CHAPTER 22

Electromagnetic Waves

CHAPTER 23

Light: Geometric Optics

Light as an Electromagnetic Wave and the Electromagnetic Spectrum

Electromagnetic waves travel at 3.00×10^8 m/s, which is the measured speed of light. This provides evidence that light is an electromagnetic wave. Moreover, electromagnetic waves demonstrate properties of light such as reflection, refraction and interference.

- The range of wavelengths of visible light is between 4.0×10^{-7} m and 7.5×10^{-7} m. The frequencies of visible light can then be determined from the relation $f\lambda = c$, where c represents the speed of light in a vacuum. They fall between 4.0×10^{14} Hz and 7.5×10^{14} Hz.
- The full range of electromagnetic frequencies, which extends well beyond visible light in both directions, makes up the **electromagnetic spectrum.** Waves of the lower frequencies at which television and radio signals are broadcast are collectively designated as **radio waves.**
- Electromagnetic waves are created by accelerating charged particles.

The Ray Model of Light

An object is visually perceived either directly, as a source of light, or indirectly by the light reflecting off of it. In many problem-solving situations, light can be assumed to travel from a source in straight-line **rays,** which are useful approximations of very narrow waves of light.

- This **ray model of light** is applicable to the reflection and refraction effects of mirrors and lenses, and it is the basis of **geometric optics.**

Reflection; Image Formation by a Plane Mirror

- The **law of reflection** states that the **angle of incidence** and the **angle of reflection** are equal angles measured on either side of a line that is normal to the reflecting boundary.

- There is a **diffuse reflection** from rough surfaces where parallel incident rays are reflected in many different directions. The light reflected from this microscopically rough printed page is an example of diffuse reflection.
- The human brain perceives an object reflected in a plane mirror as being located behind the mirror at a distance equal to its actual distance from the front of the mirror, as if the rays had traveled straight-line paths and had not been reflected. Therefore, the **image distance** from the mirror equals the **object distance** from the mirror for a plane mirror.
- Similarly, the resulting image height equals the object height. Because light rays do not travel through the mirror from the actual image location, the image that is seen is called a **virtual image.** This is distinguished from a **real image** produced by some other types of mirrors and lenses, which would appear on a screen placed at the location of the image.

Formation of Images by Spherical Mirrors

Mirror surfaces are often sections of the reflective interior or exterior surface of a sphere.

- A **convex** spherical mirror is part of the external surface of a sphere, whereas a **concave** spherical mirror is part of the internal surface of a hollow sphere.
- Light is often assumed to strike the surface of each of these mirrors in parallel rays due to the extremely large distance from the source of the light to the mirror. For a concave mirror, the reflected rays must meet at a single point to produce a sharp image. If the size of the mirror is small compared to its radius of curvature, one can assume that all of the incoming parallel rays precisely focus to a single point after they reflect. The following relations are defined and derived using that assumption.
- Where these rays meet is called the **focus.** The radius of curvature that extends to the center of the mirror is a reference line known as the **principal axis.** When the incident rays are parallel to the principal axis, the focus lies on the principal axis.
- The location of the focus on the principal axis is called the **focal point** and its distance from the mirror is called the **focal length** of the mirror. The focal point is at the midpoint of the radius of curvature of the principal axis, so $f = r/2$.
- For drawing rays to predict image formation by a spherical mirror, three lines are suggested: an incident ray parallel to the principal axis, an incident ray through the focus, and an incident ray that is perpendicular to the mirror.
- By geometry, the relationship between the focal length of a converging (concave) mirror, f, and the image and object distances from the mirror is given by the **mirror equation,** $1/d_o + 1/d_i = 1/f$, which is derived from the expression $h_i/h_o = d_i/d_o$.
- The ratio of $h_i/h_o = -d_i/d_o = m$, and is called the **lateral magnification** of a mirror. It will be positive if the object and image have the same orientation and negative if they are inverted relative to each other.
- On the reflective side of the mirror, d_o and d_i both have a positive value. On the other side of the mirror, d_o and d_i both have a negative value.

- h_o is always assumed to be positive. h_I is positive if the image is upright relative to the object, and h_I is negative if the image and object are inverted relative to each other.
- For a convex lens, the mirror equation above holds with the appropriate sign conventions.

