CHAPTER 24

The Wave Nature of Light

Debate over whether light is composed of particles or waves has stretched over several centuries of physics research. Much evidence favors the wave theory of light.

Waves Versus Particles; Huygens' Principle and Diffraction

- **Huygens' principle** considers light in an isotropic medium to exhibit wave properties.
- Every point on a light's wave front is itself the source of waves moving with the same speed, thereby creating a new wave front. This explains **diffraction,** the behavior of light bending around obstacles. The ray model of light does not account for this phenomenon.

Huygens' Principle and the Law of Refraction

Snell's law and the law of refraction as explained by the ray model of light are consistent with the wave theory of light.

- As a light wave changes media, its velocity and its wavelength will change. The wavelength in each medium depends on the index of refraction, such that $\lambda_2/\lambda_1 = n_1/n_2$. The frequency of the light wave remains the same in the two media.
- Mirages are caused by the bending of light rays resulting from the differing indices of refraction of air at different temperatures.

Interference—Young's Double-Slit Experiment

Further evidence for the wave theory comes from the behavior of **monochromatic** light—that is, light of a single wavelength directed at two narrow parallel slits in an opaque screen.

- The particle theory of light predicts that only two bright lines will appear on another screen placed on the other side of the slit-screen from the light source. However, experimental observations show a series of bright and dark parallel lines as predicted by the wave theory of light. This is an example of **wave-interference,** in which the bright lines result from **constructive interference** of rays whose paths from the two slits differ by whole number multiples of the wavelength.
- When the two path lengths differ by half a wavelength or by multiples given by $(2n - 1)\lambda/2$, dark lines resulting from **destructive interference** appear.

- The lines of this alternating pattern are called **fringes.**
- When the distance between slits, d, is much smaller than their distance from the viewing screen, constructive interference will occur at $d \sin \theta = m\lambda$, and destructive interference will occur at $d \sin \theta = (m + 1/2)\lambda$ for $m = 0, 1, 2. \ldots$
- Each interference fringe is identified by its value of m, or its **order.** The angular spacing of the fringe pattern is a function of the wavelength and the separation of the two slits.
- The slits are **coherent sources,** because the waves that pass through have the same phase relationship to each other at all times. Two separate light sources placed at the slits would not produce a fringe pattern on the screen, because they are **incoherent sources** that do not have a fixed phase relationship.

The Visible Spectrum and Dispersion

The wave theory of light retains the definition of intensity from general wave theory as energy per unit time. The wave theory defines color in terms of a wave's frequency or wavelength, since either is sufficient for uniqueness based on the equation $f\lambda = c$.

- Waves whose frequencies are slightly less than those of the visible spectrum are called **infrared,** whereas waves with slightly greater frequencies are called **ultraviolet.**
- **Dispersion** refers to refracting the many wavelengths present in white light at slightly different angles to create the full spectrum of colors. This is often observed when white light passes through prisms, water drops, and diamonds.

Diffraction by a Single Slit or Disk

The wave theory of light predicts that a point source of light pointed at an opaque disc or sphere produces a bright spot at the center of the shadow due to constructive interference of the diffracted light. It is called Poisson's spot, and it can be experimentally observed under carefully controlled conditions.

- A **diffraction pattern** is produced by a point source of light directed at a sharply defined object.
- Monochromatic light passing through a single slit undergoes destructive interference, and the angle θ to the first dark fringe on either side of the center is given by $\sin \theta = \lambda / D$, where D is the width of slit.
- Additional dark fringes due to destructive interference appear at angles given by $D \sin \theta = m\lambda$, for $m = 1, 2, 3. \ldots$

Diffraction Grating

A **diffraction grating** or **transmission grating** consists of many parallel, regularly spaced slits that allow light from a point source to pass through and produce a series of alternating bright and dark lines due to interference similar to the double-slit experiment.

- The angle for maxima is determined by the same equation used for the two-slit pattern, $\sin \theta = m\lambda / d$, for $m = 0, 1, 2. \ldots$, where d is the distance between adjacent slits.

