CHAPTER 17

Electric Potential and Electric Energy; Capacitance

Electric energy provides further quantitative descriptions of electrical phenomena beyond force and fields.

Electric Potential and Potential Difference

Since potential energy is measured as a difference of potential energies rather than as an absolute value, **electric potential energy** refers to the change in potential energy as a charge is moved between two points.

- Electric potential energy is the negative of the work done by the electric field to move the charge. Because only a difference in potential energy is measured, a zero potential energy can be assigned to either point.
- **Electric potential,** V, is defined as the quotient of potential energy to charge, such that $V_a = PE_a/q$. Further, $V_{ab} = V_a - V_b = -W_{ba}/q$. The units for electric potential difference are **volts,** equivalent to J/C, and the terms **potential difference** and **voltage** are interchangeable.
- The potential energy difference is defined as $\Delta PE = PE_b - PE_a = qV_{ba}$.

Relation Between Electric Potential and Electric Field

For a uniform electric field, the relation to electric potential is given $E = V_{ba}/d$ for a positive charge q moved from a point b to a point a separated by d meters.

- The **electric field** is a vector that can have the equivalent units V/m or N/C.
- **Electric potential** is a scalar with equivalent units of V or J/C.

Equipotential Lines

As field lines graphically represent electric fields, **equipotential lines** represent electric potential.

- The points that comprise an equipotential line have the same electric potential.
- Equipotential lines run perpendicular to field lines at any point.
- Equipotential lines are parallel to charged parallel plates, and they are in concentric circles around single charges.
- The electric potential of these lines depends on their distance from the sources of potential.

The Electron Volt, a Unit of Energy

On a microscopic scale, the **electron volt** is often used for energy. $1\ eV = 1.6 \times 10^{-19}$ J, representing the energy gained by an electron or other particle with a charge of 1.6×10^{-19} C traveling across a one-volt potential difference.

Electric Potential Due to Point Charges

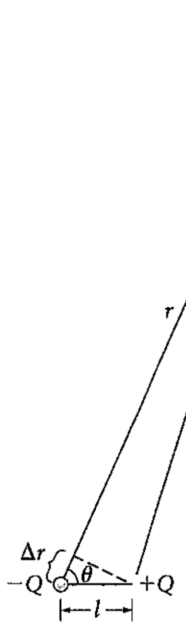
Assuming the electric potential is zero at a distance of infinity, the electric potential at a distance r from a point charge Q is given by the equation $V = kQ/r$.

- The combined electric potential of several point charges is given by the scalar sum of their individual potentials. Note that the sign for each potential must match the sign of the charge.

Electric Dipoles

An **electric dipole** consists of two equal but oppositely charged point charges in space.

- The electric potential at a point that is r meters away from the positive charge of an electric dipole, for which the charges are a distance apart, l, is given by $V = kQl \cos\theta/r^2$, where θ is the interior angle between a line from the negative charge to the positive charge and a second line from the negative charge to the point of potential. This is valid when the distance r is much greater than the distance of charge separation, l ($r >> l$).



- The product of charge and the distance of separation is called a **dipole moment,** p, so $V = kp \cos\theta/r^2$.
- This relation explains the behavior of electrically neutral polar molecules, which have a nonsymmetrical distribution of the molecular charge.

Capacitance

A **capacitor** is a charge-storing device often made from two conducting parallel plates separated by a small layer of air or a thin film.

- The plates become equally and oppositely charged when the capacitor is in a circuit pathway that has a potential difference.
- The charge quantity on each plate, Q, is proportional to the voltage applied, such that $Q = VC$. C is the **capacitance,** whose value of C depends on the capacitor's design.
- Capacitance is a function of the distance between plates, d, and the area of each plate, A, such that $C = \varepsilon_0 A/d$, where $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2/Nm^2$. ε_0 is called the permittivity of space.
- The unit of **capacitance** is C/V, also known as a **farad.**

For Additional Review

Consider why the electric dipole has 0 voltage at point P when $\theta = 90°$, seemingly contradicting the results of Example 17-4 (in Giancoli's *Physics,* page 510), in which point A is at a 90° angle and has a nonzero voltage.

