CHAPTER 13

Temperature and Kinetic Theory

Atomic Theory of Matter

The **atomic theory of matter** states that matter has a limited number of subdivisions in which the smallest indivisible piece is called an **atom.** Modern evidence for atomic theory comes from the law of definite proportions and Brownian movement.

- The **law of definite proportions** describes the atomic make-up of compounds, in which any collection of a given compound has the same proportion of constituent elements by weight as any other collection of the same compound.
- **Brownian movement** describes the continuous random motion of the atoms in all matter.
- Electrical attractive and repulsive forces keep molecules within a discrete range of distances.
- These forces affect the molecules' state of matter. For solids, attractive forces keep molecules in relatively fixed positions while vibrating in place. For liquids, molecules have more rapid movement and roll over one another. For gases, molecules have high speeds that move in every direction, filling their containers and occasionally colliding.

Temperature and Thermometers

Temperature relates to how relatively hot or cold objects are.

- Changes in temperature can change the properties of matter. **Thermometers** use these latter changes to quantify temperature changes.
- The adjustable **constant-volume gas thermometer** compensates for pressure differences and is used to provide the standard temperature scale.
- Scales for measuring temperature include Celsius (centigrade), Fahrenheit, and Kelvin. The standard conversion from Fahrenheit to Celsius is $°C = \frac{5}{9}(°F - 32)$; this can be rearranged to convert from Celsius to Fahrenheit as $°F = \frac{9}{5}°C + 32$.

Thermal Equilibrium and the Zeroth Law of Thermodynamics

Two objects initially at different temperatures that converge on a single intermediate temperature are in **thermal equilibrium.** Under this condition, no heat energy flows between the objects. In practice, the **zeroth law of**

thermodynamics implies that if two objects independently have the same temperature as a third object, then they themselves have the same temperature.

Thermal Expansion

Temperature changes can yield changes in the length and volume of an object.

- The change in length is a function of the change in temperature; $\Delta L = \alpha L_0 \Delta T$. α represents the coefficient of linear expansion for a material.
- The change in volume is a function of the change in temperature; $\Delta V = \beta V_0 \Delta T$. β represents the coefficient of volume expansion for a material.
- Barring a change of phase, the linear and volumetric expansion of nearly all materials follows these equations. An exception is water, which decreases in volume if heated from a lower temperature up to 4°C.

The Gas Laws and Absolute Temperature

Pressure, volume, and temperature are interrelated for gases.

- **Boyle's law:** Volume is inversely proportional to pressure at a constant temperature.
- **Charles's law:** Volume is directly proportional to absolute temperature at a constant pressure.
- **Gay-Lussac's law:** Pressure is directly proportional to absolute temperature at a constant volume.

The Ideal Gas Law

The three gas laws are combined in the **ideal gas law,** $PV = nRT$, where n is the number of moles of a substance and R is the universal gas constant $R = 8.315$ J/mol·K.

- The number of **moles** of a substance is equal to the ratio of a sample's mass in grams to its molecular mass in grams, where molecular mass is the sum of atomic masses for each component of a molecule.
- For the ideal gas law, the temperature scale is **Kelvin,** where K = 273.15 + °C.
- In the condition known as **standard temperature and pressure (STP),** temperature is 273 K and pressure is 1.013×10^5 N/m^2.

Ideal Gas Law in Terms of Molecules: Avogadro's Number

- The proportionality constant of the ideal gas law depends on **Avogadro's hypothesis,** which states that equal volumes of different gases whose temperatures and pressures are equal each contain an equivalent number of molecules. **Avogadro's number,** 6.02×10^{23}, represents the number of molecules in a mole.
- This allows for a restatement of the ideal gas law with Avogadro's number as $PV = NkT$, where the proportionality constant k is referred to as **Boltzmann's constant,** which equals 1.38×10^{-23} J/K, and N equals the number of molecules (not moles) in the sample.

Kinetic Theory and the Molecular Interpretation of Temperature

Kinetic theory posits that all matter is composed of atoms in random motion. Pressure is defined as a measure of the collisions of molecules against the walls of their container.

- Calculating average force using the average momentum of molecules in a gas, we can apply the ideal gas law to determine the average kinetic energy. The **average translational kinetic energy** of all molecules in a gas is equal to KE = $1/2(mv^2)$, where v is the average velocity of all of the molecules in a gas. Using a restatement of the ideal gas law, KE = $3/2kT$, where k is **Boltzmann's constant** and T is expressed in kelvins.
- These relations can be rearranged to find the **average velocity (root-mean-square velocity)** in terms of temperature and mass, $v_{rms} = \sqrt{3kT/m}$.

For Additional Review

Consider how the postulates of kinetic theory justify the component laws of the ideal gas law.

