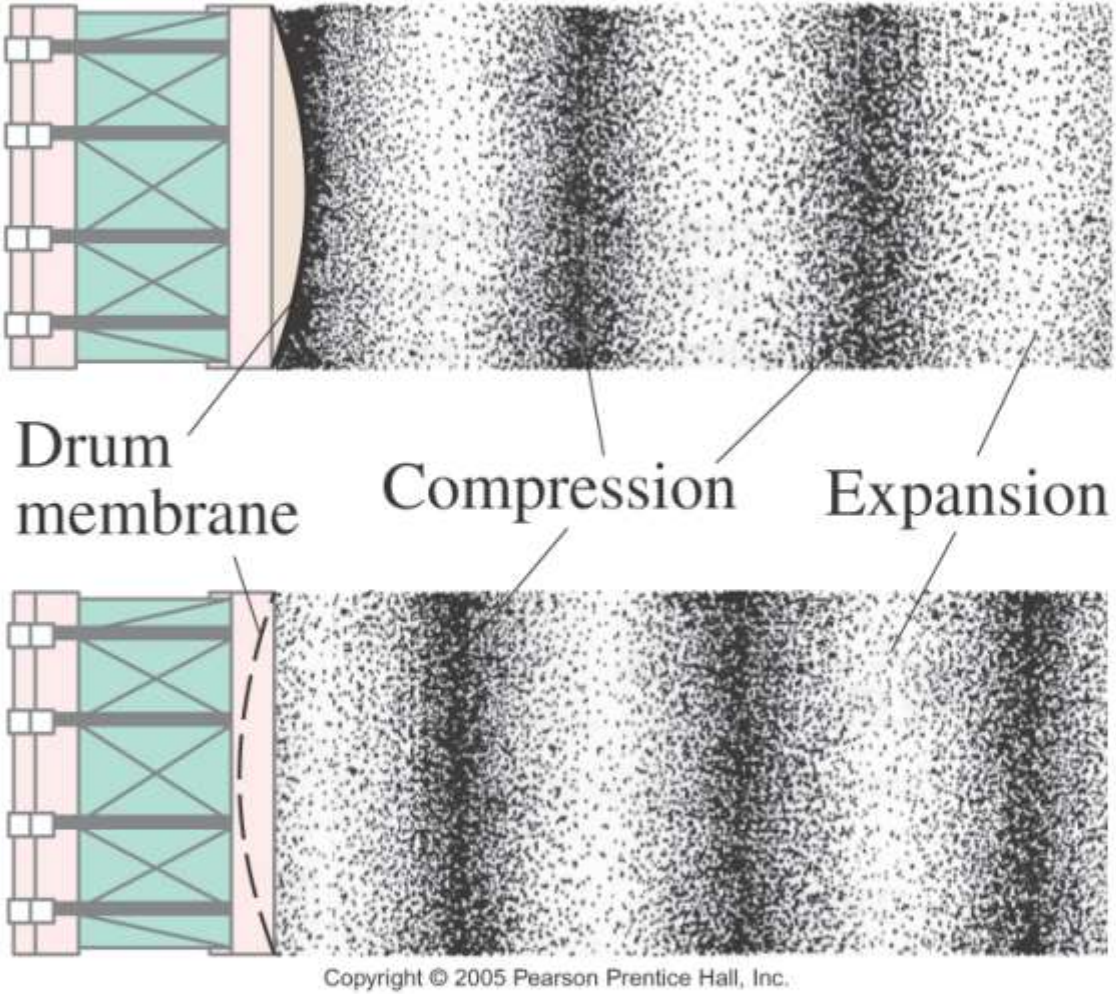
11-8 Types of Waves: Transverse and Longitudinal



The motion of particles in a wave can either be perpendicular to the wave direction (transverse) or parallel to it (longitudinal).

11-8 Types of Waves: Transverse and Longitudinal

Sound waves are longitudinal waves:



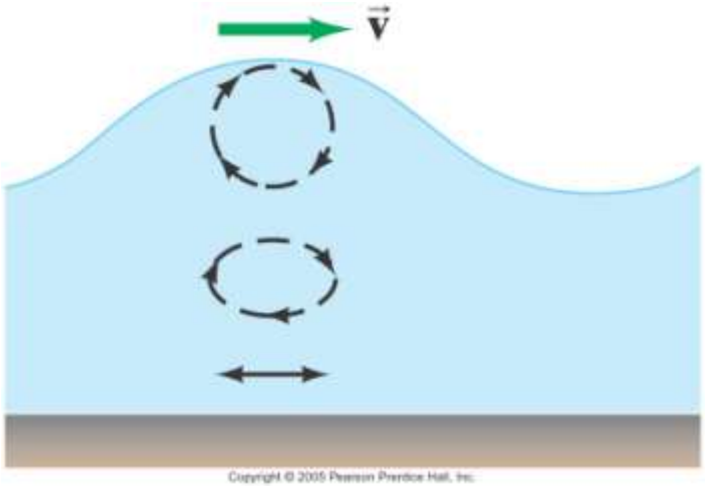
11-6 Forced Vibrations; Resonance

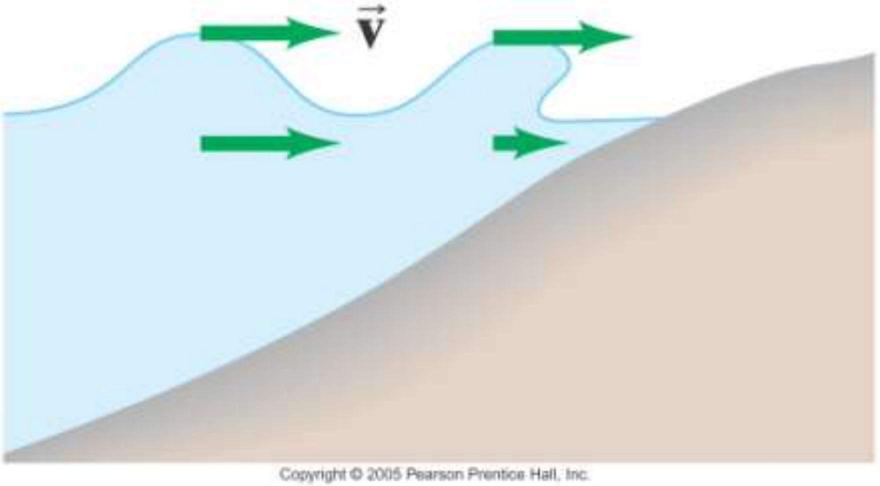
Time for a Gizmo!