Multiple-Choice Questions

1. What is the change in potential energy when a +2.5 C point charge is moved from a negative plate to a positive plate across a potential difference of 8 V?
   (A) 24 J gained
   (B) 20 J gained
   (C) 3 J gained
   (D) 0 J net change
   (E) 20 J lost

2. What is the speed of a −3.3 C charge of mass 5.4×10^{-10} kg when it is moved from a negative plate to a positive plate across a potential difference of 6.4 V?
   (A) 2.1×10^{1} m/s
   (B) 9.0×10^{2} m/s
   (C) 6.5×10^{3} m/s
   (D) 2.8×10^{5} m/s
   (E) 1.3×10^{10} m/s

3. What is the magnitude of the electric field between two parallel plates that have a potential difference of 10 V and are 25 cm apart?
   (A) 0.025 V/m
   (B) 0.25 V/m
   (C) 2.5 V/m
   (D) 4 V/m
   (E) 40 V/m

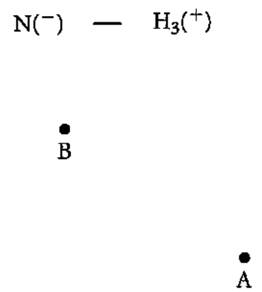
4. A −3.5 C charge that is moved from a positive plate to a negative plate gains 70 J of potential energy. What is the electric field for these parallel plates if they are 8 mm apart?
   (A) 2500 V/m
   (B) 3500 V/m
   (C) 4500 V/m
   (D) 5500 V/m
   (E) 8500 V/m

5. How much work is needed to decrease the distance between a +15 μC charge and a −20 μC charge from 1 m to 0.25 m by moving the positive charge?
   (A) −8.1 J
   (B) −2.7 J
   (C) 0 J
   (D) 2.7 J
   (E) 8.1 J

6. Determine the electric potential at a point A where $Q_1 = -20\ \mu C$ and $Q_2 = +20\ \mu C$
Q_1____10 cm______Q_2____5 cm ____A
(A) 8.0×10^1 V
(B) 1.9×10^2 V
(C) 2.4×10^6 V
(D) 5.5×10^7 V
(E) 6.7×10^{10} V

Questions 7 and 8 refer to the following diagram, which is not to scale.

This is an NH_3 molecule, which has a dipole moment of 5.0×10^{-30} C·m, arranged as shown below.



7. Find the potential at point A from where the distance between H_3 complex and A is 3.0×10^{-9} m and the interior angle at N is 60°.
(A) 9.1×10^{-4} V
(B) 2.5×10^{-3} V
(C) 8.2×10^{-3} V
(D) 7.5×10^{-2} V
(E) None of the above

8. Find the potential at point B where the distance between H_3 complex and A is 1.5×10^{-6} m and the interior angle to point N is 90°?
(A) 9.1×10^{-4} V
(B) 2.5×10^{-3} V
(C) 8.2×10^{-3} V
(D) 7.5×10^{-2} V
(E) None of the above

9. What is the charge on each plate of a capacitor whose plates are 3 cm × 3 cm and 2 mm apart when connected to a 9 V battery?
(A) 4.0×10^{-12} C
(B) 3.6×10^{-11} C
(C) 7.2×10^{-10} C
(D) 2.1×10^{-9} C
(E) 1.0×10^{-8} C

10. Which of the following are valid units for an electric field?
I. N/C
II. J/C·m
III. V/m
(A) I and II only
(B) II and III only
(C) I and III only
(D) I, II, and III
(E) None of the above

Free-Response Questions

1. Assume a zero potential at infinity for this question.
(a) Calculate the dipole moment for a polar molecule which has a potential of 0.15 V at a distance 1.24×10^{-10} m away at an interior angle of 41° from the negative charge.
(b) Assuming the charges are separated by a distance of 6.8×10^{-11} m, calculate each of the charges.
(c) Along what line would the potential be zero?

2. A $-35\ \mu C$ charge and a $+40\ \mu C$ charge are separated by 15 cm. The potential is tested at a point in space 75 cm from the negative charge. Is there a distance from the positive charge at which this point can be placed such that it has the potential is equal to 40,000 V?