Index of Refraction

The ratio of the speed of light in a vacuum to its speed in a uniform material represents that material's **index of refraction,** $n = c/v$. Since light is slower in these materials than in a vacuum, n will be greater than 1.

Refraction: Snell's Law; Total Internal Reflection; Fiber Optics

The portion of light that is not reflected at a boundary travels into a different medium and bends as a function of the incident angle and the indices of refraction of the media involved. This is a phenomenon called **refraction.**

- The **law of refraction,** or **Snell's law,** quantifies the relationship as $n_1 \sin \theta_1 = n_2 \sin \theta_2$, where θ_2 is called the **angle of refraction.**
- Snell's law suggests that for an angle θ_c, called the **critical angle,** the ray would bend such that it does not enter the new medium, but travels along the boundary. This would occur at $\sin \theta_c = (n_2/n_1) \sin 90° = n_2/n_1$. Values greater than θ_c would reflect entirely, a phenomenon **called total internal reflection.** Note that total internal reflection can only occur if $n_1 > n_2$.

Thin Lenses; Ray Tracing

A **thin lens** has two surface faces, each of which is either concave, convex, or planar, and the ratio of thickness to diameter in a thin lens is small.

- The **axis** of a thin lens is defined as the line through its center that is perpendicular to each face.
- The **focal point** is the location on the axis at which incident rays parallel to the axis converge for a converging lens, and where diverging rays would seem to originate for a diverging lens.
- The **focal length** is the distance along the axis from the center of the thin lens to the focal point. When not parallel to the axis, rays converge on the **focal plane** for a converging lens, which is perpendicular to the axis at the focal point. Rays seem to originate from the focal plane for a diverging lens in a comparable situation.
- The **power** of a lens is defined as the inverse of the focal length, $P = 1/f$, whose units are diopters, equivalent to m^{-1}.
- For drawing rays to predict image formation by a thin lens, three lines are suggested: an incident ray that is parallel to the principal axis, an incident ray that is through the focus on the near side of the lens, and an incident ray that is through the center of the lens.
- As with a mirror, a **real image** is one that can be seen on a screen or film that is placed at the location of the image, while a **virtual image** cannot.

The Lens Equation

- The relation between the focal length of a converging lens, f, and the image and object distances is given by the **lens equation,** $1/d_o + 1/d_i = 1/f$, which is derived from the relation $h_i/h_o = d_i/d_o$. For a diverging lens, the relation is such that $1/d_o - 1/d_i = -1/f$. As the distance from the lens to the object approaches infinity, the distance from the lens to the image approaches the focal length.
- The first lens equation can serve for both types of lenses when the appropriate **sign conventions** are followed, as explained below.
  1. For a converging lens, f has a positive value. For a diverging lens, f has a negative value.
  2. For an object on the same side of the lens from which the light originates (which is usually the case), d_o has a positive value. For an object on the opposite of the lens from which the light originates, d_o has a negative value.
  3. For an image on the opposite side from where the light originates, d_i has a positive value. For an image on the same of the lens as where the light originates, d_i has a negative value. This gives a positive d_i for real images and a negative d_i for virtual images.
  4. h_o is always taken to be positive. h_i is positive if the image is upright relative to the object, and h_i is negative if the image is inverted relative to the object.
- The ratio of $h_i/h_o = -d_i/d_o = m$, called the **lateral magnification** of a lens, which has a positive value if the image is upright and a negative value if it is inverted relative to the object.
- In agreement with convention #1 above, converging lenses are also called **positive lenses,** and diverging lenses are called **negative lenses.**

Problem Solving for Lenses; Problem Solving for Combinations of Lenses

- A **virtual image** will be formed when an object is between the focal point of a converging lens and the lens itself, whereas a **real image** will occur when an object is placed at a distance beyond its focal length.
- To solve for image formation by multiple lenses in combination, the image produced by the first lens can be considered the object for the next lens, and so on, to determine the characteristics of the final image.

For Additional Review

Consider problem-solving situations involving light rays traveling through combinations of both thin lens and spherical mirrors.

