CHAPTER 16

Electric Charge and Electric Field

Static Electricity; Electric Charge and Its Conservation

Early experiments with the effects of **static electricity** on different substances placed in contact or proximity provided many of the conventions for electricity used today.

- The **electric charge** of an object is a discrete quantity that can be acquired or transferred. The charges of objects affect their interaction.
- There are two types of charge, **positive** and **negative,** such that two objects of like charge repel, whereas two objects of opposite charge attract. Objects with no charge are described as **neutral.**
- Charge can be transferred or induced onto objects, but the net quantity of charge always remains constant. This is called the **law of conservation of electric charge.**

Electric Charge in the Atom

Electric charges are responsible for most forces at the microscopic level.

- The atom itself is composed partly of individual negative charges, called **electrons,** orbiting around a much larger nucleus. Positively charged protons are part of the nucleus such that the net charge of the atom is zero. Atoms that gain or lose electrons during interactions with other atoms are called **ions** and have a net nonzero charge.
- Some molecules distribute their electrons such that there is a charge difference between parts of the molecule, even though the overall charge is zero. Examples include water molecules, which are described as **polar** because of the nonuniform distribution of positive and negative charges within each molecule.

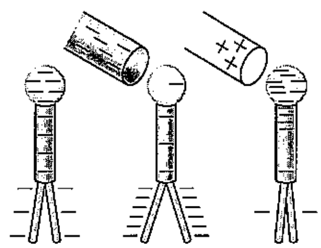
Insulators and Conductors

The ability of a material to be influenced by external charge depends on the mobility of its electrons. Materials that readily allow charge to cross them are called **conductors.** The electrons in conductors are relatively fluid in their movement within the materials. Materials whose electrons are less fluid and consequently do not allow charge to cross them are called **insulators.**

Induced Charge; The Electroscope

Charge can be imparted to neutral objects by contact or induction.

- When a charged object touches a neutral conductive object, electrons are transferred such that two objects have the same type of charge. The direction of electron transfer in such a case depends on the initial type of charge.
- When a charged object approaches a neutral conductive object without touching it, the electrons within the neutral object shift such that the near end has the opposite charge and the far end has the same charge.
- If this process were repeated, but with the far end attached to a **ground,** a charge reservoir such as the earth, the entire object would receive an overall charge opposite that of the nearby object.
- These properties are utilized by **electroscopes,** devices consisting of two metal "leaves" attached to a central conducting node for measuring charge. Charge can be imparted to the electroscope leaves, and the behavior of the leaves indicates the relative charge of other objects brought into its proximity. While the electroscope indicates whether charges are of the same or opposite type, it cannot independently identify them as positive or negative. However, the strength of charge is reflected in the angle of separation of the leaves.



(a) (b) (c)

Coulomb's Law and Its Applications

The force resulting from the interaction of charged objects is directly proportional to their charges, and it is inversely proportional to the square of their separation distance.

- Since Coulomb noted these relations, the equation (developed later) is known as Coulomb's law. $F = kQ_1Q_2/d^2$, where k represents the proportionality constant 9.0×10^9 N·m²/C², and the charges Q_1 and Q_2 are in coulombs, C. Functional units of charges are often expressed in microcoulombs, where 1 μC = 10^{-6} C. Coulomb's law can be expressed also as $F = Q_1Q_2/4\pi\varepsilon_0 d^2$, where $\varepsilon_0 = 1/4\pi k = 8.85 \times 10^{-12}$ C²/N·m², and this is called the **permittivity of free space.**
- The smallest unit of charge is called an **elementary charge,** and it has the value $e = 1.602 \times 10^{-19}$ C. A single electron has a charge of $-e$ and a single proton has a charge of $+e$. Because electrons and protons are functionally indivisible, all charges must be integer multiples of this elementary charge, and charge is thus described as **quantized.**

- While Coulomb's law gives the magnitude of force, its direction is along the line joining the charges—mutually repelling for the same charges and mutually attracting for different charges. The law simplifies charges as immobile, one-dimensional positions in space called **point charges.** It assumes that their size is small relative to their separation distance.
- Calculating the forces involving more than two particles at rest requires vectors. The force components are added in each of two perpendicular directions. If the point charges are not collinear, the magnitude of the resultant force can be determined by the Pythagorean formula. The direction of the resultant force can be found using trigonometry. Note that the signs on the components of this resultant force correspond to directions along each axis (not to the signs of the point charges).