11-8 Types of Waves: Transverse and Longitudinal

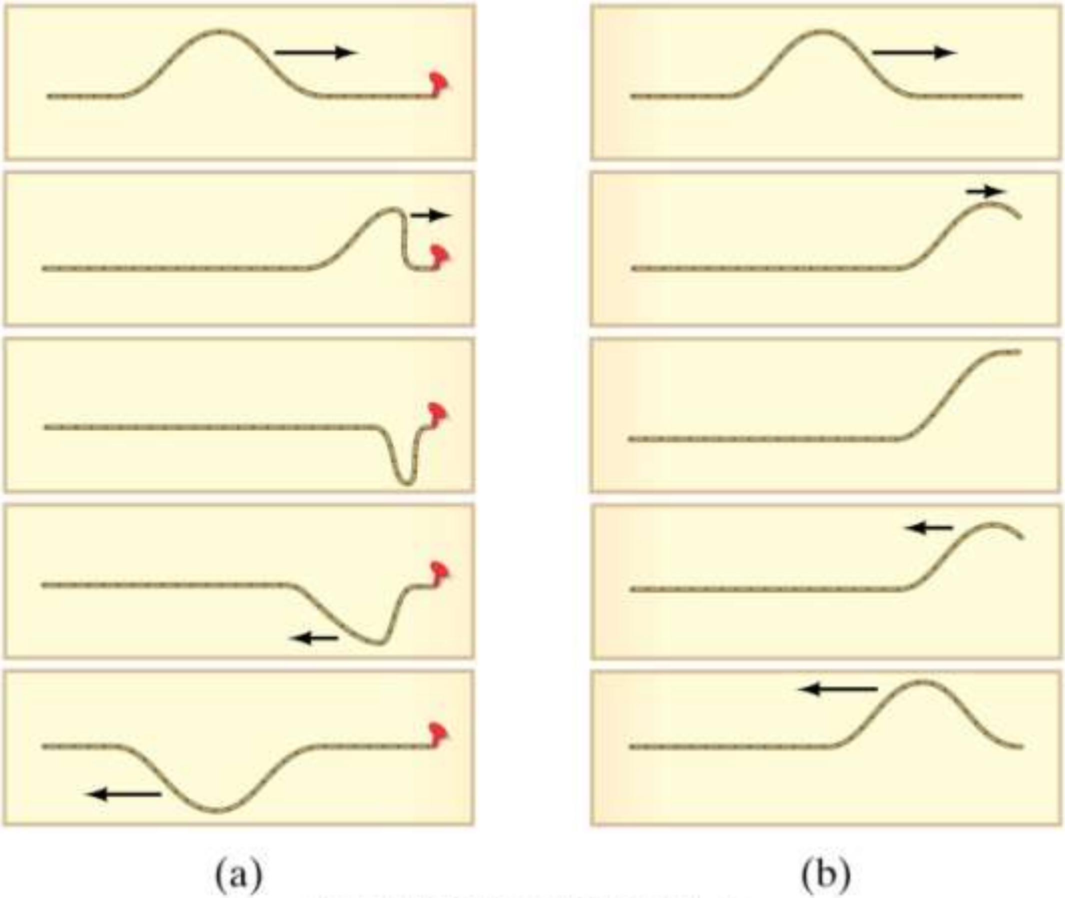
Earthquakes produce both longitudinal and transverse waves. Both types can travel through solid material, but only longitudinal waves can propagate through a fluid – in the transverse direction, a fluid has no restoring force.

Surface waves are waves that travel along the boundary between two media.





11-11 Reflection and Transmission of Waves



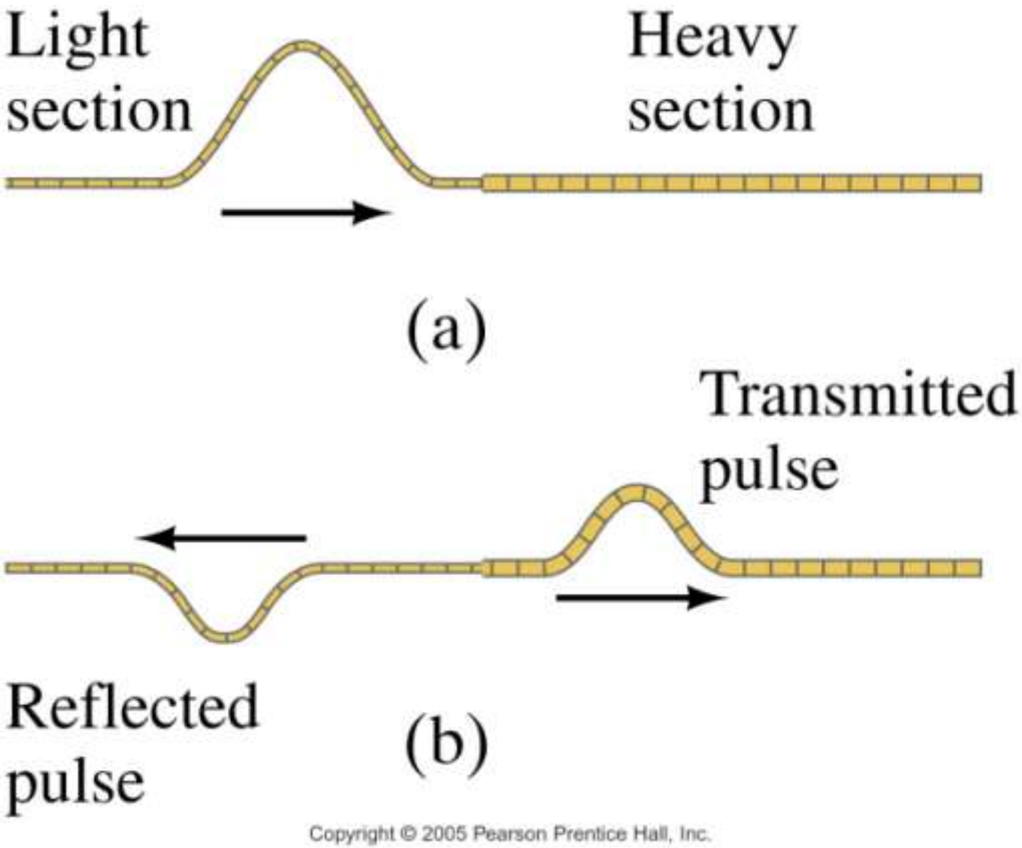
A wave reaching the end of its medium, but where the medium is still free to move, will be reflected (b), and its reflection will be upright.

