

DC Circuits

The components of electric circuits can be represented in schematic diagrams to facilitate analysis of the circuit.

Resistors in Series and in Parallel

- Resistors—either in the familiar ceramic form or other resistive devices such as light bulbs—connected sequentially in a circuit are described as being in **series**. When in series, the equivalent resistance for the circuit is the sum of the individual resistances, $R_{\text{eq}} = \Sigma R$.
- When the circuit pathway separates such that its current flows to two separate resistors, those resistors are described as being in **parallel**. When in parallel, the equivalent resistance is equal to the reciprocal of the sum of the reciprocals of the individual resistances, $1/R_{\text{eq}} = \Sigma 1/R$.
- This can be incorporated into Ohm's law $V = IR_{\text{eq}}$ relating equivalent resistance to potential and current in the entire circuit.
- The equivalent resistance of circuits containing resistors in both parallel and series should be calculated using both rules.

EMF and Terminal Voltage

The abbreviation **emf** refers to **electromotive force**, \mathcal{E} , which is the voltage resulting from the potential difference between terminals of a battery when no current flows.

- Batteries have an internal resistance, r , which affects the actual voltage, called the **terminal voltage**, which is the emf less the product of internal resistance and current, $V = \mathcal{E} - Ir$.

Kirchhoff's Rules

The method for calculating current in pathways with more than one battery is dictated by Kirchhoff's rules.

- The **junction rule**, called Kirchhoff's first rule, is based on conservation of charge. It equates the amount of charge entering a junction in a circuit pathway with the amount of charge leaving that junction.
- The **loop rule**, also called Kirchhoff's second rule, is based on conservation of energy. It equates the net loss in potential in any closed path of the circuit with an equivalent gain along the same pathway.
- These rules are combined for solving complex circuit pathways. First the junction rule provides an equation at each junction with as many variables as there

are paths entering and leaving a junction. The loop rule can be used to create as many equations as there are closed paths. Substitution or linear combination can be used to determine the current at every point along the pathway.

EMFs in Series and in Parallel; Charging a Battery

When batteries are connected in series, the voltage of the circuit is a consequence of their arrangement.

- For batteries whose currents are flowing in the same direction, their potential differences are summed.
- If their currents are flowing in the opposite direction along the pathway, the potential is the difference of the two voltages with a current in the direction of the stronger voltage. This arrangement is how batteries are charged.
- When batteries are connected in parallel, the junction rule is applicable. Net voltage does not increase or decrease, although the voltage loss due to internal resistances is lower.

Circuits Containing Capacitors in Series and in Parallel

- Capacitors connected sequentially in a circuit are said to be in **series**. When in series, the equivalent capacitance is equal to the reciprocal of the sum of the reciprocals of the individual capacitances—that is, $1/C_{\text{eq}} = \Sigma 1/C$.
- When the circuit pathway separates such that its current flows to two separate capacitors, those capacitors are said to be in **parallel**. When in parallel, the equivalent capacitance for the circuit is the sum of each individual capacitance—that is, $C_{\text{eq}} = \Sigma C$.
- These values can be incorporated into the equation $Q = C_{\text{eq}}V$ relating equivalent capacitance to potential and current in the entire circuit.
- The equivalent capacitance of circuits containing capacitors in both parallel and series should be calculated using both rules.

Circuits Containing a Resistor and a Capacitor

Circuits containing both capacitors and resistors are called **RC circuits**.

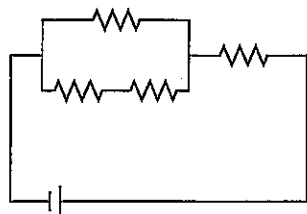
- In a circuit containing one capacitor and one resistor, the voltage across the capacitor is given by $V = \mathcal{E}(1 - e^{-t/RC})$ as a function of time as the capacitor charges. The quantity RC is called the time constant, τ , and it represents the time in seconds for the capacitor to reach 63% of full charge.
- As the capacitor discharges, the voltage falls as a function of time as shown by $V = V_0 e^{-t/RC}$.

For Additional Review

Practice Kirchhoff's rules on circuits with multiple junctions involving more than three equations to solve.

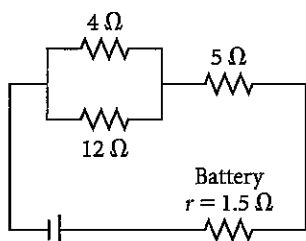
Multiple-Choice Questions

Questions 1 & 2 refer to the diagram shown below. Four identical light bulbs, each of resistance R , are connected as shown.



