CHAPTER 20

Magnetism

Magnets and Magnetic Fields

The opposite ends of a magnet are called **poles.**

- When a magnet can spin freely, as in a compass, the end pointing approximately toward geographical north is called the **north pole,** whereas the other end is the **south pole.**
- A relatively small number of materials exhibit strong magnetic properties; these materials are called **ferromagnetic.**
- In terms of force, the interactions of poles of different magnets are similar to those of two electric charges. However, unlike charges, these magnetic poles cannot be isolated.
- The poles exhibit **magnetic fields** analogous to the electric fields exhibited by two opposite charges. A magnetic field is a vector, represented by **B**. The direction of the magnetic field is tangent to the **magnetic field lines,** which graphically represent the magnetic field by extending outward from the polar surfaces and pointing from north to south. As with electric fields, the spacing of the field lines is proportional to the strength of the field. The magnitude of the magnetic field determines the torque exerted on a compass needle at that point.
- By convention, the north pole of a compass needle is said to point approximately toward the north geographic pole of the Earth, although it is actually magnetic south. The magnetic and geographic poles of the Earth are offset by about 1300 km. For a point on the Earth, the angle between the lines extending from that point to the magnetic pole and the geographic pole is called the **magnetic declination.** This angle represents the difference between the direction of magnetic north and the direction of true (geographic) north.

Electric Currents Produce Magnetism

A current-carrying wire produces a magnetic field around the wire.

- Concentric circles in a plane perpendicular to the wire represent the magnetic field graphically (with arrows denoting direction). Compass needles align tangent to arcs of the magnetic field lines circling a current-carrying wire, thus indicating the direction of the magnetic field.
- By convention, the direction of the magnetic field is determined by the **right-hand rule.** If the right hand is wrapped around the wire with the thumb pointing in the direction of the conventional current, the fingers curl around the wire in the direction of the magnetic field.

Force on an Electric Current in a Magnetic Field; Definition of B

Conversely, if a current-carrying wire is placed within a magnetic field, the direction of force exerted on the wire is perpendicular to the directions of both the current and the field.

- The direction of this force is indicated by a **second right-hand rule.** If the fingers of the right hand are pointed in the direction of the current—and then curled so that they are oriented north to south in the direction of the magnetic field—the outstretched thumb indicates the direction of force.
- The magnitude of force is a function of a wire's current, I, length in the uniform magnetic field, l, the strength of magnetic field, B, and their angular orientation, θ, such that $F = IlB \sin \theta$.
- As evident in the trigonometric component of the equation, the orientation yielding the greatest force occurs when the field and the current are perpendicular and $F = IlB$, whereas the minimum force occurs when they are parallel and $F = 0$ N.
- The unit of magnetic field is the **tesla,** equivalent to N/A·m.

Force on an Electric Charge Moving in a Magnetic Field

The magnitude of force of a magnetic field of strength B on a single moving charge, q, is a function of the velocity of the particle, v, and its angular orientation, such that $F = qvB \sin \theta$.

- As evident in the trigonometric component of the equation, the orientation yielding the greatest force will be when the velocity and the current are perpendicular and $F = qvB$. The orientation yielding the least force will occur when the velocity and current are parallel and $F = 0$ N.
- The direction of this force is given by a **third right-hand rule.** If the fingers of the right hand first are pointed in the direction of the particle's motion, and then bent in the direction of the magnetic field, the thumb points in the direction of the force on positively charged particles and in the opposite direction of the force on negatively charged particles. Note that this third right-hand rule is equivalent to the second one given above, except that the moving charges are not necessarily confined to wires.
- While in a large uniform magnetic field, a charged particle moving perpendicular to the direction of the field will follow a circular path, because the force given by the third right-hand rule is a centripetal force.

Magnetic Field Due to a Straight Wire

The strength of magnetic field due to a long straight wire is proportional to the current in the wire, I, and inversely proportional to the distance from the wire, r, such that $B = \mu_0 I / 2\pi r$, where the constant of proportionality, **the permeability of free space,** $\mu_0 = 4\pi \times 10^{-7}$ T·m/A.

Force Between Two Parallel Wires

The interactions of forces between two current-carrying wires cause attraction between parallel wires carrying current in the same direction, and they cause repulsion between parallel wires carrying current in the opposite direction.

- The force per unit length, l, of two wires of current, I_1 and I_2, that are L meters apart is given by $F/l = \mu_0 I_1 I_2 / 2\pi L$.

For Additional Review

Consider the centripetal acceleration of a charged particle moving in a circular orbit in a uniform magnetic field to determine the magnitude of its velocity.