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. **(B) is correct.** The change in potential energy is given by $\Delta PE = PE_b - PE_a = qV_{ba} = (2.5\ C)(8\ V)20\ J$, so potential energy will increase by 20 J.
2. **(D) is correct.** The change in potential energy is given by $\Delta PE = PE_b - PE_a = qV_{ba} = (-3.3\ C)(6.4\ V) = -21\ J$, so it will lose 21 J of electric potential energy. Because energy is conserved, the change in kinetic energy is equal to the change in potential energy. $1/2mv^2 = 21\ J$, so $v = \sqrt{2(21\ J)/(5.4 \times 10^{-10}\ kg)} = 2.8 \times 10^5\ m/s$.
3. **(E) is correct.** Electric field is given by $E = V_{ba}/d = 10\ V/0.25\ m = 40\ V/m$, a scalar quantity.
4. **(A) is correct.** First, the potential difference of the plates is given by $\Delta PE/q = V_{ba} = (70\ J)/(3.5\ C) = 20\ V$. Next, the quotient of voltage and distance gives the electric field, $E = V_{ba}/d = 20\ V/0.008\ m = 2500\ V/m$.
5. **(A) is correct.** The relation between work and potential difference is given by $W = qV_{ba} = q(kQ/r_b - kQ/r_a) = qk(Q/r_b - Q/r_a)$
$= (1.5 \times 10^{-5}\ C)(9.0 \times 10^9\ Nm^2/C^2)(-2.0 \times 10^{-5}\ C)[1/0.25\ m - 1/1\ m]$
$= (1.5 \times 10^{-5}\ C)(9.0 \times 10^9\ Nm^2/C^2)(-2.0 \times 10^{-5}\ C)[3\ m^{-1}] = -8.1\ J.$
6. **(C) is correct.** $V_A = V_{A1} + V_{A2} = kQ_1/r_1 + kQ_2/r_2$
$= (9.0 \times 10^9\ N{\cdot}m^2/C^2)(-2.0 \times 10^{-5}\ C)/0.15$
$+ (9.0 \times 10^9\ N{\cdot}m^2/C^2)(+2.0 \times 10^{-5}\ C)/0.05$
$= 3.6 \times 10^6\ V - 1.2 \times 10^6\ V = 2.4 \times 10^6\ V.$
This actually represents a dipole, where $\theta = 0°$, but r is not large with respect to l, so the dipole formula is not valid.
7. **(B) is correct.** The potential is given by $V = kp\cos\theta/r^2$
$= (9.0 \times 10^9\ N{\cdot}m^2/C^2)(5.0 \times 10^{-30}\ C{\cdot}m)\cos 60°/(3.0 \times 10^{-9}\ m)^2$
$= 2.5 \times 10^{-3}\ V.$
8. **(E) is the correct answer.** Because the potential is given by $V = kp\cos\theta/r^2$, without entering any values other than $\cos\theta$, it is clear that the numerator will be zero, and therefore that the potential will be zero.
9. **(B) is the correct answer.** For this capacitor design, capacitance is given by $C = \varepsilon_0 A/d = (8.85 \times 10^{-12}\ C^2/N{\cdot}m^2)(9 \times 10^{-4}\ m^2)/(2 \times 10^{-3}\ m)$
$= 4 \times 10^{-12}\ F$. Next, using the relation of charge to voltage, $Q = CV$
$= (4 \times 10^{-12}\ F)(9\ V) = 3.6 \times 10^{-11}\ C.$
10. **(D) is correct.** All three units are equivalent. Unit I and unit II are equivalent, since $N = J/m$, and units II and III are equivalent, because $V = J/C$. Units I and III are thus logically equivalent, also.

Free-Response Questions

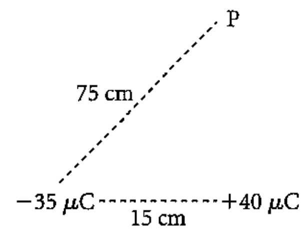
1. (a) The dipole moment, p, can be isolated from the standard equation, $V = kp\cos\theta/r^2$, such that $p = Vr^2/k\cos\theta$
$= (.15\ V)(1.24 \times 10^{-10}\ m)^2/(9.0 \times 10^9\ N{\cdot}m^2/C^2)(\cos 41°)$
$= 3.4 \times 10^{-31}\ C{\cdot}m.$

(b) $Q = p/l = 3.4 \times 10^{-31}$ C·m$/6.8 \times 10^{-11}$ m $= 5.0 \times 10^{-21}$ C, so one pole has a charge of $+5.0 \times 10^{-21}$ C, and the other has a charge of -5.0×10^{-21} C.

(c) The potential is zero at any distance much larger than the separation distance of the charges at a 90° angle from the negative charge. This happens along the line perpendicular to the line of the dipole charges. There are an infinite number of places where this happens, either above or below at a relatively large difference.

This response correctly rearranges the dipole potential formula to solve for the dipole moment in part a. The response to part b not only finds the charge based on the definition of the dipole moment, but also it states that there are two opposite charges and gives their values. The response to part c correctly points out that there are an infinite number of points, at a large enough distance away, where the potential is zero. Note that in any plane containing the dipole line there are two places ("either above or below") for a specific potential.

2. A sketch of the situation is shown below.



Without calculations, it is apparent that P must be located somewhere along a circle centered at the negative charge, Q_1, with a radius of 75 cm, and that it must form a triangle with the two point charges (unless all are collinear). The voltage at point P, $V_P = 40{,}000$ V, where
$V_P = V_{1P} + V_{2P} = kQ_1/r_1 + kQ_2/r_2$.
$40{,}000\text{ V} = (9.0 \times 10^9\text{ N·m}^2/\text{C}^2)(-3.5 \times 10^{-5}\text{ C})/0.75$
$+ (9.0 \times 10^9\text{ N·m}^2/\text{C}^2)(4.0 \times 10^{-5}\text{ C})/r_2$.
Solving for r_2, the distance is approximately 78 cm. It is possible, because the 15, 75, and 78 are possible dimensions for a triangle.

This response features a sketch demonstrating that the points must form a triangle. The correct signs are taken into account when determining the scalar quantity. Finally, the distance necessary for the given electric potential is isolated and fits the necessary dimensional criteria.