1. What is the equivalent resistance of the circuit?
 (A) $3R/2$ (D) $4R$
 (B) $3R/5$ (E) $5R/3$
 (C) $3R$
2. If the circuit has a voltage, V , what is an expression for the current in the circuit?
 (A) $5/RV$ (D) $3V/5R$
 (B) $3RV$ (E) $2V/3R$
 (C) $2R/3V$

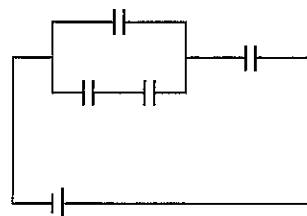
Questions 3, 4 & 5 refer to the diagram shown below. The circuit shown contains a 12.0 V battery with an internal resistance of 1.5Ω .



3. What is the equivalent resistance of the circuit?
 (A) 1.1Ω (D) 12Ω
 (B) 7.5Ω (E) 13Ω
 (C) 9.5Ω
4. What is the current of the circuit?
 (A) 25 mA (D) 1.3 A
 (B) 134 mA (E) 4.5 A
 (C) 0.9 A

5. What is the terminal voltage of this circuit?
 (A) 5.0 V (D) 12 V
 (B) 7.5 V (E) 14.5 V
 (C) 10 V

6. Four capacitors are connected as shown below.



What is the equivalent capacitance of the circuit if each capacitor has a rating of 10 pF?

- (A) 6 pF (D) 17 pF
 (B) 9 pF (E) 21 pF
 (C) 13 pF
7. Two batteries are arranged in a circuit. One battery has a voltage of 9 V, and the other has a voltage of 1.5 V. They are placed so that the positive terminal of one battery faces the positive terminal of the other in the pathway. The net voltage of the circuit is
 (A) 4 V (D) 10.5 V
 (B) 7.5 V (E) 13.5 V
 (C) 9 V
8. With a current of 1.5 A, what is the emf of a battery with an internal resistance of 6Ω and a terminal voltage of 4.5 V?
 (A) 4 V (D) 10.5 V
 (B) 7.5 V (E) 13.5 V
 (C) 9 V

9. What is the minimum number of $8\ \Omega$ resistors arranged in parallel that would be necessary to keep the current in a circuit with a 12 V battery less than 0.45 A?

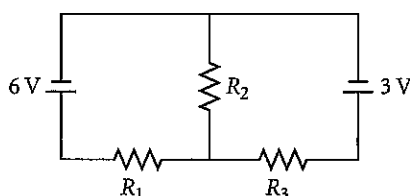
- (A) 2
- (B) 4
- (C) 8
- (D) 16
- (E) None of the above

10. What is the minimum number of $8\ \Omega$ resistors arranged in series that would be necessary to keep the current in a circuit with a 12 V battery less than 0.45 A?

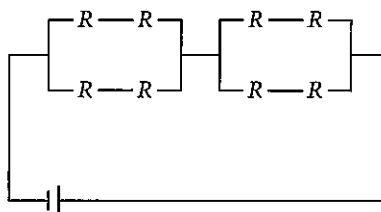
- (A) 2
- (B) 4
- (C) 8
- (D) 16
- (E) None of the above.

Free-Response Questions

1. Given $R_1 = 10\ \Omega$, $R_2 = 15\ \Omega$, $R_3 = 5\ \Omega$, find the current going through each resistor in this pathway.



2. Eight identical resistors are placed in a circuit attached to a battery as shown.



- (a) Find the equivalent resistance of this circuit.
- (b) Determine the voltage necessary for this circuit to have a current of 2.5 A if each resistor has a rating of $3\ \Omega$.
- (c) Draw a diagram of a circuit with an equivalent total resistance using any number of R ohm resistors but without any parallel branches.
- (d) Determine the equivalent resistance in the circuit presented, assuming one branch in each of the parallel circuits is removed.

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

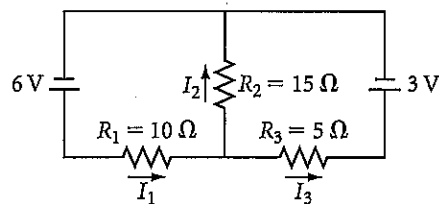
1. **(E) is correct.** First consider the two resistors in one branch in a parallel circuit. They act in series, and their equivalent resistance can be summed. So the parallel circuit can be considered in which one branch has a resistance of R while the other has a resistance of $2R$. $1/R + 1/2R = 3/2R$, such that the

entire parallel portion has an equivalent resistance of $2R/3$. Finally, that can be considered in series with the remaining resistor, so $2R/3 + R = 5R/3$.