- When compared with the fringe pattern produced by two slits, the maxima produced by the diffraction grating are brighter and narrower, allowing for more accurate wavelength measurement.
- Light of more than one wavelength passing through a diffraction grating will produce separate (and sometimes overlapping) patterns for the different wavelengths. Only along the center line (at $m = 0$) will all of the patterns have a common maximum (bright fringe).
- White light passing through a diffraction grating will show the patterns for all visible wavelengths, which produces a continuous **spectrum.**

The Spectrometer and Spectroscopy

Diffraction gratings are the principle components of **spectrometers,** which indicate the wavelengths of light sources by allowing precise measurement of the angle at which diffraction peaks are produced.

- Wavelengths are given by $\lambda = (d/m) \sin \theta$, where m is the order and d is the distance between slits in the diffraction grating. Spectrometers can be used for identifying atoms or molecules in gases and solids because the wavelengths which determine the spacing of the maxima are related to the atomic energy levels.
- When heated or when a large current is applied, a rarefied gas produces a **line spectrum** composed of a unique pattern of certain discrete wavelengths.
- When heated, a solid emits many wavelengths in a **continuous spectrum.**
- Dark lines on a continuous spectrum are called **absorption lines,** corresponding to wavelengths which are absorbed by specific atoms and molecules between the source and the spectrometer. Absorption spectra are often used in astronomy and biology.

Interference by Thin Films

The constructive interference of light waves from the two surfaces of a thin film produces the colors visible on soap bubbles and on thin films of oil illuminated by white light. A colorful region appears where light of that color is reinforced by constructive interference (but light of other colors is not).

- Light that hits the upper surface is partially reflected and partially transmitted through the medium to be reflected later by the inner surface of the medium.
- The wavelength of the light and the thickness of the medium determine where the interference of the light reflected from the upper and lower surfaces is constructive or destructive.

Polarization

Plane-polarization describes the situation in which the oscillations of a transverse electromagnetic wave are in a single plane. A plane-polarized transverse wave cannot travel through a slit perpendicular to its plane of oscillation. If a source emits light with vibrations in many planes, it is **unpolarized.**

- The intensity of a plane-polarized electromagnetic wave that travels through a polarizer is a function of the initial intensity, I_0, and the angle between the initial plane of polarization and the polarizer, θ, such that $I = I_0 \cos^2 \theta$.

- Unpolarized light can be made plane-polarized with a **Polaroid sheet** (also called a **polarizing filter**). A Polaroid sheet can be rotated to determine if light is polarized and, if so, to find its plane of polarization. Two Polaroid sheets with perpendicular planes of polarization will not allow any light to pass through.

For Additional Review

Consider how an astronomer studying a distant star can determine whether the absorption lines on a continuous spectrum recorded by a spectrometer represent wavelengths of elements present in the outer layer of the star or of elements present in the Earth's atmosphere.

Multiple-Choice Questions

1. What is the frequency of a violet light that has a wavelength of 4.3×10^{-7} m?
   (A) 5.6×10^{11} Hz
   (B) 3.1×10^{12} Hz
   (C) 6.6×10^{12} Hz
   (D) 5.6×10^{13} Hz
   (E) 7.0×10^{14} Hz

2. What is the percentage of error if a 104.5 MHz FM radio signal is measured to have a 3.06-meter wavelength?
   (A) 1.5%
   (B) 4.6%
   (C) 6.7%
   (D) 12%
   (E) 16%

3. Which of the following are accounted for by both the wave theory of light and the particle theory of light?
   I. Diffraction
   II. Refraction
   III. Double slit interference patterns
   (A) I only
   (B) II only
   (C) III only
   (D) I and III
   (E) I, II, and III

4. Monochromatic light from a point source projected on an opaque screen with two parallel slits spaced 2.5×10^{-5} meters apart has a first order bright fringe that is 1.7° from the center. What is its wavelength?
   (A) 530 nm
   (B) 610 nm
   (C) 640 nm
   (D) 690 nm
   (E) 740 nm

5. Sources of radiation whose emitted waves have the same phase relationship at all times are described as
   (A) continuous
   (B) coherent
   (C) dispersed
   (D) interfered
   (E) monochromatic

6. Which of the following is NOT an advantage of using a diffraction grating over a double slit for monochromatic light?
   (A) The maxima are brighter.
   (B) The maxima are narrower.
   (C) More precise wavelength measurements can be obtained.
   (D) A continuous spectrum is always produced.
   (E) More destructive interference is produced.

