CHAPTER 21

Electromagnetic Induction and Faraday's Law; AC Circuits

Induced EMF

While a steady magnetic field produces no current in a wire, Faraday's experiments showed that a changing magnetic field produces what is called an **induced current** in a wire.

- This process of producing an emf is called **electromagnetic induction.** It will occur when either the wire or the magnet changes positions relative to the other.

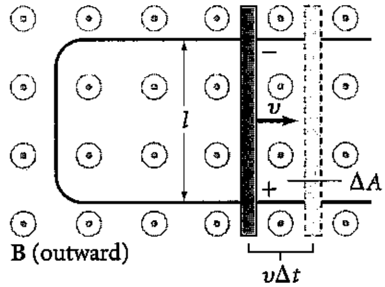
Faraday's Law of Induction; Lenz's Law

The magnitude of an induced emf depends on the area of a coil, A, passing through a magnetic field, B, as well as the rate at which the magnetic field changes and the angle at which the magnetic field passes through the line perpendicular to the plane of the coil. That is, induced emf is a measure of the rate that **magnetic flux** changes.

- Magnetic flux is given by the equation $\Phi_B = BA\cos\theta$, whose units are called **webers.** Values of theta imply the flux is strongest when the magnetic field is perpendicular to the plane of the coil—and weakest (zero) when the magnetic field and the plane of the coil are parallel.
- **Faraday's law of induction** quantifies the emf in terms of the change in magnetic flux over time, t, such that if the coil has N loops, $\mathscr{E} = -N\Delta\Phi_B/\Delta t$.
- **Lenz's law** states that the direction of current for an induced emf will be in the opposite direction of the change in flux that induced it.
- An induced emf will occur if the strength of the magnetic field is changed, the loop's area is changed, or the angle at which they meet is changed. All of these events will cause a change in the magnetic flux through the loop.

EMF Induced in a Moving Conductor; Changing Magnetic Flux Produces an Electric Field

Just as emf can be induced into a coil by a change in area, it can be induced into a conducting pathway in which one section moves, as seen in the diagram on the next page.



- As one end of length, l, that makes up the rectangular pathway travels at a velocity, v, a conducting pathway is maintained while changing one of its dimensions—and consequently, its area.
- If this occurs in a plane perpendicular to a magnetic field, B, the emf induced is equal to the product of B, v, and l if they are all perpendicular, $\mathcal{E} = Blv$.
- The electric field within that moving conducting portion is therefore $E = vB$.

Electric Generators and Counter EMF

Electric generators induce current by rotating a coil in a magnetic field.

- In this arrangement, the direction of current changes twice each rotation, and the magnitude of the current oscillates relative to the orientation of the plane of the loop to the magnetic field. This creates an alternating current with an output emf such that $\mathcal{E} = NBA\omega \sin \omega t$, where N is the number of loops, B is the magnetic field, A is the area of the loop, and ω is the angular velocity. The angular velocity can also be expressed in terms of frequency as $\omega = 2\pi f$.
- The **counter** or **back emf** of a motor is the induced emf generated by its operation which opposes its motion in accordance with Lenz's law.

Transformers; Transmission of Power

A **transformer** is a pair of interacting coils used to change an AC voltage.

- These two coils are known as the **primary** and **secondary coils.** An alternating current traveling through the primary coil produces a magnetic field that induces a current of the same frequency in the secondary coil. Voltage differences between the coils can be affected by manipulating the number of loops in each coil.
- The voltage induced in the secondary coil is a function of the number of loops in the secondary coil and the rate of change of the magnetic flux, $V_S = N_S \, \Delta\Phi_B/\Delta t$. Similarly, the voltage input in the primary coil is related to the number of loops in the primary coil and to the rate of change of the magnetic flux, $V_P = N_P \, \Delta\Phi_B/\Delta t$. By combining these two expressions, the ratio of voltages is equal to the ratio of the numbers of loops. $V_S/V_P = N_S/N_P$. This is called the **transformer equation.** It assumes an "ideal" transformer where little or no flux is lost.
- When the secondary coil has more loops than the primary coil, the induced voltage will be greater in the secondary coil than in the primary coil. This arrangement is called a **step-up transformer.** The converse arrangement,