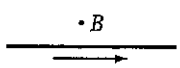
Multiple-Choice Questions

1. At which of the following angles from parallel to the direction of the magnetic field does a wire carrying 8.00 A of current have to be oriented so that its 0.500 m length subject to a 2.00 T magnetic field experiences a force of 2.00 N?
(A) 0.00°
(B) 14.5°
(C) 45.0°
(D) 63.5°
(E) 90.0°

2. What is the magnitude of force on a particle of charge $+3\ \mu C$ that moves through a magnetic field of 1.5 T at 7.8×10^4 m/s at 30° from the parallel?
(A) 0.18 N
(B) 0.24 N
(C) 0.32 N
(D) 0.38 N
(E) 0.50 N

Questions 3 & 4 refer to the diagram shown below.

A point, B, is 1 meter away from a wire that carries a 100 amp current in the indicated direction.



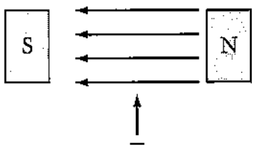
3. What is the magnitude of the magnetic field at point B?
(A) 4.0×10^{-3} T
(B) 8.0×10^{-4} T
(C) 2.0×10^{-5} T
(D) 4.0×10^{-6} T
(E) 8.0×10^{-7} T

4. What is the direction of the magnetic field at point B?
(A) To the left in the plane of the page
(B) To the right in the plane of the page
(C) Out of the page
(D) Into the page
(E) Upward in the plane of the page

5. What is the force between two 1.5 m parallel wires that are 10 cm apart each carrying a current of 9 A in the same direction?
(A) 9.8×10^{-2} N
(B) 2.1×10^{-3} N
(C) 6.1×10^{-3} N
(D) 9.8×10^{-3} N
(E) 2.4×10^{-4} N

6. The force on a current-carrying wire is 5.5 N when a length of it is subjected to a uniform magnetic field at a 50° angle from parallel. What is the maximum possible force that can be attained from the same length of wire carrying the same current in a magnetic field of the same strength?
(A) 5.5 N
(B) 6.0 N
(C) 6.6 N
(D) 7.2 N
(E) 7.8 N

7. What is the direction of the force on a negative charge that travels through a magnetic field, as shown below?



(A) Out of the page
(B) Into the page
(C) Upward in the plane of the page
(D) To the left in the plane of the page
(E) To the right in the plane of the page

8. The force exerted by a uniform magnetic field on an electron is 1.7 N while the electron moves at a constant velocity. The electron travels at an angle of 17° to the direction of the field. What is the minimum possible magnitude of force on the electron while traveling at the same velocity in a magnetic field of the same strength?
(A) 0.61 N
(B) 0.95 N
(C) 1.3 N
(D) 1.7 N
(E) None of the above

9. As the length of a current-carrying wire within a uniform magnetic field doubles, the force on the wire
(A) quadruples
(B) doubles
(C) stays the same
(D) is halved
(E) is quartered

10. A force is exerted between two parallel wires carrying currents in the same direction. As the current in each wire doubles, for the exerted force between these wires to remain the same, the distance between the wires would have to
(A) quadruple
(B) double
(C) stay the same
(D) be halved
(E) be quartered

Free-Response Questions

1. The force exerted by a magnetic field on a wire of length l, carrying current, I, is 20.8 N. The magnetic field makes an angle of 66° to the direction of the current.
   (a) In terms of current and length, what is the strength of the magnetic field?
   (b) A $+10.0\ \mu C$ charge passes through a small magnetic field which has the same strength as in part (a) above. The charge is traveling with a velocity, v, that is perpendicular to the direction of the field. Find the force on this charge in terms of the results from part (a).

2. A current carrying long straight wire has a force of 47.25 N on the 35 cm of its length that is placed perpendicular to a magnetic field of strength 7.5 T.
   (a) If two wires like this one were not subject to a magnetic field and were placed parallel and 0.68 cm apart with currents running in opposite directions, what would be the magnitude and direction of force per unit length?
   (b) More generally, to maintain a constant force between two stationary wires with variable currents, how would the currents have to be related? Explain first in terms of variables, and then give a specific numerical example.

