16 Direct Current Circuits

16-1 Current and Resistance

Vocabulary **Current:** The amount of charge that passes through an area in a given amount of time.

$$\textbf{current} = \frac{\textbf{charge}}{\textbf{time}} \qquad \text{or} \qquad I = \frac{\Delta q}{\Delta t}$$

The SI unit for current is the **ampere (A),** which equals one **coulomb per second (C/s).**

In conductors, such as metal wires, electrons are relatively free to move, and can carry energy throughout a circuit. This energy comes from a source such as a **battery** that converts chemical energy into electrical energy for use in the circuit. As energy is transformed in a battery, a potential difference, V, develops across the battery's terminals. This potential difference is called an **electromotive force,** or **EMF**. In this book, voltage between the terminals of a battery is simply referred to as potential difference.

Vocabulary **Resistance:** An opposition to the flow of charge.

For a given source voltage, the resistance of a circuit determines how much charge will flow in the circuit. When charge passes through a resistance, some electrical energy is changed to other forms. This is produced by a potential difference across the resistance.

$$\textbf{potential difference} = \textbf{(current)(resistance)} \qquad \text{or} \qquad V = IR$$

The SI unit for resistance is the **ohm (Ω)**, which equals one **volt per amp (V/A).**

Sometimes it is not desirable to use wires that have a high resistance, because considerable energy losses occur when charge flows through a resistor. However, in any device that produces heat, such as a toaster, high resistance is needed or else the toaster would not get hot. Therefore, a heating element made with superconducting wires would be useless.

The resistance of a wire depends upon the type of material that the wire is made of, its length, and its cross-sectional area. The longer the wire, the more resistant it is to the flow of charge. The larger the cross-sectional area of the wire, the less resistant it is to charge flow. Temperature also affects the resistance of a wire. The hotter the wire, the more resistant it becomes to the flow of charge. This means that more current will flow through a toaster when it is first turned on than when the coils are glowing red hot.

Solved Examples

Example 1: Household current in a circuit cannot generally exceed 15 A for safety reasons. What is the maximum amount of charge that could flow through this circuit in a house during the course of a 24.0-h day?

Solution: Because the unit ampere means coulombs per second, 24.0 h must be converted in 86 400 s.

Given: $I = 15\text{ A}$, $\Delta t = 86\,400\text{ s}$

Unknown: $\Delta q = ?$

Original equation: $I = \frac{\Delta q}{\Delta t}$

Solve: $\Delta q = I\Delta t = (15\text{ A})(86\,400\text{ s}) = \mathbf{1.3 \times 10^6\ C}$

Example 2: What is the resistance of the heating element in a car lock de-icer that contains a 1.5-V battery supplying a current of 0.5 A to the circuit?

Given: $V = 1.5\text{ V}$, $I = 0.5\text{ A}$

Unknown: $R = ?$

Original equation: $V = IR$

Solve: $R = \frac{V}{I} = \frac{1.5\text{ V}}{0.5\text{ A}} = \mathbf{3\ \Omega}$

Practice Exercises

Exercise 1: Arthur is going trick-or-treating for Halloween so he puts new batteries in his flashlight before leaving the house. Until the batteries die, it draws 0.500 A of current, allowing a total of 5400. C of charge to flow through the circuit. How long will Arthur be able to use the flashlight before the batteries' energy is depleted?

Answer: ____________________

Exercise 2: Fabian's car radio will run from the 12-V car battery that produces a current of 0.20 A even when the car is turned off. The car battery will no longer operate when it has lost 1.2×10^6 J of energy. If Fabian gets out of the car and leaves the radio on by mistake, how long will it take for the car battery to go completely dead (that is, lose all energy)?

Answer: ____________________

Exercise 3: While cooking dinner, Dinah's oven uses a 220.-V line and draws 8.00 A of current when heated to its maximum temperature. What is the resistance of the oven when it is fully heated?