A wave hitting an obstacle will be reflected (a), and its reflection will be inverted.

11-11 Reflection and Transmission of Waves

Time for a Gizmo!

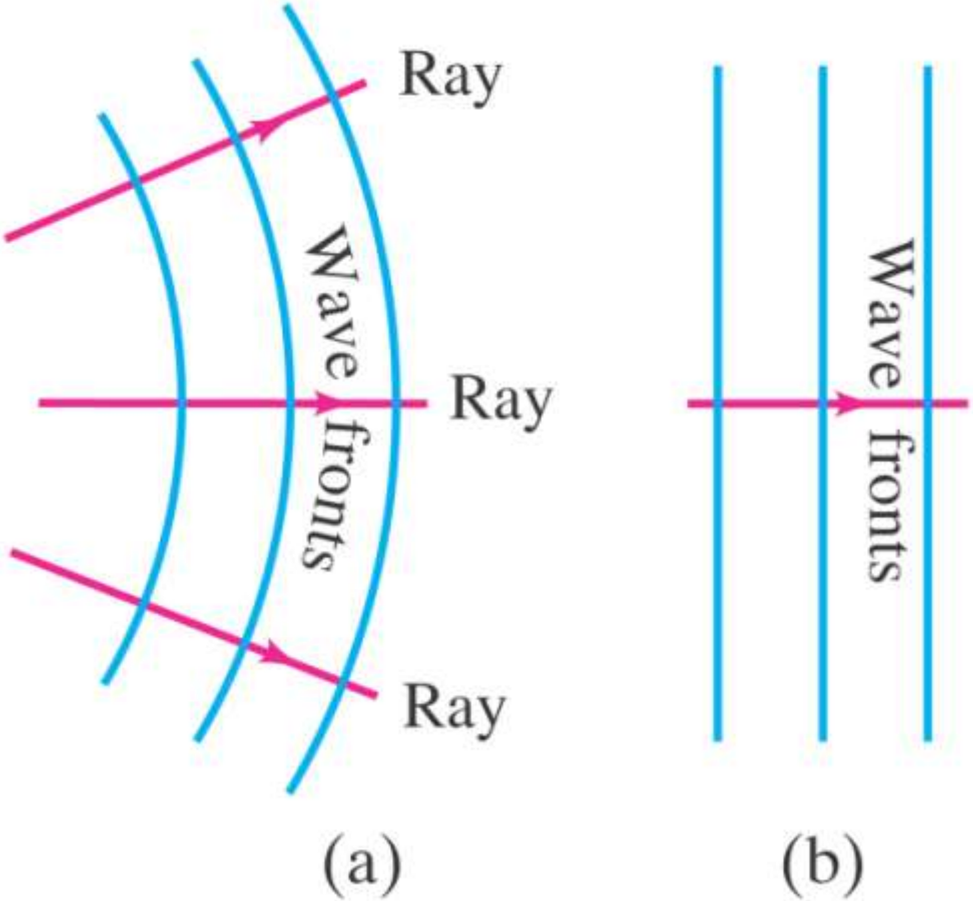
11-11 Reflection and Transmission of Waves



A wave encountering a denser medium will be partly reflected and partly transmitted; if the wave speed is less in the denser medium, the wavelength will be shorter.