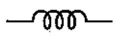
designed to induce a lower voltage in the secondary coil, is called a **step-down transformer.**

- The current for each coil can be deduced from the power equation and the transformer equation, such that $I_S/I_P = N_P/N_S$.
- Because induction is dependent on a changing magnetic flux, transformers will only operate with an AC current or with a pulsed DC current. With a steady current, the flux remains constant and induction does not occur.

Inductance

The induction in transformers is a specialized case of **mutual inductance,** in which an alternating current-carrying coil induces a current into another coil whose emf, $\mathscr{E}_2$, is proportional to the rate of change of the current, I_1, such that $\mathscr{E}_2 = -M\Delta I_1/\Delta t$, where the constant M depends on the material and shape of the coils.

- **Self-inductance** describes the induction of a magnetic field by an AC current in a lone coil, such that the emf is in the opposite direction as the change in flux. This emf is $\mathscr{E} = -L\Delta I/\Delta t$, where L depends on the material and shape of the coils.
- A self-inducting coil is called an **inductor** and is seen in circuit diagrams such as the one below.



- L and M have the units of henrys, where $1\ \text{H} = 1\ \Omega\cdot\text{s} = \text{V}\cdot\text{s/A}$.

For Additional Review

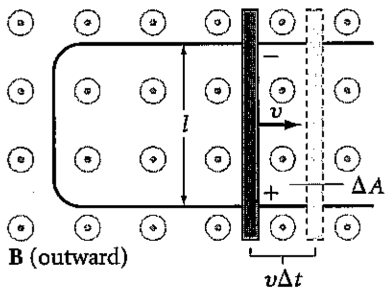
Contrast the inductance by coils in transformers with more general cases of mutual inductance.

Multiple-Choice Questions

1. Find the magnitude of induced emf in a circular coil with 25 loops and a radius 7 cm. The coil begins completely outside any magnetic field, and then during the next 10 seconds it moves completely into a uniform 15 T magnetic field. Assume that the plane of the loop and the direction of motion are both perpendicular to the magnetic field.
   (A) 0 V
   (B) 0.23 V
   (C) 0.58 V
   (D) 1.2 V
   (E) 3.6 V

2. What are possible dimensions for a rectangular coil that has a magnetic flux of 9.5 webers when in a magnetic field of strength 19 T at an angle of 60° from the perpendicular to the plane of the coil?
   (A) 80 cm × 70 cm
   (B) 360 cm × 30 cm
   (C) 120 cm × 80 cm
   (D) 1000 cm × 100 cm
   (E) 250 cm × 40 cm

Questions 3 & 4 refer to the diagram shown below.



3. What is the emf induced when a 18 cm conducting rod attached to a U-shaped conductor moves at a speed of 1 cm/s perpendicular to a magnetic field of strength 37.5 T?
   (A) 3.0 V
   (B) 2.1 V
   (C) 0.07 V
   (D) 0.03 V
   (E) 0.01 V

4. What is the strength of the electric field produced in the conductor by the change in flux?
   (A) 0.38 N/C
   (B) 0.75 N/C
   (C) 1.5 N/C
   (D) 3.8 N/C
   (E) 6.8 N/C

5. What is the maximum total output emf of a 50 Hz alternating current generator with a rotating 500-loop coil with an area of 0.68 m^2 in a 0.70 T magnetic field?
   (A) 33 V
   (B) 675 V
   (C) 1,150 V
   (D) 25,000 V
   (E) 75,000 V

6. How many loops must there be in the primary coil for a step-down transformer to convert 2400 V AC to 240 V AC in which the secondary coil has 76 coils?
   (A) 1200 loops
   (B) 760 loops
   (C) 610 loops
   (D) 120 loops
   (E) 12 loops