Multiple-Choice Questions

1. What is the height of an image produced by a 5 cm high object placed 15 cm from the mirrored surface of a concave spherical mirror whose focal length is 10 cm?
   (A) 30 cm, upright
   (B) 10 cm, upright
   (C) 10 cm, inverted
   (D) 15 cm, inverted
   (E) 30 cm, inverted

2. What is the height of an object that produces an 18 cm high inverted image when it is placed 8 cm from the mirrored surface of a concave spherical mirror and has an image distance of 24 cm?
   (A) 6 cm, upright
   (B) 52 cm, upright
   (C) 52 cm, inverted
   (D) 18 cm, inverted
   (E) 6 cm, inverted

3. What is the ratio of the radius of curvature of a spherical convex mirror to its focal length?
   (A) $-1/2$
   (B) $1/2$
   (C) -1
   (D) 2
   (E) -2

4. For a giant spherical convex mirror with a focal length of 17 m, what would be the image distance of an object that is placed 9 m from the center of the mirror along the principle axis?
   (A) 5.9 m, on the same side of the mirror
   (B) 5.9 m, on the opposite side of the mirror
   (C) 19 m, on the same side of the mirror
   (D) 19 m, on the opposite side of the mirror
   (E) No image will be formed.

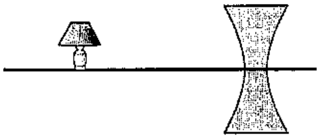
5. A ray of light travels from one substance into another medium at 33.0° from the normal and is refracted at 38° from the normal. What is the associated critical angle for a light ray traveling between these substances?
   (A) 45°
   (B) 62°
   (C) 74°
   (D) 85°
   (E) 90°

6. At which of the following angles from the normal will total internal reflection occur for a light ray traveling between two substances in which the incident substance has twice the index of refraction as the refracting substance.
   (A) 21°
   (B) 24°
   (C) 27°
   (D) 30°
   (E) 33°

7. What is the absolute height and type of the image produced when an 11 cm high object is placed 10 cm from a converging lens of focal length 5 cm?
   (A) 11 cm, real
   (B) 10 cm, virtual
   (C) 10 cm, real
   (D) 11 cm, virtual
   (E) 5 cm, real

Questions 8 & 9 refer to the diagram shown below.

An upright object placed 25 cm away from a diverging lens produces an image that is 10 cm from the center of the lens and on the same side of the lens.



8. What would be the focal length of the diverging lens?
   (A) -8 cm
   (B) -11 cm
   (C) -17 cm
   (D) -21 cm
   (E) -35 cm

9. What is the orientation and type of image formed by the diverging lens?
   (A) Upright and real
   (B) Upright and virtual
   (C) Inverted and real
   (D) Inverted and virtual
   (E) Not enough information is provided to determine the orientation and type of image.

10. Which of the following is NOT an accurate sign convention for geometric optics when dealing with thin lenses?
   (A) Focal length is considered negative for a diverging lens.
   (B) The image distance is positive when on the same side from which light is approaching.
   (C) The object distance is positive when on the same side from which light is approaching.
   (D) The image distance is always positive for a real image.
   (E) The height of the object is always considered positive.

Free-Response Questions

1. A person is standing 2.35 meters from a full-length plane mirror. His eye level is 1.55 meters from the ground. An object with a height of 25 cm is 1 meter away from the mirror. What is the perceived distance and the actual distance from the person's eye to the top of the object? Draw a diagram demonstrating your solution method.
2. Two converging lenses are placed 28 cm from each other collinear along their axes. Their focal lengths are both 8 cm. A 6 cm high object is placed 12 cm from the center of one of the two lenses, on the far side of both lenses.
   (a) What is the resulting image distance from the original object when both lenses are used together to produce the final image?
   (b) What will be the height of the final image produced, and will it be inverted or upright?
   (c) Will the FINAL image be real or virtual? Why?