7. The spectrum produced by a prism when white light travels through it is caused by
   (A) making the ultraviolet and infrared wavelengths visible
   (B) the bending of each wavelength of visible light through a slightly different angle
   (C) the diffraction of light by an obstacle
   (D) a total internal reflection within the prism
   (E) none of the above

8. What is the distance between adjacent slits of a diffraction grating if the third order bright fringe for light with a 450 nm wavelength has an angle of 11.5° from the centerline?
   (A) 9.0×10^{-5} m
   (B) 1.4×10^{-6} m
   (C) 6.8×10^{-6} m
   (D) 2.7×10^{-7} m
   (E) 3.0×10^{-8} m

9. In terms of wavelength, how far apart are the path length differences for adjacent constructive and destructive interference fringes in the double-slit experiment?
   (A) $\lambda/2$
   (B) 2λ
   (C) λ
   (D) $4\lambda/3$
   (E) $3\lambda/2$

10. A light of wavelength 640 nm in air enters a new medium. If its wavelength in the new medium is 375 nm, the index of refraction is
    (A) 0.59
    (B) 1.0
    (C) 1.4
    (D) 1.7
    (E) Not enough information is provided to determine the index of refraction.

Free-Response Questions

1. What will be the angle and distance between the first- and second-order bright fringes when monochromatic light of wavelength 630 nm is projected onto an opaque screen with two parallel slits spaced 3.15×10^{-6} meters apart and onto a viewing screen that is 2.00 meters away?

2. (a) Determine how far offset the angles of second-order fringes are for two wavelengths of light—one of 550 nm and one of 680 nm—that pass through a diffraction grating which has 900 slits per centimeter.
   (b) State two sets of orders at which the maxima of these wavelengths coincide, and show your method.

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. (E) is correct. The relationship between frequency and wavelength for an electromagnetic wave, which includes light, is given by $f\lambda = c$, therefore $f = c/\lambda = (3.00 \times 10^8 \text{ m/s})/(4.2 \times 10^{-7} \text{ m}) = 7.0 \times 10^{14}$ Hz.

2. (C) is correct. The product of wavelength and frequency should be equal to $c = 3.00 \times 10^8$ m, because for an electromagnetic wave $f\lambda = c$.
Here, $(104.5 \times 10^6 \text{ s}^{-1})(3.06 \text{ m}) = 3.20 \times 10^8$ m.
The percentage of error is $0.20/3.00 = 0.067$ or 6.7%.

3. **(B) is correct.** Wave theory could not account for diffraction or the interference patterns of Young's experiments. Both wave theory and ray theory account for refraction, and are consistent with Snell's law.
4. **(E) is correct.** The relationship $\sin \theta_1 = m\lambda / d$ can be rearranged in terms of wavelength, so $\lambda = d \sin \theta_1 / m = (2.5 \times 10^{-5}\text{ m}) \sin (1.7°)/(1)$ $= 7.4 \times 10^{-7}\text{ m} = 740\text{ nm}$.
5. **(B) is correct.** By definition, coherent sources are those whose emitted waves have the same phase relationship at all times. Other sources are described as incoherent.
6. **(D) is correct.** The maxima produced by a diffraction grating are both narrower and brighter, due to more destructive interference being produced, allowing for more accurate measurement of wavelengths, so answers *A*, *B*, *C*, and *E* are true and interrelated. A continuous spectrum will not be produced by a monochromatic light source.
7. **(B) is correct.** A prism bends each of the wavelengths that compose white light such that the individual wavelengths of visible light are separated into the continuous spectrum of visible colors.
8. **(C) is correct.** The equation for a diffraction grating is given by $\sin \theta = m\lambda / D$. Therefore, the distance between slits $D = m\lambda / \sin \theta$ $= (3)(4.5 \times 10^{-7}\text{ m})/\sin (11.5°) = 6.8 \times 10^{-6}\text{ m}$.
9. **(A) is correct.** The equation for constructive interference is given by $d \sin \theta = m\lambda$, whereas destructive interference is given by $d \sin \theta = [m + (1/2)]\lambda$ and are thus offset by $\lambda / 2$.
10. **(D) is correct.** A consequence of the law of refraction, as confirmed by the wave theory of light, is that $\lambda_2/\lambda_1 = n_1/n_2$. Recall the index of refraction of air is 1. So $n_2 = n_1 \lambda_1/\lambda_2 = (1)(6.40 \times 10^{-7}\text{ m})/(3.75 \times 10^{-7}\text{ m}) = 1.7$.