The Electric Field

Electrical forces do not require contact for objects to influence each other's motion. As with gravitational force, this circumstance is described as "action at a distance."

- The force radiating from an electrical charge creates a **field** around the charge. The interaction between point charges is explained by the behavior of each charge in the field of the other charge.
- The strength of a field at a point in space can be measured by its force imposed on a **test charge,** which is a relatively insignificant positive charge placed at that point. The strength of the **electric field** at a point in space is the ratio of the force on a test charge at that point to the magnitude of the test charge, so $E = F/q$.
- Combining this with Coulomb's law, we find that the electric field at a distance r from a single point charge Q has a magnitude of $E = kQ/r^2$.
- For several point charges, the total strength of their electric fields at a point can be determined by summing their vector components, $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \cdots$, which is called the **superposition principle.**

Field Lines

Electric fields are represented graphically with arrows extending away from positive charges or toward negative charges.

- Electric field lines signify the direction of force caused by the electric field. The number of field lines in a given region is proportional to the magnitude of the force there.
- The field's direction at any point is tangent to the field line at that point.
- The strength of forces is indicated by the proximity of field lines to each other, such that they are closer together nearer the charge source.
- For oppositely charged parallel plates, the magnitude of the electric field is constant everywhere between the plates, so the field lines are evenly spaced.

Electric Fields and Conductors

- When a conductor is placed inside the electric field produced by stationary charges, the strength of the electric field inside the conductor is zero. The electrons within the conductor arrange themselves by induction so that $E = 0$ everywhere inside the conductor. However, the external field caused by the charge continues from the exterior surface of the conductor as though the conductor were not present. The direction of the field is perpendicular to the external surface of a conductor.

For Additional Review

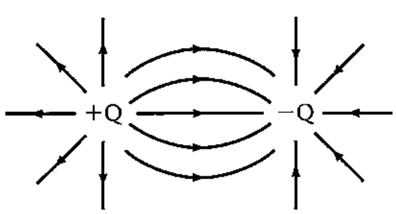
Calculate the field strength due to several charges where no more than two charges are collinear.

Multiple-Choice Questions

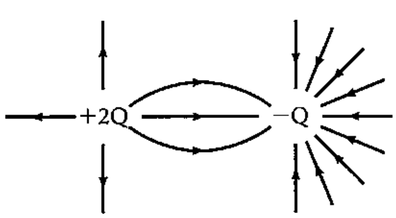
1. What is the magnitude of force between two negative point charges, one of -2×10^{-5} C, the other of -4×10^{-5} C, that are 4.3×10^{-4} m apart?
   (A) 4.8×10^{6} N
   (B) 7.2×10^{6} N
   (C) 3.9×10^{7} N
   (D) 8.9×10^{7} N
   (E) 4.3×10^{8} N

2. Which of the following is NOT a consistent representation of charges and their field lines?

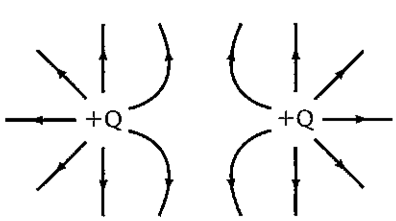
I.



II.



III.