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. **(C) is correct.** From the mirror equation, $1/d_o + 1/d_i = 1/f$, the relationship can be developed for lateral magnification. First, $1/15 + 1/d_i = 1/10$ cm, $1/d_i = 1/30$ cm, $d_i = 30$ cm. Therefore, $m = -d_i/d_o = -30$ cm$/15$ cm, so $h_i = \left(\frac{-30\text{ cm}}{15\text{ cm}}\right)(5\text{ cm}) = -10$ cm. The sign indicates that it is inverted.
2. **(A) is correct.** Using the lateral magnification relation, $m = h_i/h_o = -d_i/d_o$, $-18\text{ cm}/h_o = -(24\text{ cm})/(8\text{ cm})$, so $h_o = 6$ cm. The sign indicates that the image is upright.
3. **(D) is correct.** The radius of curvature is twice the distance of the focal distance. For a convex mirror, both are taken to be negative quantities, but the quotient of two negative numbers is a positive quantity. Therefore, $-2f/-f = 2$.
4. **(B) is correct.** The focal length of a convex mirror is taken as a negative value, $f = -17$ m. Using the mirror equation, $1/d_o + 1/d_i = 1/f$, $1/d_i = 1/(-17\text{ m}) - 1/(9\text{ m})$, so $d_i = -5.9$ m. Because it is a negative quantity, the distance image appears to be on the opposite side of the convex mirror.

5. **(B) is correct.** From the given information, an equation can be derived from Snell's law, $n_1 \sin \theta_1 = n_2 \sin \theta_1$, $n_1 \sin 33.0° = n_2 \sin 38°$. Rearranging this, $\sin 33.0°/\sin 38° = n_2/n_1 = 0.88$. By definition, the critical angle is given by $\sin \theta_c = n_2/n_1$, so $\theta_c = \sin^{-1} n_2/n_1 = \sin^{-1}(0.88) = 62°$

6. **(E) is correct.** From Snell's law, $n_1 \sin \theta_1 = n_2 \sin \theta_2$, total internal reflection will occur for values of $\theta > \theta_c$, where $\sin \theta_c = n_2/n_1 \sin 90° = n_2/n_1$. From the information given, $n_2/n_1 = 1/2$, so $\theta_c = 30°$, and only one choice fits the necessary criteria, $\theta > \theta_c$.

7. **(A) is correct.** The lens equation can be utilized here. Thus, $1/10$ cm + $1/d_i = 1/5$ cm, so $d_i = 10$ cm, and the height would be given by $h_i/h_o = -d_i/d_o$, so $h_i = h_o(-d_i/d_o) = 11 \text{ cm } (-10 \text{ cm}/10 \text{ cm}) = -11$ cm. This means that the image, while inverted, is 11 cm in height, and real.

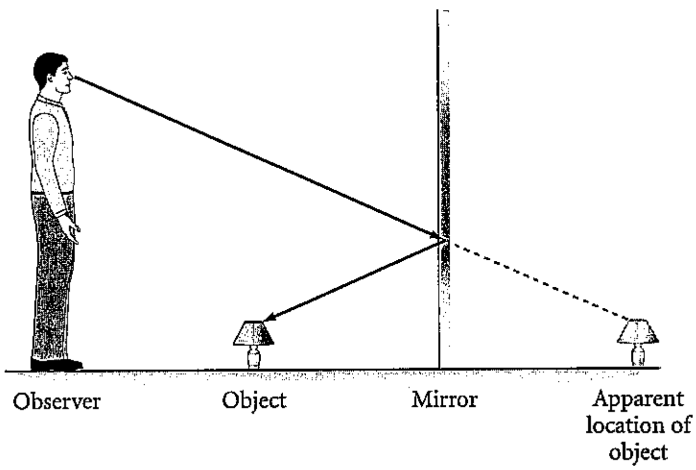
8. **(C) is correct.** The image distance is taken to be a negative quantity when on the same side as where the light comes from. Also, f is negative for a diverging lens in the lens equation, $1/d_o + 1/d_i = 1/f$, so $1/25 \text{ cm} - 1/10 \text{ cm} = 1/f$, so $f = -17$ cm, and the focal length is 17 cm.

9. **(B) is correct.** From the relation, $h_i/h_o = -d_i/d_o$, so $h_i = h_o(-d_i/d_o) = h_o(-(-10 \text{ cm})/(25 \text{ cm})) = 2h_o/5$, which is a positive value, and therefore upright. The image is virtual because the image distance is negative.