11-11 Reflection and Transmission of Waves

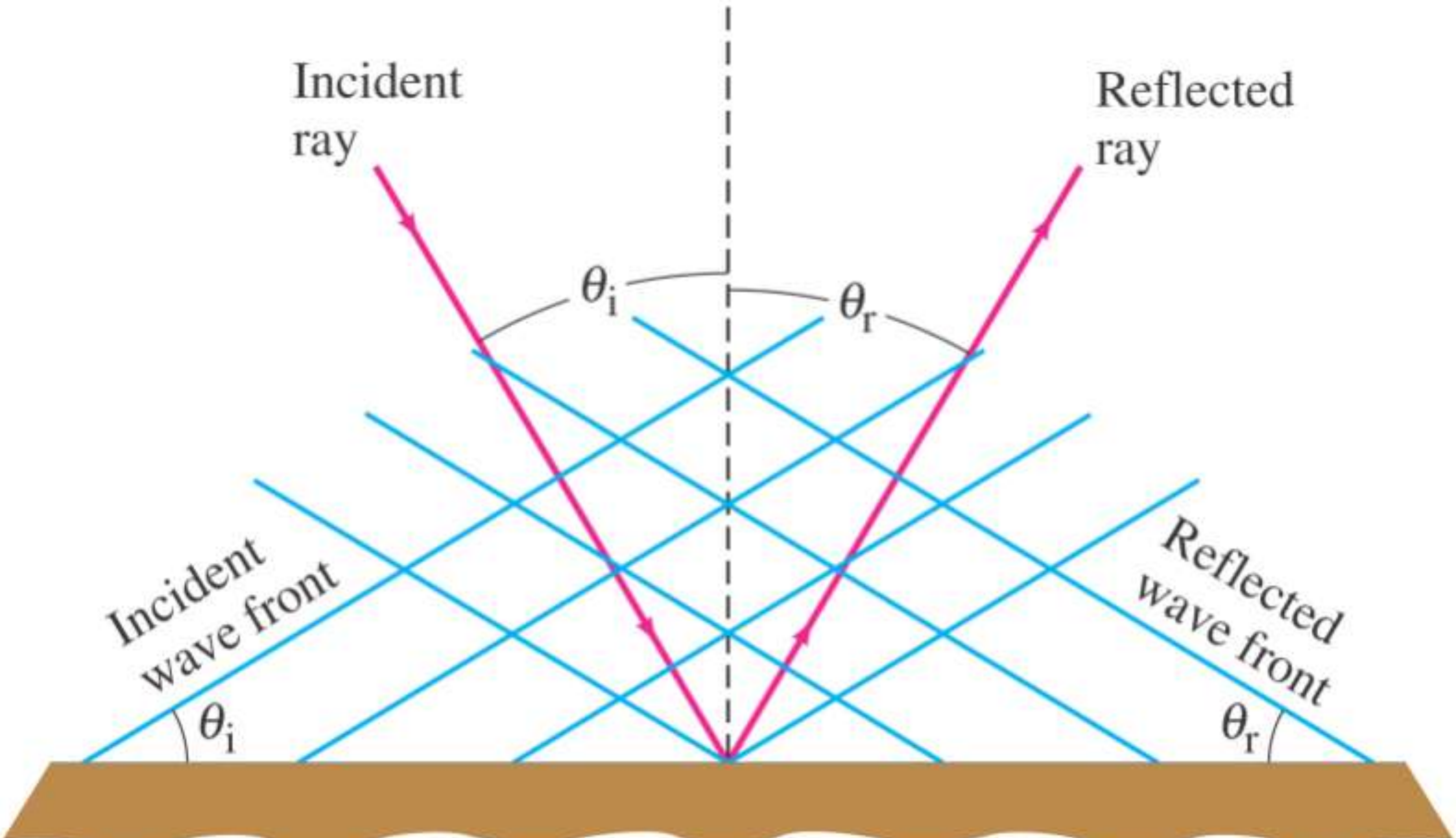
Two- or three-dimensional waves can be represented by wave fronts, which are curves of surfaces where all the waves have the same phase.



Lines perpendicular to the wave fronts are called rays; they point in the direction of propagation of the wave.

11-11 Reflection and Transmission of Waves

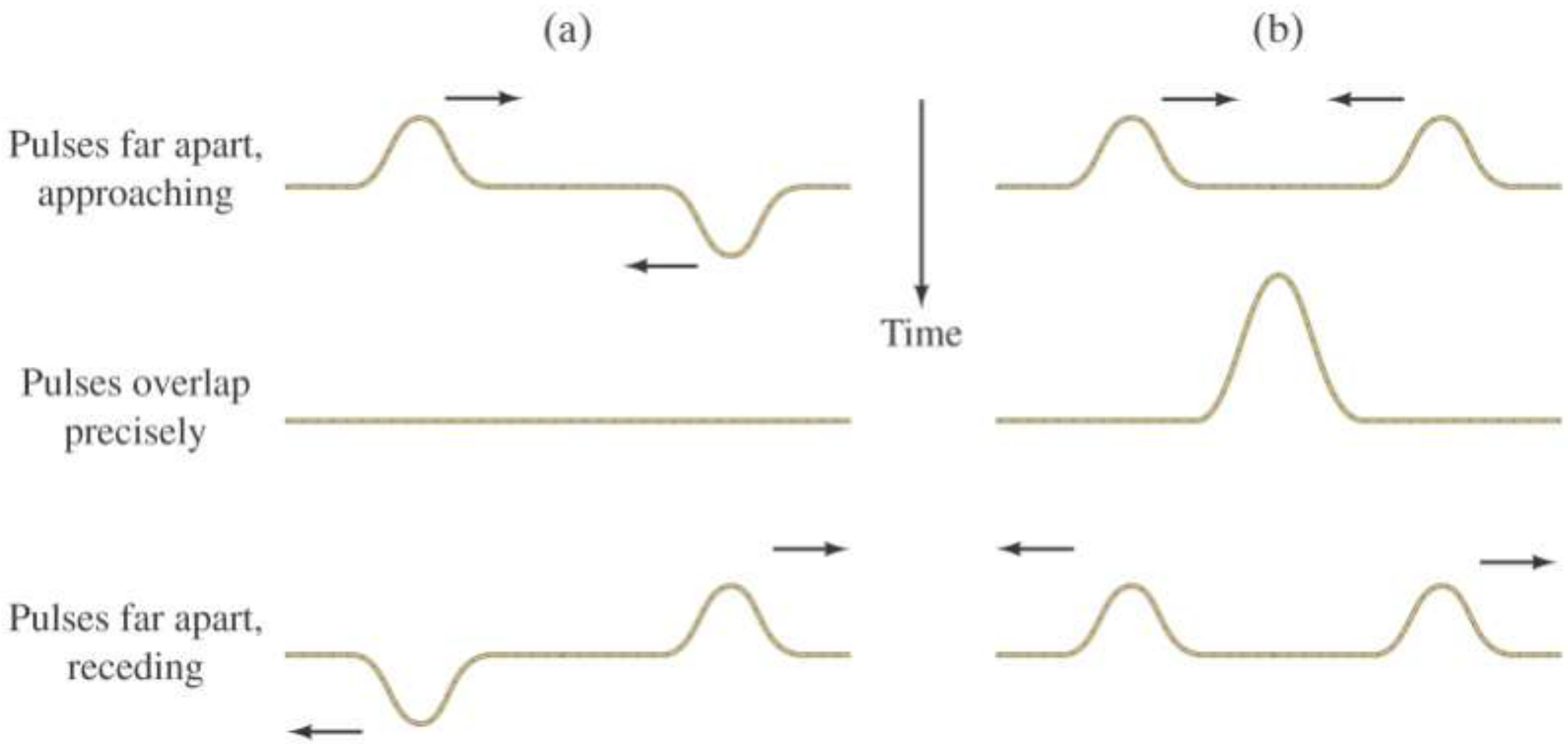
The **law of reflection**: the angle of **incidence** equals the angle of **reflection**.