   (A) I only
   (B) II only
   (C) III only
   (D) I and II only
   (E) II and III only

3. A negative charge of $-5.0\ \mu$C is equidistant from two positive charges as shown below. One positive charge is $+3.0\ \mu$C, the other is $+4.7\ \mu$C, and each is 0.01 m away from the negative charge. What is the net electrostatic force on the central charge if all three point charges are collinear?
   $+4.7\ \mu$C ——— $-5.0\ \mu$C ——— $+3.0\ \mu$C
   (A) 3500 N to the left
   (B) 700 N to the left
   (C) 700 N to the right
   (D) 3500 N to the right
   (E) 0 N

4. Find the magnitude of an electric field 1 mm from $+6.0 \times 10^{-3}$ C point charge.
   (A) 5.4×10^{2} N/C
   (B) 5.4×10^{4} N/C
   (C) 5.4×10^{7} N/C
   (D) 5.4×10^{11} N/C
   (E) 5.4×10^{13} N/C

5. What is the electric field halfway between two collinear negative charges separated by 6 cm. One charge is $-2.1\ \mu C$, whereas the other is $-1.3\ \mu C$.
   (A) 2.6×10^5 N/C
   (B) 6.3×10^5 N/C
   (C) 3.2×10^6 N/C
   (D) 8.0×10^6 N/C
   (E) 1.1×10^7 N/C

6. A metal rod of unknown charge charges an electroscope by induction. As a second metal rod, also of unknown charge, approaches the charged electroscope, the electroscope leaves separate more. This means that
   (A) the charge of both rods is negative
   (B) the charge of both rods is positive
   (C) the charge of the first rod is positive, and the charge of the second is negative
   (D) the charge of the first rod is negative, and the charge of the second is positive
   (E) Not enough information is given to determine the answer.

7. The electric field halfway between two positive point charges of 1.7×10^{-5} C is
   (A) 34 N/C
   (B) 17 N/C
   (C) 0 N/C
   (D) −17 N/C
   (E) −34 N/C

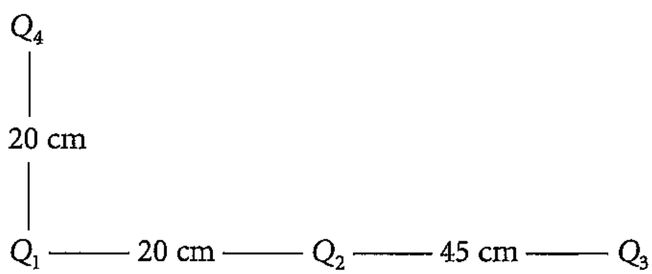
8. As the charge doubles for two oppositely charged particles, initially $+Q$ and $-Q$, the strength of the electric field halfway between them
   (A) is quartered
   (B) is halved
   (C) remains the same
   (D) is doubled
   (E) is quadrupled

9. As the distance between two identical charges is halved, the magnitude of the force between them
   (A) is quartered
   (B) is halved
   (C) remains the same
   (D) is doubled
   (E) is quadrupled

10. What is the electric force of two electrons 10^{-10} m apart?
   (A) 2.3×10^{-8} N
   (B) 4.6×10^{-8} N
   (C) 9.2×10^{-8} N
   (D) 2.3×10^{-9} N
   (E) 4.6×10^{-10} N