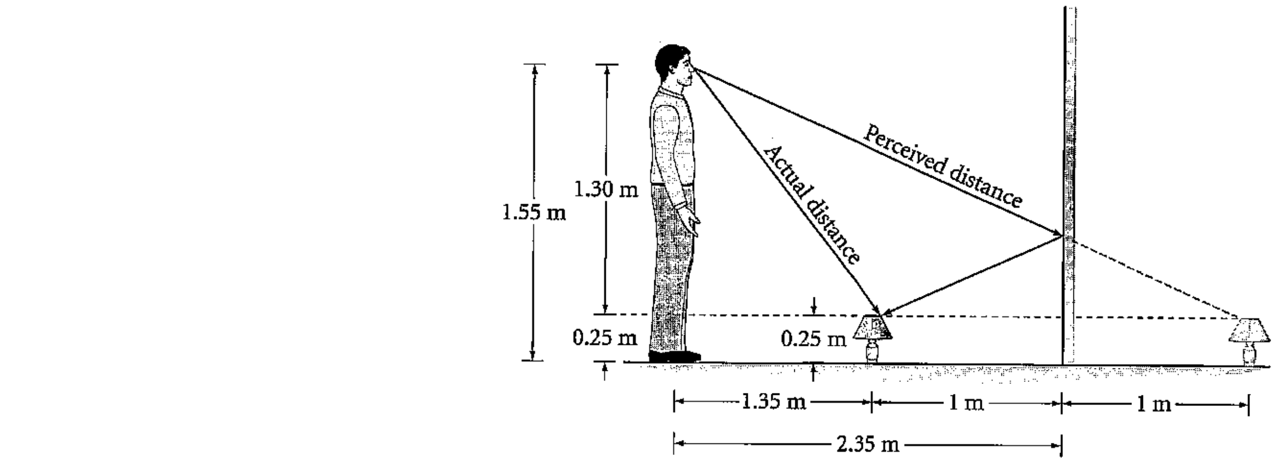
10. **(B) is correct.** By convention, the image distance is negative when on the same side from which light is approaching, and positive otherwise.

Free-Response Questions

1. Here is a basic diagram (not to scale):



The fact that the angle of incidence and the angle of reflection are the same can be utilized to form geometric relations to solve for the value sought.



From the Pythagorean theorem in the figure above,
Perceived distance $= \sqrt{(3.35\text{ m})^2 + (1.30\text{ m})^2} = 3.59$ m.

The actual distance is given, from the Pythagorean theorem, by the relation $x = \sqrt{(1.35\text{ m})^2 + (1.3\text{ m})^2} = 1.87$ m.

This response correctly depicts reflective optical phenomenon based on the provided evidence. From the diagram, the appropriate geometrical and optical relationships are successfully developed and applied. Further, the distinction between apparent and actual distance is demonstrated using the Pythagorean theorem, although an equivalent trigonometric explanation would be equally acceptable.

2. (a) For combinations of lenses, the image of the first lens can be considered the object for the second lens. The distance of the image formed for the first lens can be found from the lens equation $1/d_o + 1/d_i = 1/f$, so $1/12$ cm $+ 1/d_i = 1/8$ cm, so $d_i = 24$ cm. The height would be given by $h_i/h_o = -d_i/d_o$, so $h_i = h_o(-d_i/d_o) = 6$ cm$(-24$ cm$/12$ cm$) = -12$ cm. The sign indicates that the image is inverted. For the second lens, we again use $1/d_o + 1/d_i = 1/f$, with the new object distance, 28 cm $-$ 24 cm $=$ 4 cm. Using $1/d_o + 1/d_i = 1/f$, $1/4$ cm $+ 1/d_i = 1/8$ cm, so $d_i = -8$ cm. The image distance will be along the axis between the two lenses, 8 cm from the second lens and 20 cm from the first lens. Therefore, the final image is 32 cm from the original object.

   (b) The height of the image of the first lens will be the height of the object for the second lens, so $h_i/h_o = -d_i/d_o$, so $h_i = h_o(-d_i/d_o) = -12$ cm. For the second lens, $h_i = h_o(-d_i/d_o) = -12$ cm $(-(-8)$ cm$/4$ cm$) = -24$ cm and the image will be inverted from the initial object.

   (c) Because image distance is positive for a real image and negative for a virtual image, a virtual image will be produced by the second lens. This image could not be seen on a screen placed at the image location.

This response demonstrates an understanding of the lens equations, as well as of the method for solving a combination of lenses question. The responses to parts a and b apply this method. The response to part b demonstrates an understanding of the sign conventions for upright and inverted images. The response to part c shows an understanding of the sign conventions for image distance and for real and virtual images which justifies the answer.