Answer: ____________________

Exercise 4: Justine's hair dryer has a resistance of 9.00 Ω when first turned on. a) How much current does the hair dryer draw from the 110.-V line in Justine's house? b) What happens to the resistance of the hair dryer as it runs for a long time?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 5: Camille takes her pocket calculator out of her bookbag as she gets ready to do her physics homework. In the calculator, a 0.160-C charge encounters 19.0 Ω of resistance every 2.00 seconds. What is the potential difference of the battery?

Answer: ____________________

16-2 Capacitance

Vocabulary **Capacitor:** A device that stores charge on conductors that are separated by an insulator.

Capacitance is a measure of the amount of charge stored on the conductors, for a given potential difference.

$$\textbf{capacitance} = \frac{\textbf{amount of charge}}{\textbf{potential difference}} \quad \text{or} \quad C = \frac{\Delta q}{V}$$

The SI unit for capacitance is the **farad (F)**, which equals one **coulomb per volt (C/V).**

A capacitor may be used in a circuit by storing charge on two parallel plates and then periodically releasing it into the circuit, creating an intermittent flow of charge.

Solved Examples

Example 3: The first capacitor was invented by Pieter van Musschenbroek in 1745 when he and his assistant stored charge in a device called a Leyden jar. If 5×10^{-4} C of charge were stored in the jar over a potential difference of 10 000 V, what was the capacitance of the Leyden jar? (When van Musschenbroek touched the jar, he received such a large jolt that he exclaimed he would not try the experiment again for all the kingdom of France!)

Given: $\Delta q = 5 \times 10^{-4}\ \text{C}$, $V = 10\,000\ \text{V}$

Unknown: $C = ?$

Original equation: $C = \frac{\Delta q}{V}$

Solve: $C = \frac{\Delta q}{V} = \frac{5 \times 10^{-4}\ \text{C}}{10\,000\ \text{V}} = \mathbf{5 \times 10^{-8}\ F}$

Example 4: Lydia pushes the shutter button of her camera and the flash unit releases the 4.5×10^{-3} C of charge that was stored in a 500.-μF capacitor. What is the potential difference across the plates of the capacitor inside the flash?

Solution: The term μ (micro) means 10^{-6}, so a μF means 10^{-6} farad.

Given: $\Delta q = 4.5 \times 10^{-3}\ \text{C}$, $C = 500. \times 10^{-6}\ \text{F}$

Unknown: $V = ?$

Original equation: $C = \frac{\Delta q}{V}$

Solve: $V = \frac{\Delta q}{C} = \frac{4.5 \times 10^{-3}\ \text{C}}{500. \times 10^{-6}\ \text{F}} = \mathbf{9.0\ V}$

Practice Exercises

Exercise 6: The nervous system of the human body contains axons whose membranes act as small capacitors. A membrane is capable of storing 1.2×10^{-9} C of charge across a potential difference of 0.070 V before discharging nerve impulses through the body. What is the capacitance of one of these axon membranes?

Answer: ____________________

Exercise 7: During a lightning storm, the separation between the clouds and the earth acts as a giant capacitor with a capacitance of 2500 μF. If the transmitting tower of radio station KBOZ is hit by a bolt of lightning carrying 50. C of charge, what is the potential difference between the cloud and the tower?

Answer: ____________________

Exercise 8: Dr. Frankenstein brings his monster to life with electroshock treatment by discharging a 50.-μF capacitor through the monster's neck across a potential difference of 24 V. How much charge flows into the monster to make him come alive?

Answer: ____________________

Exercise 9: On Saturday nights, Greg likes to go the Frisco Disco, where he can dance under the strobe light. The strobe contains a 200-μF capacitor that stores charge over a 1000-V potential difference. If the strobe flashes 4 times each second, what is the current flow created by the strobe's capacitor?

Answer: ____________________

16-3 Power

Vocabulary **Power:** The amount of work done in a given unit of time.