11-12 Interference; Principle of Superposition

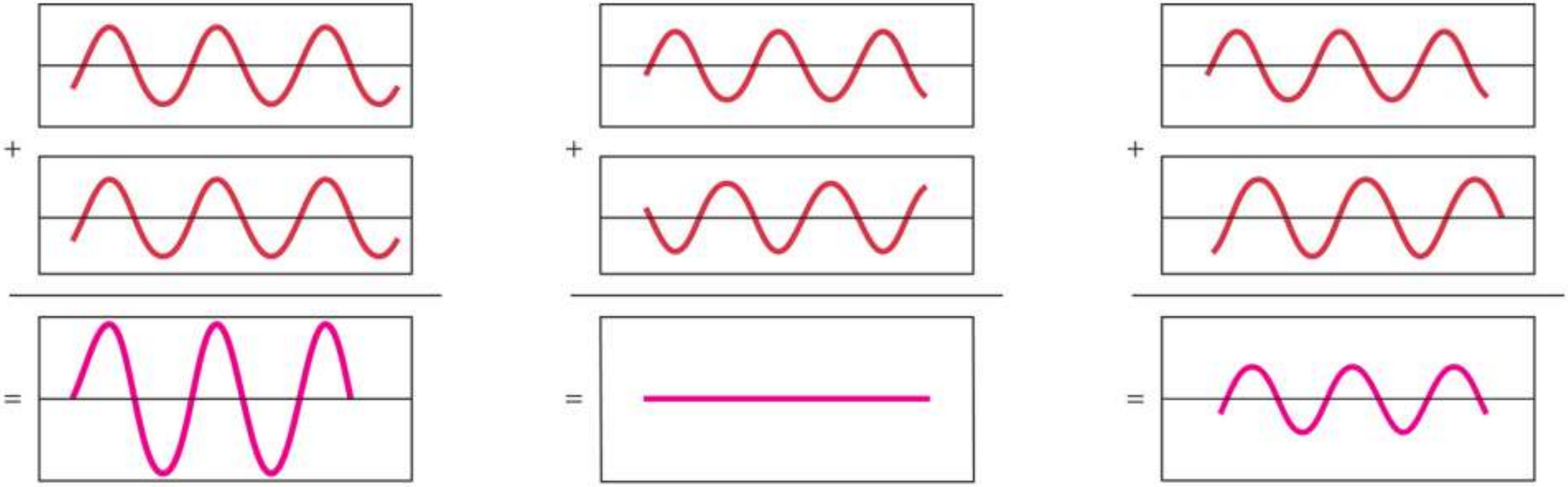
The **superposition** principle says that when two waves pass through the same point, the **displacement** is the arithmetic **sum** of the individual displacements.

In the figure below, (a) exhibits **destructive** interference and (b) exhibits **constructive** interference.



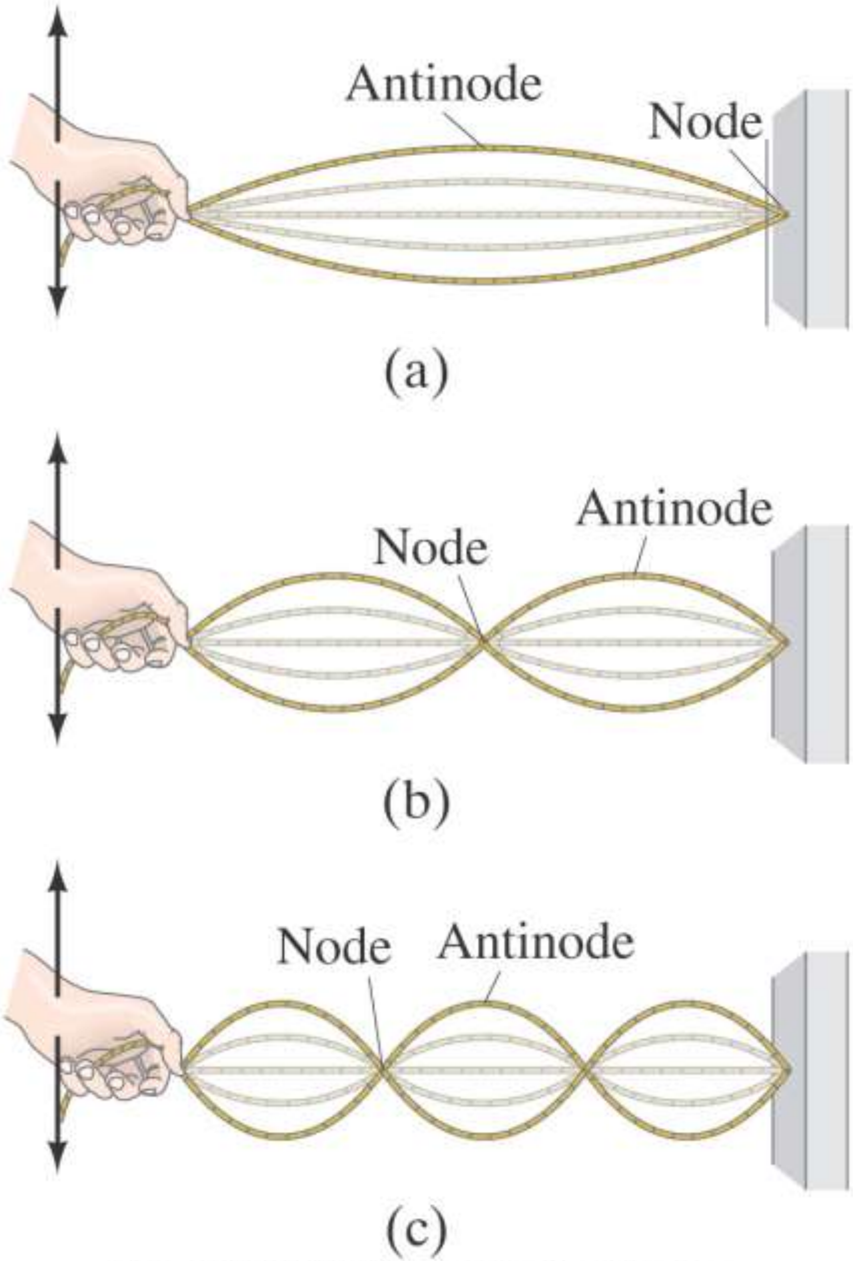
11-12 Interference; Principle of Superposition