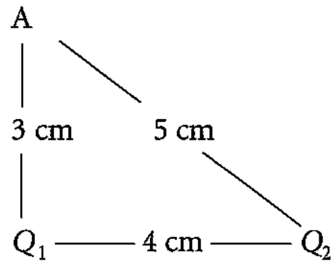
Free-Response Questions

1. Calculate the net forces on particle Q_2 as shown below.



$Q_1 = +3.2\ \mu C$
$Q_2 = -1.4\ \mu C$
$Q_3 = -2.2\ \mu C$
$Q_4 = +0.4\ \mu C$

2. Calculate the electric field at point A as shown below.



$Q_1 = +4.2\ \mu C$
$Q_2 = -8.7\ \mu C$

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. **(C) is correct.** Force between two point charges is given by $F = kQ_1Q_2/d^2$ $= (9.0 \times 10^9\ N{\cdot}m^2/C^2)(2 \times 10^{-5}\ C)(4 \times 10^{-5}\ C)/(4.3 \times 10^{-4}\ m)^2$ $= 3.9 \times 10^7\ N$.
2. **(B) is correct.** The number of field lines should reflect the relative strength of a charge. In Figure II, the field lines are reversed, with twice as many heading to the $-Q$ charge as the $+2Q$ charge. Figures I and III are accurate.
3. **(B) is correct.** Consider the force between $+4.7\ \mu C$ and $-5.0\ \mu C$ as F_{21} and the force between $+3.0\ \mu C$ and $-5.0\ \mu C$ as F_{23}.
   $F_{21} = (9.0 \times 10^9\ N{\cdot}m^2/C^2)(5.0 \times 10^{-6}\ C)(4.7 \times 10^{-6}\ C)/(10^{-2}\ m)^2$
   $= 2.1 \times 10^3\ N$,
   $F_{23} = (9.0 \times 10^9\ N{\cdot}m^2/C^2)(5.0 \times 10^{-6} C)(3.0 \times 10^{-6}\ C)/(10^{-2} m)^2$
   $= 1.4 \times 10^3\ N$.
   Force is additive, but the direction of the force must be accounted for. Each positive charge exerts an attractive force on the negative charge in opposite directions, with the stronger positive charge exerting a stronger force.
   Thus $F = F_{21} + F_{23} = 2.1 \times 10^3\ N - 1.4 \times 10^3\ N = 700\ N$ in the direction of the $+4.7\ \mu C$ charge.
4. **(E) is correct.** The electric field is given by $E = kQ/r^2$ $= (9.0 \times 10^9\ N{\cdot}m^2/C^2)(6.0 \times 10^{-3}\ C)/(1.0 \times 10^{-3}\ m)^2 = 5.4 \times 10^{13}\ N/C$.
5. **(D) is correct.** The field from each charge is pulling in its own direction. As such, $E = kQ_1/r_1^2 - kQ_2/r_2^2$ in the direction of the stronger charge.

$$E = \left(9.0 \times 10^9\ \frac{N{\cdot}m^2}{C^2}\right)\left[\frac{(2.1 \times 10^{-6}\ C)}{(3.0 \times 10^{-2}\ m)^2} - \frac{(1.3 \times 10^{-6} C)}{(3.0 \times 10^{-2}\ m)^2}\right]$$

$$E = \left(9.0 \times 10^9\ \frac{N{\cdot}m^2}{C^2}\right)[2.3 \times 10^{-3}\ C/m^2 - 1.4 \times 10^{-3} C/m^2]$$

$$E = 8.0 \times 10^6\ N/C.$$

6. (E) is correct. The behavior of this electroscope indicates that the two rods have opposite charges, but the specific charge on each rod cannot be determined.

7. (C) is correct. Without any calculation, the field strength is 0 N/C because the fields from the two charges exactly cancel there. Note that the fields do *not* cancel at other points that are equidistant from the two charges, because the vectors are not colinear.

8. (D) is the correct answer. As the superposition principle states, the electrical field is equal to the sum of electrical fields. Halfway between the charges, the electrical field is in the same direction for each. Initially, the electric field is $E = kQ/r^2 + kQ/r^2 = k2Q/r^2$. If the charge doubles, the strength of the electric field is $E = k2Q/r^2 + k2Q/r^2 = k4Q/r^2$. As such, the electric field has doubled.

9. (E) is correct. By Coulomb's law, $F = kQ_1Q_2/d^2$, as the distance between the charges is halved, $F = kQ_1Q_2/(d/2)^2 = 4kQ_1Q_2/d^2$. As such the force quadruples.