As seen in the previous chapter, electrical work is done when an amount of charge, Δq, is transferred across a potential difference, V, or $W = \Delta qV$. The faster this transfer of charge occurs, the more power is generated in the circuit.

$$\textbf{Power} = \frac{\textbf{work}}{\textbf{elapsed time}} \quad \text{or} \quad P = \frac{W}{\Delta t} = \frac{\Delta qV}{\Delta t} = IV$$

Therefore, as current is drawn in a circuit to power an appliance, a potential difference occurs across the appliance.

The SI unit for electrical power is the **watt (W),** which equals one **joule per second (J/s).**

Solved Examples

Example 5: The lighter in Bryce's car has a resistance of 4.0 Ω. a) How much current does the lighter draw when it is run off the car's 12-V battery? b) How much power does the lighter use?

a. *Given:* $R = 4.0\ \Omega$, $V = 12\ \text{V}$ — *Unknown:* $I = ?$ — *Original equation:* $V = IR$

Solve: $I = \frac{V}{R} = \frac{12\ V}{4.0\ \Omega} = \mathbf{3.0\ A}$

b. *Given:* $I = 3.0\ \text{A}$, $V = 12\ \text{V}$ — *Unknown:* $P = ?$ — *Original equation:* $P = IV$

Solve: $P = IV = (3.0\ \text{A})(12\ \text{V}) = \mathbf{36\ W}$

Example 6: A 120.-V outlet in Carol's college dorm room is wired with a circuit breaker on a 5-A line so that students cannot overload the circuit. a) If Carol tries to iron a blouse for class with her 700-W iron, will she trip the circuit breaker? b) What is the resistance of the iron?

Solution: A circuit breaker is a switch that automatically turns a circuit off if the current is too high.

a. *Given:* $P = 700.\ \text{W}$, $V = 120.\ \text{V}$ — *Unknown:* $I = ?$ — *Original equation:* $P = IV$

Solve: $I = \frac{P}{V} = \frac{700.\ \text{W}}{120.\ \text{V}} = \mathbf{5.83\ A}$ Yes, she will!

It may be difficult to see how a watt/volt equals an amp until you begin to break down the units.

$$\frac{\text{watt}}{\text{volt}} = \frac{\text{joule/second}}{\text{joule/coulomb}} = \frac{\text{coulomb}}{\text{second}} = \text{amp}$$

b. Now find the resistance using $V = IR$.

Given: $V = 120.\ \text{V}$
$I = 5.83\ \text{A}$

Unknown: $R = ?$
Original equation: $V = IR$

Solve: $R = \frac{V}{I} = \frac{120.\ \text{V}}{5.83\ \text{A}} = \mathbf{20.5\ \Omega}$

Example 7: The Garcias like to keep their 40.0-W front porch light on at night to welcome visitors. If the light is on from 6 p.m. until 7 a.m., and the Garcias pay 8.00¢ per kWh, how much does it cost to run the light for this amount of time each week?

Solution: First, convert the power units to kilowatts, kW, because the cost of household energy is measured in kWh. 40.0 W = 0.0400 kW

Next, determine how long the light is left on each week. From 6 P.M. until 7 A.M. is 13 h. Operating 7 days a week means that the light is on for a total of 91.0 hours.

Given: $P = 0.0400\ \text{kW}$
$\Delta t = 91.0\ \text{h}$

Unknown: $W = ?$ Cost = ?
Original equation: $P = \frac{W}{\Delta t}$

Solve: $W = P\Delta t = (0.0400\ \text{kW})(91.0\ \text{h}) = \mathbf{3.64\ kWh}$

$$\text{Cost} = \frac{8.00¢}{1.00\ \text{kWh}}(3.64\ \text{kWh}) = \mathbf{29.1¢}$$

Therefore, it costs the Garcias about 29¢ to run the light all night for an entire week, or a little over $15 per year.

Practice Exercises

Exercise 10: How much power is used by a contact lens heating unit that draws 0.070 A of current from a 120-V line?