Multiple-Choice Questions

1. In the Kelvin scale, 50 degrees Fahrenheit is equal to
   (A) 10 K
   (B) 122 K
   (C) 283 K
   (D) 300 K
   (E) 395 K

2. How many moles of an ideal monatomic gas are in a fixed container of size 0.50 m^3 at 2.0×10^5 N/m^2 and 10°C?
   (A) 1.7×10^1 mol
   (B) 4.2×10^1 mol
   (C) 1.8×10^2 mol
   (D) 8.8×10^2 mol
   (E) 4.4×10^3 mol

3. An aluminum block is 51.250 m long at 20.0°C. If the coefficient for linear expansion of aluminum is 25.0×10^{-6}/C°, what is its length at 0°C?
   (A) 51.181 m
   (B) 51.224 m
   (C) 51.250 m
   (D) 51.276 m
   (E) 51.323 m

4. An empty glass jug has a capacity of 2.00 L at 15°C. If the coefficient for volume expansion for Pyrex is $\beta = 27 \times 10^{-6}$/C°, what is its change in capacity at 50°C?
   (A) 1.89×10^{-3} L
   (B) 5.33×10^{-2} L
   (C) 4.20×10^{-1} L
   (D) 6.32×10^{-4} L
   (E) 6.21×10^{-5} L

5. An ideal-gas-filled sphere of volume 0.05 m^3 inflates to 0.7 m^3 when heated from 20°C to 30°C. If the final pressure is 2.05×10^5 N/m^2, what was the initial pressure?
   (A) 1.51×10^4 N/m^2
   (B) 9.05×10^4 N/m^2
   (C) 1.32×10^5 N/m^2
   (D) 2.78×10^6 N/m^2
   (E) 3.81×10^6 N/m^2

6. The product of Boltzmann's constant and temperature will be expressed in what units?
   (A) kilograms
   (B) joules
   (C) newtons
   (D) kelvins
   (E) moles

7. If the pressure of an ideal gas held in a vessel is halved while its volume is quadrupled, the temperature will be
   (A) half of its original value
   (B) the same as its original value
   (C) twice its original value
   (D) one quarter of its original value
   (E) four times its original value

8. How many kilograms of carbon dioxide (CO_2) are in a fixed container of size 0.25 m^3 at STP?
   (A) 0.050 kg
   (B) 0.153 kg
   (C) 0.221 kg
   (D) 0.304 kg
   (E) 0.491 kg

9. What will be the ratio of linear expansion between two identical objects whose temperature increases by 50°C and decreases by 50°C?
   (A) 1
   (B) 0.5
   (C) 0
   (D) −0.5
   (E) −1

10. An ideal-gas-filled sphere of radius 10 cm expands to 20 cm when heated from 0°C to 10°C. If the final pressure is 1.5×10^5 N/m^2, what was the initial pressure?
    (A) 8.4×10^4 N/m^2
    (B) 3.4×10^5 N/m^2
    (C) 7.8×10^5 N/m^2
    (D) 1.2×10^6 N/m^2
    (E) 4.8×10^6 N/m^2

Free-Response Questions

1. (a) For an ideal gas, what is the atomic mass unit for the atom that makes up the diatomic molecule that has a root-mean-square velocity of 502 m/s at 10°C?
   (b) What is its average kinetic energy for each molecule?
   (c) How many moles of this molecule would fill 10 m^3 when held at STP?

2. A 5.0 L aluminum container holds 4.8 L of gasoline at 15°C. The coefficient for volume expansion for aluminum is $\beta = 75 \times 10^{-6}$/C° and for gasoline is $\beta = 950 \times 10^{-6}$/C°.
   (a) Will the container overflow if heated to 80°C?
   (b) Assume the same initial conditions given above, except that now the container is not aluminum. In terms of an unknown coefficient of volume expansion, what is the minimum initial size for the new container to hold the gasoline at 80°C?

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. (C) is correct. Fahrenheit must be converted to Celsius first before the conversion to kelvins. °C = 5/9(°F − 32), °C = 5/9(50° − 32) = 5/9(18) = 10°C, K = 273 + °C = 283 K.

2. (B) is correct. Using the ideal gas law, $n = PV/RT$ $= (0.50\text{ m}^3)(2.0 \times 10^5\text{ N/m}^2)/(8.315\text{ J/mol·K})(283\text{ K}) = 4.2 \times 10^1$ moles.

3. (B) is correct. The relationship between temperature and change in length is given by $\Delta L = \alpha L_o \Delta T$.

$\Delta L = (25.0 \times 10^{-6}/\text{C}°)(51.250 \text{ m})(-20 \text{ C}°) = -0.026 \text{ m}$.
Therefore, $L = L_o + \Delta L = 51.250 \text{ m} + -0.026 \text{ m} = 51.224$.

4. (A) is correct. Change in volume is given by $\Delta V = \beta V_o \Delta T$ $= (27 \times 10^{-6}/\text{C}°)(2.00 \text{ L})(35°\text{C}) = 0.00189 \text{ L}$, which in scientific notation is 1.89×10^{-3} L.