10. (A) is correct. Force between two point charges is given by $F = kQ_1Q_2/d^2$
$= (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\ \text{C})(1.6 \times 10^{-19}\ \text{C})/(10^{-10}\ \text{m})^2$
$= 2.3 \times 10^{-8}$ N.

Free-Response Questions

1. First, from trigonometry, the angle between Q_2 and Q_4 must be 45°. Then, reference axes must be established for each, here assuming up and right to be positive and down and left to be negative. Next, the forces for each must be calculated.
$F_{23} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.4 \times 10^{-6}\ \text{C})(2.2 \times 10^{-6}\ \text{C})/(0.45\ \text{m})^2$
$= 0.14$ N (repulsion to the left, so -0.14 N)
$F_{21} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.4 \times 10^{-6}\ \text{C})(3.2 \times 10^{-6}\ \text{C})/(0.2\ \text{m})^2$
$= 1.0$ N (attraction to the left, so -1.0 N)
$F_{24} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.4 \times 10^{-6}\ \text{C})(4.0 \times 10^{-7}\ \text{C})/(0.28\ \text{m})$
$= 0.064$ N (attraction to the left, so $-.064$ N).
Then combine these facts, breaking the forces into vectors.
$F_x = F_{23} + F_{21} + F_{24} \cos\theta = -0.14\ \text{N} + -1.0\ \text{N} + -0.045\ \text{N} = -1.2\ \text{N}$,
and $F_y = F_{24} \sin\theta = 0.045$ N.
The magnitude and direction can be determined using
$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(-1.2)^2 + (0.045\ \text{N})^2} = 1.2$ N
and $\tan\theta = F_y/F_x = 0.045\ \text{N}/-1.2\ \text{N} = 178°$.
This is understandable, since the preponderance of force comes from the $-x$ direction.

   This response would receive full credit because it correctly computes the forces between the charges, separates and sums each force into its correct vector components, and converts this into the correct magnitude and direction of force.

2. The field from each point charge must be computed before the vector components can be summed:
$E_{A1} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.2 \times 10^{-6}\ \text{C})/(3.0 \times 10^{-2}\ \text{m})^2 = 4.2 \times 10^7$ N/C
$E_{A2} = (9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(8.7 \times 10^{-6}\ \text{C})/(5.0 \times 10^{-2}\ \text{m})^2 = 3.1 \times 10^7$ N/C.

$E_{Ax} = E_{A2} \cos\theta$, where θ is the angle at Q_2. By trigonometric definitions, $\cos\theta$ = adjacent/hypotenuse = 4/5, even if θ is not explicitly presented (although it could be determined if necessary) and $E_{Ax} = (3.1 \times 10^7 \text{ N/C})(0.8) = 2.5 \times 10^7$ N/C. $E_{Ay} = E_{A1} + -E_{A2} \sin\theta$ where θ is the angle at Q_2. Note that E_{A1Y} is upward (positive) and E_{A2Y} is downward (negative). Again, by trigonometric definitions, $\sin\theta$ = opposite/hypotenuse = 3/5 = 0.6 and
$E_{Ay} = 4.2 \times 10^7 \text{ N/C} + -(3.1 \times 10^7 \text{ N/C})(0.6) = 2.3 \times 10^7$ N/C.
For magnitude and direction,
$E = \sqrt{E_{Ax}^2 + E_{Ay}^2} = \sqrt{(2.5 \times 10^7 \text{ N/C})^2 + (2.3 \times 10^7 \text{ N/C})^2} = 3.4 \times 10^7$ N/C
where $\tan\theta = E_{Ay}/E_{Ax} = 2.3 \times 10^7 \text{ N/C}/2.5 \times 10^7$ N/C. $\theta = 43°$.

This response would receive full credit because it correctly computes the field at the point due to each charge, separates and sums each field into its correct vector components, and converts this into the correct magnitude and direction for the electric field.