These figures show the sum of two waves. In (a) they add **constructively**; in (b) they add **destructively**; and in (c) they add **partially destructively**.



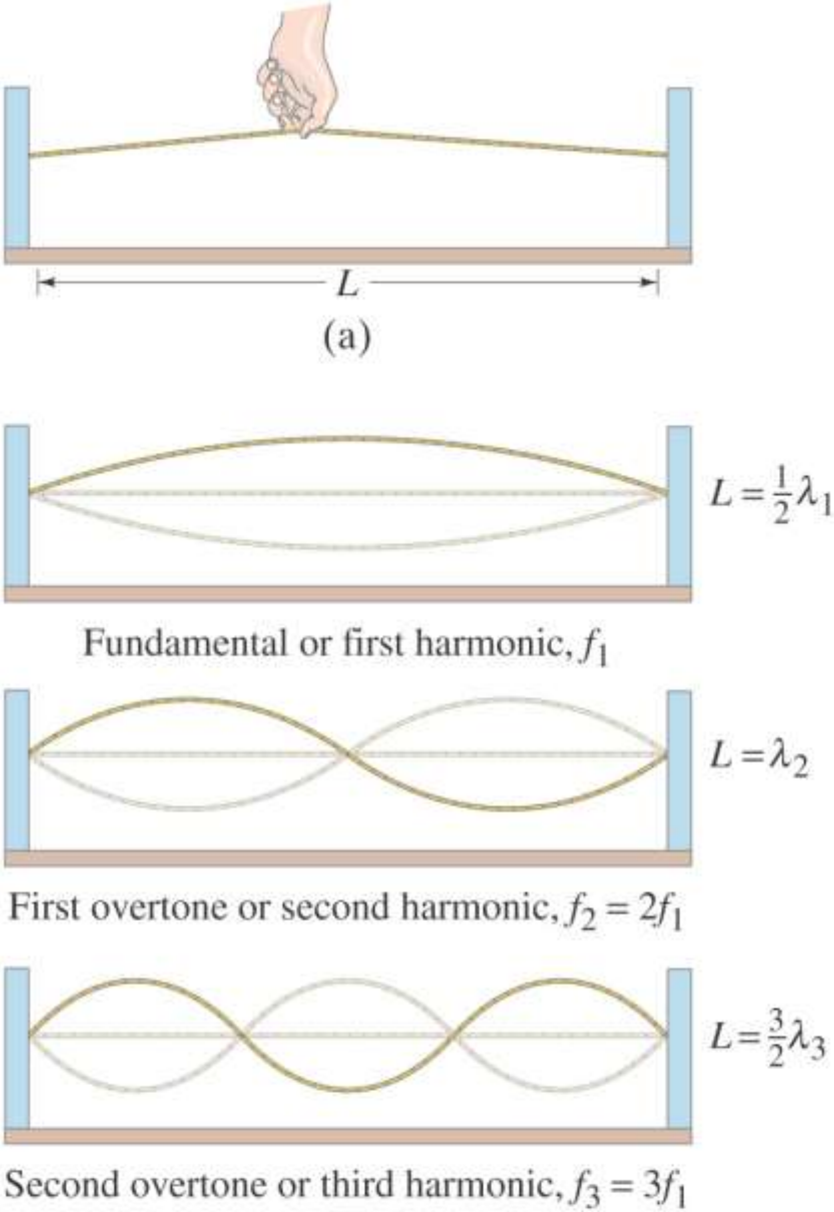
(a) (b) (c)

11-13 Standing Waves; Resonance



Standing waves occur when both ends of a string are **fixed**. In that case, only waves which are **motionless** at the ends of the string can persist. There are **nodes**, where the amplitude is always zero, and **antinodes**, where the amplitude varies from zero to the maximum value.

11-13 Standing Waves; Resonance



The **frequencies** of the standing waves on a particular string are called **resonant** frequencies.

They are also referred to as the **fundamental** and **harmonics**.