Answer: ______________

Exercise 11: Celeste's air conditioner uses 2160 W of power as a current of 9.0 A passes through it. a) What is the voltage drop when the air conditioner is running? b) How does this compare to the usual household voltage? c) What would happen if Celeste tried connecting her air conditioner to a usual 120-V line?

Answer: **a.** ____________________

Answer: **b.** ____________________

Answer: **c.** ____________________

Exercise 12: Which has more resistance when plugged into a 120.-V line, a 1400.-W microwave oven or a 150.-W electric can opener?

Answer: ____________________

Exercise 13: Valerie's 180-W electric rollers are plugged into a 120-V line in her bedroom. a) What current do the electric rollers draw? b) What is the resistance of the rollers when they are heated? c) Combining the equations just used, derive an equation that relates power to voltage and resistance.

Answer: **a.** ____________________

Answer: **b.** ____________________

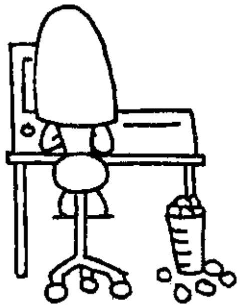
Answer: **c.** ____________________

Exercise 14: Mrs. Olsen leaves her 0.900-kW electric coffee maker on each day as she heads off to work at 6 A.M. because she likes to come home to a hot cup of coffee at 6 P.M. a) If the electric company charges Mrs. Olsen $0.100 per kWh, how much does running the coffee maker cost her each day? b) What is the yearly cost to run the coffee maker?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 15: While writing this book, the author spent about 1000 h working on her personal computer that has a power input of 60.0 W. Seventy additional hours were spent with the 60.0-W computer and the 240.-W printer running. How much did it cost for the energy use of these two devices, at a cost of $0.100 per kWh?



Answer: ____________________

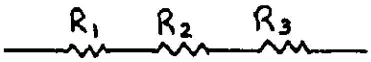
16.4 Series and Parallel Circuits

When multiple resistors are used in a circuit, the total resistance in the circuit must be found before finding the current. Resistors can be combined in a circuit in series or in parallel.

Resistors in Series

When connected in series, the total resistance, R_T, is equal to

$$R_T = R_1 + R_2 + R_3 + \ldots$$



In series, the total resistance is always *larger* than any individual resistance.

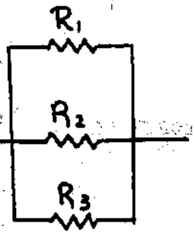
Current in series resistors: In series circuits, charge has only one path through which to flow. Therefore, the current passing through each resistor in series is the same.

Potential difference across series resistors: As charge passes through each of the resistors, it loses some energy. This means that there will be a potential difference across each resistor. The sum of all the potential differences equals the potential difference across the battery, assuming negligible resistance in the connecting wires.

Resistors in Parallel

When connected in parallel, the total resistance, R_T, is equal to

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$



Don't forget! After finding a common denominator and determining the sum of these fractions, flip over the answer to determine R_T.

In parallel circuits, the total resistance is always *smaller* than any individual resistance.

Current in parallel resistors: In parallel circuits, there is more than one possible path and current divides itself according to the resistance of each path. Since current will take the "path of least resistance," the smallest resistor will allow the most current through, while the largest resistor will allow the least current through. The sum of the currents in each parallel resistor equals the original current entering the branches.

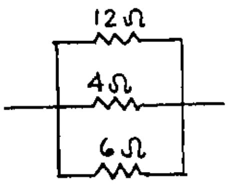
Potential difference in parallel resistors: The potential difference across each of the resistors in a parallel combination is the same. If there are no other resistors in the circuit, it is equal to the potential difference across the battery, assuming negligible resistance in the connecting wires.

Solved Examples

Example 8: Find the total resistance of the three resistors connected in series.