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. **(B) is correct.** The equation for strength of the magnetic field per unit, $F/l = IB \sin \theta$, can be rearranged so that $\theta = \sin^{-1}[F/IlB]$ $= \sin^{-1}[2\text{ N}/(8\text{ A})(0.5\text{ m})(2\text{ T})] = 14°$.
2. **(A) is correct.** The magnitude of force on a charged particle is given by $F = qvB \sin \theta = (3.0 \times 10^{-6}\text{ C})(7.8 \times 10^{4}\text{ m/s})(1.5\text{ T})(\sin 30°) = 0.18\text{ N}$
3. **(C) is correct.** The magnitude of a magnetic field at a point due to a long straight wire is given by $B = \mu_0 I/2\pi r$, where the constant of proportionality $\mu_0 = 4\pi \times 10^{-7}\text{ T·m/A}$. So $B = (4\pi \times 10^{-7}\text{ T·m/A})(100\text{ A})/(2\pi)(1\text{ m}) = 2.0 \times 10^{-5}\text{ T}$.
4. **(C) is correct.** By the appropriate right-hand rule, the thumb is pointed in the direction of current, and the fingers curl around the wire indicating the direction of the magnetic field. The direction would therefore be out of the page.
5. **(E) is correct.** The attractive force between two parallel wires carrying currents in the same direction is given by $F = \mu_0 I_1 I_2 l/2\pi L$, where the constant of proportionality $\mu_0 = 4\pi \times 10^{-7}$.

   Here, $F = \dfrac{(4\pi \times 10^{-7}\text{ T·m/A})(9\text{ A})(9\text{ A})(1.5\text{ m})}{2\pi(0.1\text{ m})} = 2.4 \times 10^{-4}\text{ N}$.
6. **(D) is correct.** If the force on a current-carrying wire is given by $F = IlB \sin \theta$, the maximum force will occur when the angle between them is 90°. Therefore if $5.5\text{ N} = IlB \sin 50°$, $F_{max} = IlB = 5.5\text{ N}/\sin 50° = 7.2\text{ N}$.
7. **(B) is correct.** The fingers point in the direction of motion, then curl north to south. The thumb then points in the direction of the deflecting force on a positive charge, which is opposite of the direction of the deflecting force on a negative charge.
8. **(E) is correct.** The magnitude of force on a charge traveling at a constant velocity in a uniform magnetic field is given by $F = qvB \sin \theta$. Note that the minimum force will occur when the directions of the velocity and the magnetic field are parallel ($\theta = 0°$). At this orientation, the force is 0 N.
9. **(B) is correct.** The force on the length of a current-carrying wire in a uniform magnetic field is given by $F = IlB \sin \theta$. As the length (l) doubles, the force (F) also doubles, assuming the other variables remain the same.
10. **(A) is correct.** The force between two current-carrying wires is given by $F = (\mu_0/2\pi)(I_1 I_2/L)l$. As the current in each doubles, the distance between them (L) would have to quadruple for the force between them to stay the same, because $F = (\mu_0/2\pi)(2I_1)(2I_2)/4L)l = (\mu_0/2\pi)(I_1 I_2/L)l$.

Free-Response Questions

1. (a) The force on a current-carrying wire is given by $F = IlB \sin \theta$. From the given information, $20.8\text{ N} = IlB \sin 66°$, so $B = 20.8\text{ N}/Il \sin 66° = 22.8\text{ N}/Il$.

(b) The force on a charge traveling through the magnetic field is given by $F = qvB \sin \theta$. Because the charge and magnetic field meet at a 90° angle, $F = qvB$. From the results of (a), $F = (22.8 \times 10^{-5}\ \text{N·C})v/Il$.

This response correctly uses the equations for force on a current-carrying wire in a uniform magnetic field and on a charge traveling through a uniform magnetic field. Responses to parts a and b combine variable and numeric terms for the appropriate solution, as well as interrelated manipulation of the force equations.

2. (a) The current has to be determined first. From the background to the problem, the force equation $F = IlB \sin \theta$ can be used, so

$$I = \frac{F}{lB \sin \theta} = \frac{47.25\ \text{N}}{(0.35\ \text{m})(7.5\ \text{T})(\sin 90°)} = 18\ \text{amps}.$$

This can be applied to the equation for the force-per-unit length between two long straight wires,

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi L} = \frac{(4\pi \times 10^{-7}\ \text{T·m/A})(18\ \text{A})(18\ \text{A})}{2\pi(0.0068\ \text{m})} = 9.5 \times 10^{-3}\ \text{N/m}.$$

Because of their antiparallel arrangement, the wires repulse each other by the right-hand rule.

(b) For two straight parallel wires, $F/l = \mu_0 I_1 I_2/2\pi L$. If the wires are stationary, the separation distance (l) is constant. The product $I_1 I_2$ would have to be constant to maintain a constant force. That is, the two currents would be inversely proportional. If I_1 increased by a factor of n, I_2 would have to decrease by a factor of n. In practice, this means that the force between two wires remains constant if the current in one is doubled while the current in the other is halved. The force would also remain constant if the current in one were quadrupled while the current in the other were quartered.

This response correctly uses the background facts of one situation to apply it to another. The response to part a demonstrates how the necessary information about the current can be extracted and used for the same wire subjected to different conditions. Further, it employs the right-hand rule to determine the direction of force. The response to part b shows a mathematical understanding of the relationship between the currents in this force equation, and it gives examples of its consequences.