11-13 Standing Waves; Resonance

The **wavelengths** and **frequencies** of standing waves are:

$$\lambda_n = \frac{2L}{n}, \qquad n = 1, 2, 3, \cdots \tag{11-19a}$$

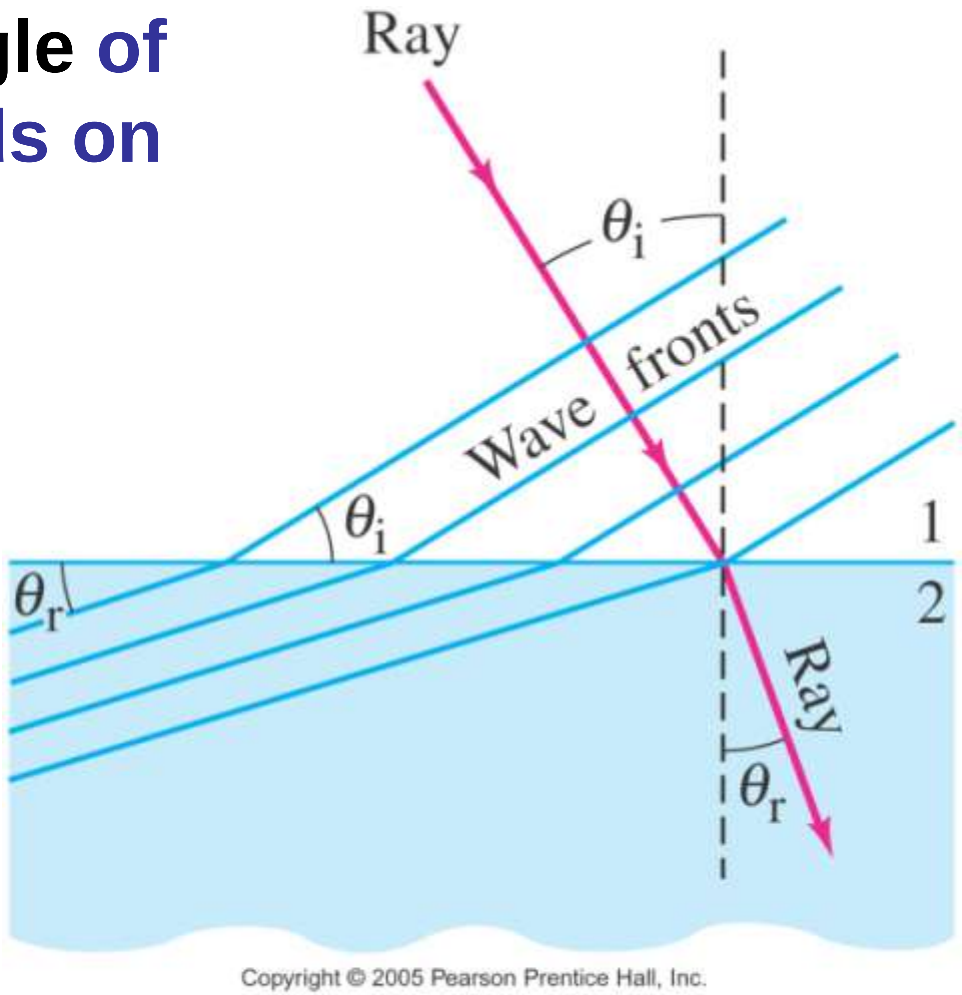
$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} = nf_1, \qquad n = 1, 2, 3, \cdots \tag{11-19b}$$

11-14 Refraction

If the wave enters a medium where the wave **speed** is different, it will be **refracted** – its wave fronts and rays will change direction.

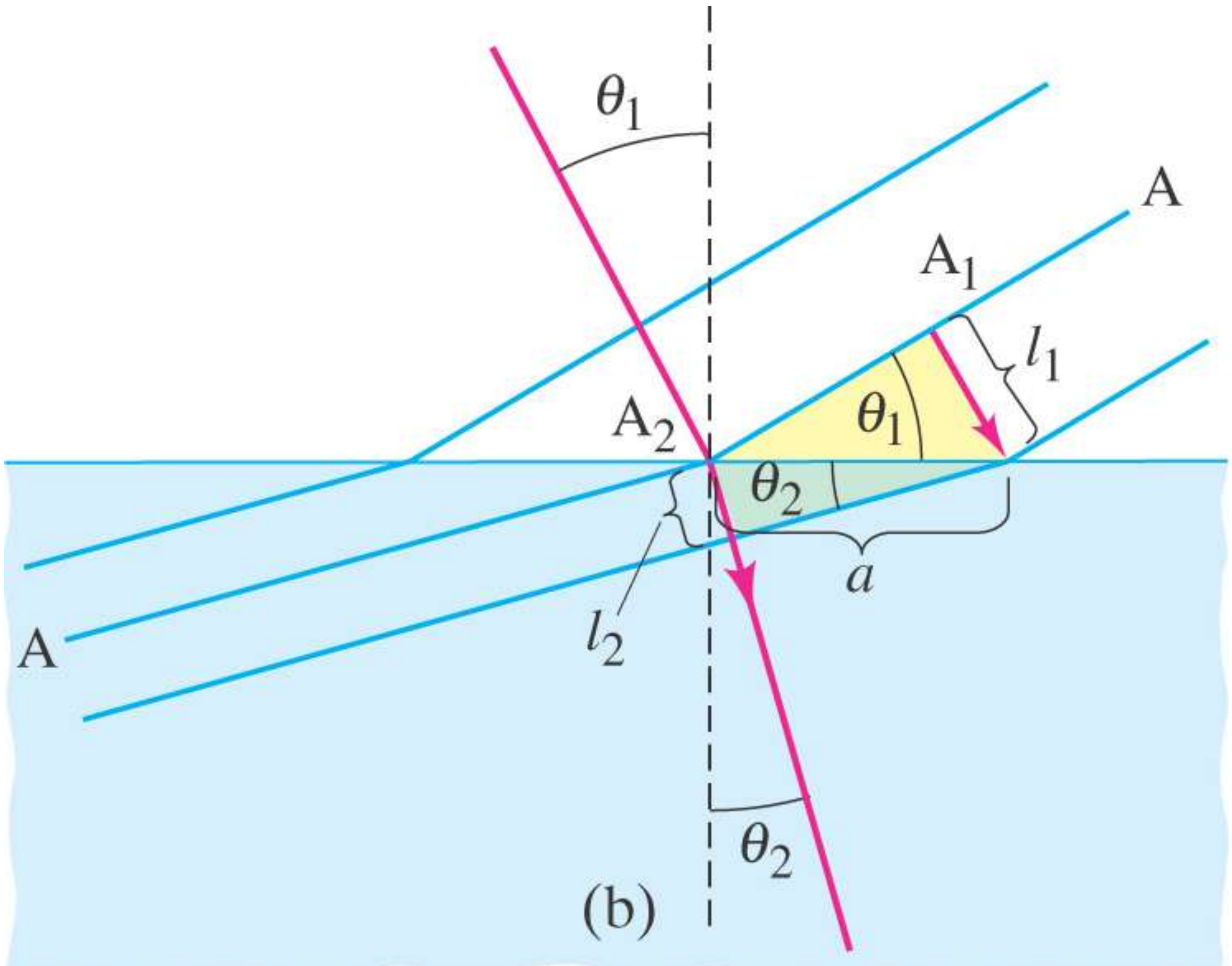
We can calculate the **angle** of refraction, which depends on both **wave speeds**:

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} \qquad (11\text{-}20)$$

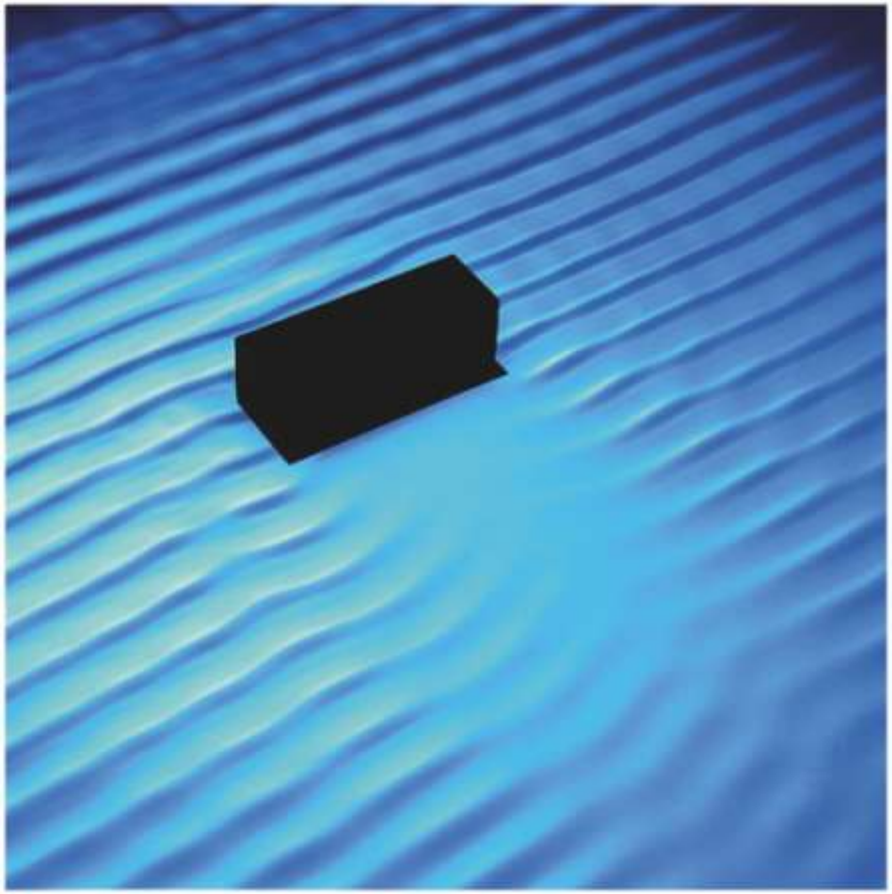


11-14 Refraction

The **law of refraction** works both ways – a wave going from a **slower** medium to a **faster** one would follow the red line in the other direction.



11-15 Diffraction



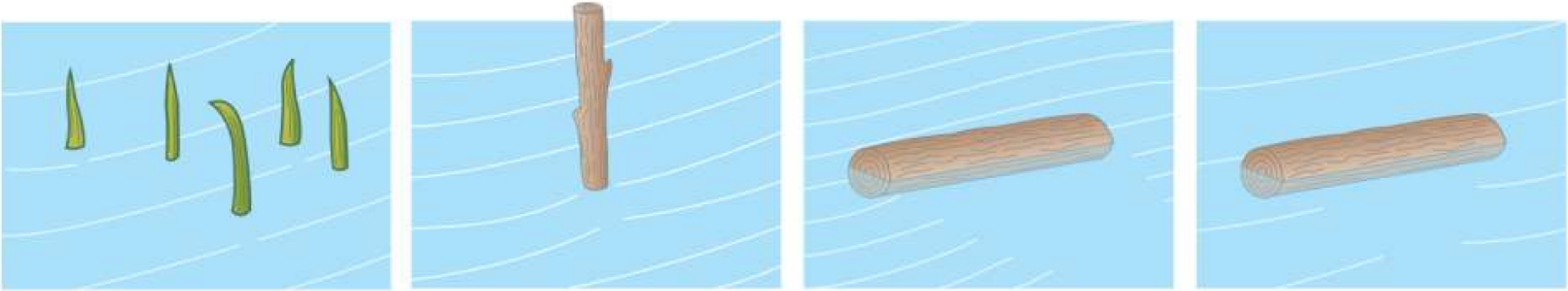
When waves encounter an **obstacle**, they bend around it, leaving a "shadow region." This is called **diffraction**.

11-15 Diffraction

Time for a Gizmo!

11-15 Diffraction

The amount of diffraction depends on the size of the obstacle compared to the wavelength. If the obstacle is much smaller than the wavelength, the wave is barely affected (a). If the object is comparable to, or larger than, the wavelength, diffraction is much more significant (b, c, d).



(a) Water waves passing blades of grass

(b) Stick in water

(c) Short-wavelength waves passing log

(d) Long-wavelength waves passing log