Solve: $R_T = R_1 + R_2 + R_3 = 12\ \Omega + 4\ \Omega + 6\ \Omega = \mathbf{22\ \Omega}$

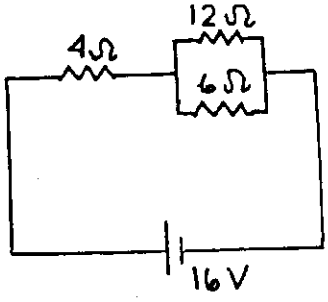
Example 9: Find the total resistance of the same three resistors now connected in parallel.



Solve: $\dfrac{1}{R_T} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} = \dfrac{1}{12\ \Omega} + \dfrac{1}{4\ \Omega} + \dfrac{1}{6\ \Omega}$

$$\frac{1}{R_T} = \frac{1}{12\ \Omega} + \frac{3}{12\ \Omega} + \frac{2}{12\ \Omega} = \frac{6}{12\ \Omega} = \frac{1}{2\ \Omega} \quad R_T = \mathbf{2\ \Omega}$$

Example 10: Find the total current passing through the circuit.



This circuit contains resistors in parallel that are then combined with a resistor in series. Always begin solving such a resistor combination by working from the inside out. In other words, first determine the equivalent resistance of the two resistors in parallel before combining this total resistance with the one in series.

Look first at the parallel combination.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{12\ \Omega} + \frac{1}{6\ \Omega} = \frac{1}{12\ \Omega} + \frac{2}{12\ \Omega} = \frac{3}{12\ \Omega} = \frac{1}{4\ \Omega}$$

$R_T = \mathbf{4\ \Omega}$

Now, combine this equivalent resistance with the resistor in series.

$$R_T = R_1 + R_2 = 4\ \Omega + 4\ \Omega = \mathbf{8\ \Omega}$$

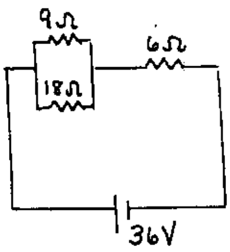
To find the current flowing through the circuit, use this total resistance in combination with the potential difference from the battery.

Given: $V = 16\ \text{V}$
$R = 8\ \Omega$

Unknown: $I = ?$
Original equation: $V = IR$

Solve: $I = \dfrac{V}{R} = \dfrac{16\ \text{V}}{8\ \Omega} = \mathbf{2\ A}$

Example 11: Find the current in the 9-Ω resistor.



For the parallel branch

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{9\ \Omega} + \frac{1}{18\ \Omega} = \frac{2}{18\ \Omega} + \frac{1}{18\ \Omega} = \frac{3}{18\ \Omega} = \frac{1}{6\ \Omega}$$

$R_T = \mathbf{6\ \Omega}$

Combining with the series resistor

$$R_T = R_1 + R_2 = 6\ \Omega + 6\ \Omega = \mathbf{12\ \Omega}$$

Given: $V = 36\text{ V}$
$R = 12\ \Omega$

Unknown: $I = ?$
Original equation: $V = IR$

Solve: $I = \dfrac{V}{R} = \dfrac{36\text{ V}}{12\ \Omega} = \mathbf{3\ A}$

This 3 A is the current through the entire circuit. Use this current to find the potential difference across the parallel combination. Remember, the potential difference across resistors wired in parallel is the same regardless of which path is taken. Because the resistors in parallel have a combined resistance of 6 Ω, you find the potential difference across the parallel branch as follows.

Given: $R = 6\ \Omega$
$I = 3\text{ A}$

Unknown: $V = ?$
Original equation: $V = IR$

Solve: $V = IR = (3\text{ A})(6\ \Omega) = \mathbf{18\ V}$

Therefore, the potential difference across both the top and the bottom branches is 18 V. Now use this 18-V drop to determine the current in the 9-Ω resistor.