Free-Response Questions

1. The angle would be given by $\sin \theta = m\lambda / d$, where m is the order.
For the first order, the angle is given by $\theta_1 = \sin^{-1}(m\lambda / d)$
$= \sin^{-1}[(1)(6.30 \times 10^{-7}\text{ m})/(3.15 \times 10^{-6}\text{ m})] = 11.5°$.
For the second order, the angle is given by $\theta_2 = \sin^{-1}(m\lambda / d)$
$= \sin^{-1}[(2)(6.30 \times 10^{-7}\text{ m})/(3.15 \times 10^{-6}\text{ m})] = 23.6°$.
Therefore, the angular separation of these two fringes is $23.6° - 11.5° = 12.1°$. Using trigonometry, the distance between the fringes can be found, since $\tan \theta = (\text{distance from center})/(\text{distance from screen})$. Therefore, $d_c = d_s \tan \theta_1 = (2.00\text{ m}) \tan 11.5° = .407\text{ m}$, and $d_c = d_s \tan \theta_2$ $= (2.00\text{ m}) \tan 23.6° = .874\text{ m}$, so the distance between the fringes is $0.874\text{ m} - 0.407\text{ m} = 0.467\text{ m}$.

This response correctly rearranges the equation for maxima to solve for angle, and it correctly converts units of nanometers to meters. After these angles are found, the trigonometric definition of tangent is introduced to convert the angle measure to its projection onto a screen in terms of the screen's distance.

2. (a) The relationship between angle, order, wavelength, and distance between diffraction grating slits is given by $\sin \theta = m\lambda / D$. A grating with 900 slits/cm = 90,000 slits/m, are spaced 1.1×10^{-5} meters apart. For the 550 nm wavelength of light the second order spectral line is given by
$\theta = \sin^{-1}(m\lambda / d) = \sin^{-1}(2)(5.5 \times 10^{-7}\ \text{m})/1.1 \times 10^{-5}\ \text{m} = 5.7°$.
For the 680 nm wavelength of light the second order spectral line is given by
$\theta = \sin^{-1}(m\lambda / d) = \sin^{-1}(2)(6.8 \times 10^{-7}\ \text{m})/1.1 \times 10^{-5}\ \text{m} = 7.1°$.
The angular difference is 1.4°.

(b) As with any wavelengths, the principle maxima at $m = 0$ coincide. By constructing a table with the equations for each, it would also occur to two significant figures when the fourth order of the 550 nm wavelength and the third order of the 680 nm wavelength coincide or when the fifth order of the 550 nm wavelength and the fourth order of the 680 nm wavelength coincide.

m	$\sin^{-1}[(m)(5.5 \times 10^{-7}\ \text{m})/(1.1 \times 10^{-5}\ \text{m})]$	$\sin^{-1}[(m)(6.8 \times 10^{-7}\ \text{m})/(1.1 \times 10^{-5}\ \text{m})]$
1	2.9°	3.5°
2	5.7°	7.1°
3	8.6°	11°
4	11°	14°
5	14°	18°

This response correctly uses the equation for diffraction gratings and shows an understanding of how spectral lines may coincide. The response to part a correctly demonstrates how to convert a description of a grating in terms of slits per unit length into distance between slits; the student then applies this to a rearranged version of the equation $\sin \theta = m\lambda / D$. *The response to part b demonstrates one method for determining at what orders they interact. Other methods include finding the integers A and B for which* $(5.5 \times 10^{-7}\ \text{m})A = (6.8 \times 10^{-7}\ \text{m})B$, *and seeing if those values produce sensible values for* θ. *Here, three values (including* $m = 0$, *which is a valid response) are actually listed.*