7. How much current must be in the primary coil for a step-up transformer to convert 120 V AC to 240 V AC where the secondary coil of 20 loops which has 0.125 A?
   (A) 0.013 A
   (B) 0.025 A
   (C) 0.125 A
   (D) 0.250 A
   (E) 1.25 A

8. What is the change in magnetic flux in a coil of area 5 m^2 as its orientation relative to the perpendicular of a uniform 3.0 T magnetic field changes from 45° to 90°?
   (A) −11 Wb
   (B) 0 Wb
   (C) 7.5 Wb
   (D) 11 Wb
   (E) 21 Wb

9. What are the units for the product of two values whose respective units are henrys and amperes?
   (A) Ω/s
   (B) V·s
   (C) A^2/s
   (D) Ω·V
   (E) V/s

10. What is the induced emf produced when the radius of one loop of wire in a 12 T magnetic field changes from 2 cm to 9 cm in 3 seconds? Assume that the plane of the loop remains parallel to the direction of the magnetic field.
    (A) 0 V
    (B) 0.32 V
    (C) 0.96 V
    (D) 1.25 V
    (E) 2.21 V

Free-Response Questions

1. (a) A transformer has a primary coil with 36 loops, a current of 0.900 A and a voltage of 7 V. Using the transformer equation, what is the voltage of the secondary coil if its current is 0.675 A?
   (b) With the same primary coil as described in (a), how many loops would have to be in the secondary coil to have a current of 2.7 A?
   (c) Name the type of transformer associated with the induction in each (a) and (b).
   (b) Describe a practical application of a transformer or of mutually inducing coils.
   (e) Why are transformers dependent on an AC current?

2. (a) Explain how Faraday's law is consistent with Lenz's law.
   (b) What does Faraday's law predict will differ if the plane of a coil is shifted from perpendicular to parallel to a magnetic field, as opposed to the reverse action over the same time period?
   (c) How many loops would be necessary in a coil to produce 9 V when its plane is perpendicular to a 0.375 T magnetic field as its area is halved from 0.50 m^2 to 0.25 m^2 in 2 minutes?

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. (C) is correct. First, magnetic flux has to be determined. Because the loop begins outside of any magnetic field, the initial magnetic flux is zero. The final flux is given by $\Phi_B = BA = (15\ \text{T})(\pi(0.07\ \text{m})^2) = 0.23$ Wb. Next, using Faraday's law, $\mathscr{E} = -N\Delta\Phi_B/\Delta t = -25(0.23\ \text{Wb})/10\ \text{s} = -0.58$ V, which has a magnitude of 0.58 V.

2. (E) is correct. Rearranging the expression for magnetic flux, $A = \Phi_B/B \cos\theta = 9.5/19 \cos 60° = 1\ \text{m}^2$. Because the coil is rectangular, the product of its dimensions must equal 1 m^2. Of the choices, only 250 cm × 40 cm = 10000 cm^2 = 1 m^2.

3. (C) is correct. The motional emf induced when the conductor's velocity, length and the magnetic field are all perpendicular is given by the equation $E = Blv = (37.5\ \text{T})(0.18\ \text{m})(0.01\ \text{m/s}) = 0.07$ V.

4. (A) is correct. The electric field is given by $E = vB = (0.01\ \text{m/s})(37.5\ \text{T}) = 0.375$ N/C, which is 0.38 N/C to two significant figures.

5. (E) is correct. To find angular velocity, $\omega = 2\pi f = 2\pi(50\ \text{s}^{-1}) = 314\ \text{s}^{-1}$. Next, while $\mathscr{E} = NBA\,\omega \sin \omega t$, its maximum will occur at a time t such that $\sin \omega t = 1$. As such, the maximum $\mathscr{E} = NBA\,\omega$ $= (500)(0.70\ \text{T})(0.68\ \text{m}^2)(314\ \text{s}^{-1}) = 75{,}000$ V.