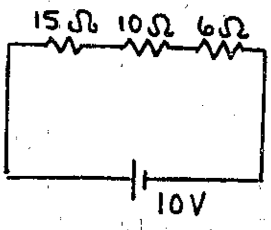
Given: $V = 18\text{ V}$
$R = 9\ \Omega$

Unknown: $I = ?$
Original equation: $V = IR$

Solve: $I = \dfrac{V}{R} = \dfrac{18\text{ V}}{9\ \Omega} = \mathbf{2\ A}$

Practice Exercises

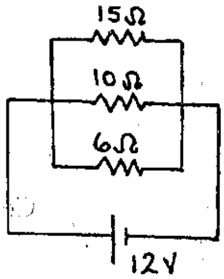
Exercise 16: Using the diagram, a) find the total resistance in the circuit. b) Find the total current through the circuit.



Answer: **a.** ____________________

Answer: **b.** ____________________

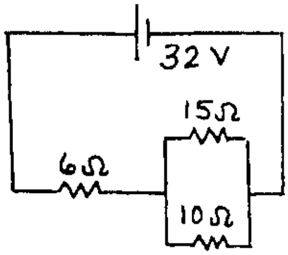
Exercise 17: Using the diagram, a) find the total resistance in the circuit. b) Find the total current through the circuit.



Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 18: Using the diagram, a) find the total resistance in the circuit. b) Find the total current through the circuit.



Answer: **a.** ____________________

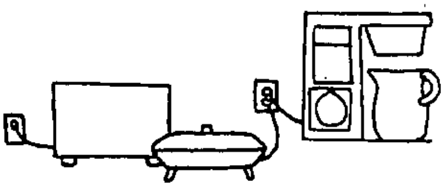
Answer: **b.** ____________________

Exercise 19: Old-fashioned holiday lights were connected in series across a 120-V household line. a) If a string of these lights consists of 12 bulbs, what is the potential difference across each bulb? b) If the bulbs were connected in parallel, what would be the potential difference across each bulb?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 20: Before going to work each morning, Gene runs his 18-Ω toaster, 11-Ω electric frying pan, and 14-Ω electric coffee maker, all at the same time. The three are connected in parallel across a 120-V line. a) What is the current through each appliance? b) If a household circuit could carry a maximum current of 15 A, would Gene be able to run all of these appliances at the same time?



Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 21: Timmy is playing with a new electronics kit he has received for his birthday. He takes out four resistors with resistances of 15 Ω, 20 Ω, 20 Ω, and 30 Ω. a) How would Timmy have to wire the resistors so that they would allow the maximum amount of current to be drawn? Calculate the total resistance in this circuit. b) How must he wire the resistors so that they draw a minimum amount of current? Calculate the total resistance in this circuit.

Answer: a. ____________________

Answer: b. ____________________

Exercise 22: Farmer Crockett is preparing tomato seedlings for his spring planting by growing the small plants over five 46-Ω strip heaters wired in parallel. a) How much current does each heater draw from a 120-V line? b) How much current do they draw all together?

Answer: a. ____________________

Answer: b. ____________________

Additional Exercises

A-1: Otto accidently leaves his automobile headlights on overnight and is unable to start his car in the morning. Each of the two headlights connected in parallel draws 2.00 A of current from the 12.0-V battery. If the battery stores 7.50×10^5 J of energy, how long will it take for the headlights to go off? b) Why are the headlights connected in parallel?

A-2: Officer Moynihan is patrolling his beat with a 4.5-V flashlight whose lightbulb has a resistance of 12 Ω. How much current does the flashlight draw?

A-3: Each night before falling asleep, Linus turns on his electric blanket that is plugged into the 120.-V electrical outlet. A current of 1.20 A flows through the blanket. a) What is the blanket's resistance? b) Does Linus want his electric blanket to have a high resistance or a low resistance? Why?

A-4: Herbert had just suffered a heart attack but he was revived in the hospital emergency room with a device called a defibrillator. (The paddles of a defibrillator supply a short pulse of high voltage to restart the heart.) The defibrillator contains a 20.-μF capacitor that releases 0.15 C of charge. a) What is the potential difference between the defribrillator paddles during the discharge? b) Why do you think doctors yell "Clear!" to the attendants before discharging the defibrillator?