- 2. (D) is correct. From Ohm's law, $I = V/R_{eq}$. If $R_{eq} = 5R/3$, $I = 3V/5R$.
- 3. (C) is correct. The equivalent resistance of the circuit can be determined by calculating the resistances in parallel $1/4 \Omega + 1/12 \Omega = 1/3 \Omega$, and then by summing the resistances in series $3 \Omega + 5 \Omega + 1.5 \Omega = 9.5 \Omega$.
- 4. (D) is correct. This answer requires a correct calculation of equivalent resistance, since (from Ohm's law) $I = V/R_{eq} = 12 \text{ V}/9.5 \Omega = 1.3 \text{ A}$.
- 5. (C) is correct. The terminal voltage is given by $V = \mathcal{E} - Ir = 12 \text{ V} - (1.3 \text{ A})(1.5 \Omega) = 10 \text{ V}$.
- 6. (A) is correct. First consider the two capacitors in one branch of the parallel circuit. They act in series, and their equivalent capacitance is the reciprocal of the sum of their reciprocals. So the parallel circuit has one branch with a capacitance of 10 pF , and the other branch has a capacitance of 5 pF . Since $10 \text{ pF} + 5 \text{ pF} = 15 \text{ pF}$, the entire parallel portion has an equivalent capacitance of 15 pF . Finally, that can be considered in series with the remaining capacitor, so $1/C_{eq} = 1/15 \text{ pF} + 1/10 \text{ pF} = 5/30 \text{ pF} = 1/6 \text{ pF}$. Thus $C_{eq} = 6 \text{ pF}$.
- 7. (B) is correct. When batteries are placed such that their currents flow in opposite directions, their net voltage is their difference—with a current in the direction dictated by the stronger battery. This arrangement is used in recharging batteries.
- 8. (E) is correct. The emf will be given by $\mathcal{E} = V + Ir = 4.5 \text{ V} + (1.5 \text{ A})(6 \Omega) = 13.5 \text{ V}$.
- 9. (E) is correct. From Ohm's law, $R_{eq} = V/I = 12 \text{ V}/0.45 \text{ A} = 27 \Omega$. Arranging 8Ω resistors in parallel would yield a $R_{eq} \leq 8 \Omega$. There is therefore no possible arrangement to make this occur.
- 10. (B) is correct. From Ohm's law, $R_{eq} = V/I = 12 \text{ V}/0.45 \text{ A} = 27 \Omega$. Since $R_{eq} = \Sigma R$, it would take at least four 8Ω resistors to accomplish this.

Free-Response Questions

1. Applying Kirchhoff's rules, the current directions can be assigned at a junction, and from the junction rule, $I_1 = I_2 + I_3$. One possible example is shown below.



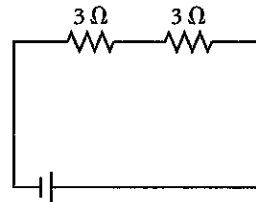
Next, the loop rule can be applied along closed paths, and the following relations can be derived from the circuit pathways as defined.

$$\begin{aligned}
 6 - 10I_1 - 15I_2 &= 0 \\
 3 + 5I_3 - 15I_2 &= 0 \\
 6 - 10I_1 - 5I_3 - 3 &= 0
 \end{aligned}$$

Combining any two of these equations with the junction rule (in which $I_1 = I_2 + I_3$) and using either substitution or linear combination, $I_1 = 0.22$ A, $I_2 = 0.27$ A, and $I_3 = 0.05$ A.

This response would get full credit, because currents are assigned and the junction rule is successfully used to develop a preliminary equation at the junction. Two equations derived from the loop rule are necessary so that there are as many linearly independent equations as there are variables. Finally, these equations are correctly solved, and the answers are consistent with the current arrows.

2. (a) In each parallel branch, there is an equivalent resistance of $2R$. For each parallel circuit, $1/R_{\text{eq}} = 1/2R + 1/2R = 2/2R = 1/R$, so each parallel portion has an equivalent resistance of R . This happens twice in series, so $R_{\text{eq}} = 2R$.
- (b) From Ohm's law, $V = IR_{\text{eq}}$, $V = (2.5 \text{ A})(6 \Omega) = 15 \text{ V}$.
- (c) The simplest accurate response is shown below.



- (d) If one branch is removed, the circuit is effectively four resistors in series, and the resistance is $4R$, or 12Ω , if 3Ω is substituted.

This response to part a provides a functional understanding of the series and parallel equations for equivalent resistance, and the response to part b provides a functional understanding of Ohm's law. The response to part c displays an ability to construct a simple circuit based on equivalent resistance, which is the converse of part a. The response to part d displays a broader understanding of the information necessary for answering part a.