6. (B) is correct. The ratio of voltages to loops is given by the transformer equation, $V_S/V_P = N_S/N_P$. Therefore, 240 V/2400 V = 76 loops/N_P and N_P = 760 loops.

7. (D) is correct. The ratio of voltages to loops is given by the transformer equation, $V_S/V_P = N_S/N_P$. Therefore, 240 V/120 V = 20 loops/N_P and

$N_P = 10$ loops. Next, the relationship between current and number of coils in a transformer is given by $I_S/I_P = N_P/N_S$. In this case, $0.125\ A/I_P = 10$ loops/20 loops and $I_P = 0.250$ A.

8. (A) is correct. Flux is given by $\Phi_B = BA\cos\theta$.
At 45°, $\Phi_B = (3.0\ \text{T})(5\ \text{m}^2)\cos 45° = 11$ Wb.
At 90°, $\Phi_B = 0$. $\Delta\Phi_B = 0\ \text{Wb} - 11\ \text{Wb} = -11$ Wb.

9. (B) is correct. The henry, the unit for mutual inductance, is equivalent to V·s/A. As such, H·A = V·s.

10. (A) is correct. An emf is not induced if the plane of the coil is parallel to the lines of the magnetic field, since the magnetic flux through the coil does not change when the coil shrinks.

Free-Response Questions

1. (a) Using the $I_S/I_P = N_P/N_S$ equation, 0.675 A/0.900 A = 36 loops/N_S, meaning N_S is 48 loops. Using $V_S/V_P = N_S/N_P$, V_S/7 V = 48 loops/36 loops, so $V_S = 9.33$ V.
   (b) Using the $I_S/I_P = N_P/N_S$ equation, 2.70 A/0.900 A = 36 loops/N_S, $N_S = 12$ loops.
   (c) Part (a) is called a step-up transformer; part (b) is called a step-down transformer.
   (d) Transformers are used to transmit electricity over long distances, and they minimize power loss due to the resistance of transmission wires by lowering the current.
   (e) Transformers require an AC current, because the alternation of the current's direction in the primary coil gives rise to an induced current (also alternating) in the secondary coil. A DC current would not induce a current in the secondary loop because the flux through the secondary coil would remain constant. A change in flux is required for electromagnetic induction.

   This response would receive full credit because it correctly applies the transformer equation to parts a and b, although there are other methods; the response to part c has both types of transformers correctly labeled; for the response to part d, any relevant example would suffice, including non-contacting battery rechargers; and for the response to part e, both descriptions of current type are correct and correctly applied.

2. (a) The negative sign of Faraday's law indicates the direction of the induced emf, consistent with Lenz's law, which states that induced emf produces a current whose magnetic field is in the opposite direction of the change in flux.
   (b) Faraday's law accounts for the rate of change of magnetic flux. When the plane of a coil is perpendicular to a magnetic field, flux is maximized, and when the plane is parallel, flux is zero. Change in flux in this case is a negative quantity, yielding a positive voltage from Faraday's law. In the

opposite case, change in flux in this case is a positive quantity, yielding a negative voltage from Faraday's law.

(c) Initially, $\Phi_B = BA = (0.375\ \text{T})(0.50\ \text{m}^2) = 0.1875$ Wb. After the change in area, $\Phi_B = (0.375\ \text{T})(0.25\ \text{m}^2) = 0.09375$.
$\Delta\Phi_B = 0.09375\ \text{Wb} - 0.1875\ \text{Wb} = -0.09375$ Wb.
From Faraday's law, $E = -N\,\Delta\Phi_B/\Delta t$, $N = -E\Delta t/\Delta\Phi_B$
$= (-9\ \text{V})(120\ \text{s})/-0.09375\ \text{Wb} = 11{,}520$ loops.

This response receives full credit because it correctly states both Faraday's and Lenz's laws for the response to part a; it uses the definition of magnetic flux as it applies to Faraday's law for the response to part b; the response to part c requires correct algebraic manipulation, as suggested in part b, to get a positive value for the number of loops.