A-5: Sherm is typing his term paper on a computer that contains a high-speed switch, controlled with a small 100×10^{-12} F speed-up capacitor. What is the current flow created by the capacitor if it discharges every 0.1 s across a potential difference of 5 V?

A-6: Every Sunday morning Stuart makes "breakfast in bed" for his wife. However, because the household wires can only carry a maximum current of 15 A from the 120.-V line, it is difficult to run all of the appliances simultaneously without blowing a fuse. What is the most power Stuart may use while cooking, before blowing a fuse?

A-7: In the previous exercise, a) how much current will Stuart draw if he tries to run the 700.-W toaster and 1000.-W coffee maker at the same time? b) Will this cause him to blow the fuse?

A-8: Xiaoyi's aquarium operates for 24.0 h a day and contains a 5.0-W heater, two 20.0-W lightbulbs, and a 35.0-W electric filter. If Xiaoyi pays $0.100 per kWh for her electricity bill, how much will it cost to maintain the aquarium for 30.0 days?

A-9: The average power plant, running at full capacity, puts out 500. MW of power. If the power company charges its customers $0.10 per kWh, what is the revenue brought in by the power plant each day?

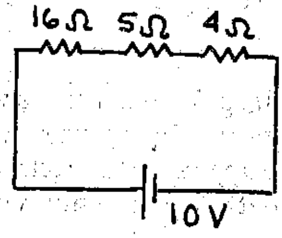
A-10: Horace has invented a unique pair of reading glasses that have two small light bulbs at the bottom wired in series, so that he can see the newspaper when he is reading at night. Each of the bulbs has a resistance of 2.00 Ω, and the system runs off a 3.20-V battery. How much current is drawn by Horace's reading glasses?

A-11: Jay has two 8-Ω stereo speakers wired in series in the front of his car connected to the 4.0-V output of the stereo. a) What is the current through each of the speakers? b) In his garage, Jay finds two more old speakers with resistances of 4 Ω and 16 Ω. He wires each in parallel with the 8-Ω combination. What is the new current through the 8-Ω speakers? c) If the loudness of each speaker is proportional to the amount of power used, how has the loudness of the two 8-Ω speakers changed?

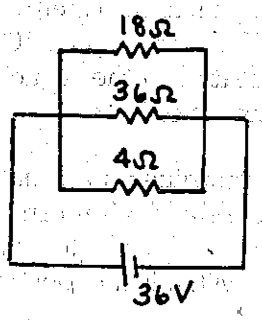
A-12: Find a) the total resistance in circuit A below. b) Find the total current through the circuit.

A-13: Find a) the total resistance in circuit B below. b) Find the total current through the circuit.

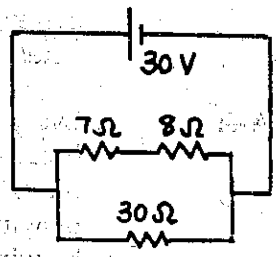
A-14: Find a) the total resistance in circuit C below. b) Find the total current through the circuit.



Circuit A



Circuit B

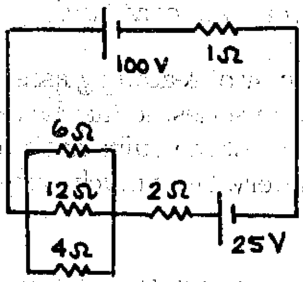


Circuit C

Challenge Exercises for Further Study

B-1: An 800.-W submersible electric heater is put into a 20.0 °C hottub until the 50.0-kg of tub water has warmed up to 70.0 °C. How long will it take for the heater to heat the tub water? (c_{water} = 4187 J/kg°C)

B-2: Find the total current in the circuit in the diagram.



B-3: In exercise A-10, the light bulbs are rated for 5 h of use before they burn out. If the battery can supply 5184 J to the circuit, which occurs first, energy depletion in the battery or failure of a bulb?