5. (D) is correct. The ideal gas law applies to both states of this situation, which is $PV = nRT$. nR remains constant, so $P_1 V_1/T_1 = P_2 V_2/T_2$. Rearranged, this is

$$P_1 = \frac{P_2 V_2 T_1}{T_2 V_1}$$

$$P_1 = \frac{(2.05 \times 10^5 \text{ N/m}^2)(0.7 \text{ m}^3)(293 \text{ K})}{(303 \text{ K})(0.05 \text{ m}^3)} = 2.78 \times 10^6 \text{ N/m}^2.$$

6. (B) is correct. Boltzmann's constant, k, has the units J/K, so the product kT, where T is in kelvins, will be in joules.

7. (C) is correct. This relies on the laws of proportion underlying the ideal gas law. Pressure and volume are inversely proportional to each other, and their product is directly proportional to temperature. For PV/T to remain constant, if pressure is halved and volume is quadrupled, temperature must double. To check, $(P/2)(4V)/2T = PV/T$.

8. (E) is correct. At STP,

$$n = \frac{PV}{RT} = \frac{(1.013 \times 10^5 \text{ N/m}^2)(0.25 \text{ m}^3)}{(8.315 \text{ J/mol·K})(273 \text{ K})} = 11.16 \text{ mol}.$$

The mass = (moles)(molecular mass) = (11.16 mol)(44.01 g/mol) = 491 g or 0.491 kg.

9. (E) is correct. The ratio of linear expansion will be $\Delta L_1/\Delta L_2$ $= \alpha L_o \Delta T_1/\alpha L_o \Delta T_2 = \Delta T_1/\Delta T_2$. Even without the precise starting and finishing temperatures, the change in Celsius will be the same as in kelvins, as such can be $\Delta L_1/\Delta L_2 = (50 \text{ K})/(-50 \text{ K}) = -1$.

10. (D) is correct. $P_1 V_1/T_1 = P_2 V_2/T_2$ can be applied here. First, the volumes must be found. With a given radius, $V_1 = 4/3\pi r^3$ $= 4/3\pi(0.10)^3 = 0.001 \text{ m}^3$ and $V_2 = 4/3\pi r^3 = 4/3\pi(0.20 \text{ m})^3 = 0.008 \text{ m}^3$. Therefore, $P_1(0.001 \text{ m}^3)/273 \text{ K} = (1.5 \times 10^5 \text{ N/m}^2)(0.008 \text{ m}^3)/283 \text{ K}$, so $P_1 = 1.2 \times 10^6 \text{ N/m}^2$.

Free-Response Questions

1. (a) The root-mean-square velocity is given by $v_{rms} = \sqrt{3\,kT/m}$. With the information given, $502 \text{ m/s} = \sqrt{3(1.38 \times 10^{-23} \text{ J/K})(283)/m}$, so $m = 4.65 \times 10^{-26}$ kg. The mass of the molecule is given by $4.65 \times 10^{-26} \text{ kg}/(1.66 \times 10^{-27} \text{ kg/u})$. The mass of the molecule is 28.01 u, so for each atom 14 amu, and so the element is Nitrogen.
   (b) The average KE is $3/2kT = 3/2(1.38 \times 10^{-23} \text{ J/K})(283 \text{ K})$ $= 5.86 \times 10^{-21}$ J.
   (c) Using the ideal gas law, $n = PV/RT$ $= (1.013 \times 10^5 \text{ N/m}^2)(10 \text{ m}^3)/(8.315 \text{ J/mol·K})(273 \text{ K}) = 446$ moles.

The response to part a correctly applies the root-mean-square velocity equation, converting to the atomic mass units. The average kinetic energy as a function of temperature is determined in the response to part b. In both cases, the temperatures are expressed in kelvins. The response to part c applies the ideal gas law.

2. (a) The change in volume for the container is given by $\Delta V = \beta V_0 \Delta T = (75 \times 10^{-6}/\text{C}°)(5\text{ L})(65°\text{C}) = 2.4 \times 10^{-2}$ L. The change in volume for the gasoline is given by $\Delta V = \beta V_0 \Delta T = (950 \times 10^{-6}/\text{C}°)(4.8\text{ L})(65°\text{C}) = 3.0 \times 10^{-1}$ L meaning $V_f = V_0 + \Delta V = 5.1$ L. The container does not expand enough to match the expansion of the fluid, so it will overflow.
   (b) For the new container to hold the change in volume of gasoline, we use the results of part (a) $V = V_0(1 + \beta\Delta T)$, $V_0 = 5.1\text{ L}/(1 + \beta\ 65°\text{C})$.

This response uses the given coefficient of volumetric expansion in conjunction with the applicable equation to determine the change of volume in both the liquid and the container in the response to part a. The response to part b applies this same equation to solve for initial rather than for final volume.