CHAPTER 2

Describing Motion: Kinematics in One Dimension

The field of **mechanics** encompasses **kinematics,** which describes the motion of objects, and **dynamics,** which details the forces and energy underlying motion. **Translational motion** refers to motion without rotation. Strictly linear motion can be viewed in one dimension.

Reference Frames and Displacement

A **frame of reference** describes the orientation of an object's motion with regard to its position, distance, speed, and direction.

- The reference frame is used to anchor **coordinate axes,** which place these values in a dimensional framework. The number of axes corresponds to the number of dimensions necessary to express information regarding motion, and these axes are successively named by letter: x, y, z. For one-dimensional motion, an x axis is sufficient to express the position of an object.
- The change in an object's position is referred to as its **displacement.** Thus, distance traveled, which refers to the total measure of an object's path in transit from its starting to finishing positions, differs from displacement, which refers strictly to the difference in the starting and finishing positions.
- A **vector** is a quantity that has both a magnitude and a direction. In one dimension, the sign of a vector can be used to indicate its direction.

Average Velocity and Instantaneous Velocity

- **Average speed** denotes the ratio of the distance traveled by an object to the corresponding time interval; average speed is a scalar quantity.
- **Average velocity** denotes the ratio of the displacement of an object to the corresponding time interval; average velocity is a vector quantity, because it refers to the magnitude and direction of the object's motion, $v_{avg} = \Delta x/\Delta t$. In one-dimensional motion, the sign of the velocity vector indicates its direction within its defined frame of reference.
- **Instantaneous velocity** is the velocity at a precise instant of time, equivalent to the limit of the average velocity as the time interval approaches zero,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}.$$

Acceleration

Acceleration quantifies an object's change in velocity.

- **Average acceleration** is the ratio of the change in velocity to the time interval over which the velocity change occurs. Because velocity is a vector, acceleration is a vector. In one-dimensional motion, the sign of the acceleration vector indicates its direction within its defined frame of reference.
- **Instantaneous acceleration** is the acceleration at a precise instant of time, equivalent to the limit of the average acceleration as the time interval approaches zero,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}.$$

- **Deceleration** refers to an acceleration vector in the opposite direction from the velocity vector.

Motion at Constant Acceleration

For an object moving at a constant acceleration, called **uniformly accelerated motion,** there are many relatively simple equations relating time, velocity, and/or displacement:

- An object's velocity is a function of its acceleration, initial velocity and time, $v = v_0 + at$.
- The average velocity is given by the average of the initial and final velocities, $v_{avg} = (v_0 + v)/2$.
- An object's final position is a function of its acceleration, initial velocity, initial position and time, $x = x_0 + v_0 t + 1/2(at^2)$.
- An object's velocity is a function of its acceleration, initial velocity, and initial and final positions, $v = v_0^2 + 2a(x - x_0)$.

Falling Objects

All objects falling vertically experience uniform acceleration at the rate of 9.8 m/s^2 near the Earth's surface assuming negligible air resistance.

- The distance an object travels is directly proportional to the square of time from when the initial velocity is zero. The rate of change in velocity is referred to as **acceleration due to gravity.** Free-falling objects experiencing this acceleration, symbolized by g, trace a path given by kinematics equations where $a = g$.
- The sign of the acceleration must be consistent with the signs of velocity and displacement for the object's motion.

Graphical Analysis of Linear Motion

Linear motion can be represented in two dimensions as a graph whose axes represent time and position or time and velocity.

- On both graphical schemes, time is represented on the horizontal, and the value expressed on the vertical is a function of time. As such, time is the independent variable, and position or velocity is the dependent variable.

- The quotient of the change in the dependent variable divided by the change in the independent variable is **slope.** The slope of a position vs. time graph at a time, *t*, indicates the velocity at that time. Slope can be calculated easily for a linear portion of a graph, but the slope for a curved portion of the graph is given by the slope of a tangent line at that point.
- Displacement over a time interval can be determined from a velocity vs. time graph by calculating the area under a curve during that time interval. If the curve consists of a series of linear segments, the area under the curve can be determined by summing the rectangles and triangles that make up that area.
- For other graphs, finding the precise area under a curve requires calculus. It can be estimated by summing the areas of thin rectangles that make up the area under a curve, where the accuracy depends on the width of the rectangles.

For Additional Review

Compare the way an acceleration vs. time graph can provide information about the change in velocity and the way a velocity vs. time graph can provide information about acceleration.

Multiple-Choice Questions

1. Which of the following values are vectors?
   I. Instantaneous velocity
   II. Distance traveled
   III. Average acceleration
   (A) II only
   (B) I and II
   (C) II and III
   (D) I and III
   (E) I, II, and III

2. For an object that travels 20 kilometers north and then 15 kilometers south, what is the ratio of the distance traveled to the displacement?
   (A) 0
   (B) 1/7
   (C) 1
   (D) 7
   (E) 35

3. For an object that travels 60 meters east and then 60 meters west in fifteen minutes, what is the ratio of average velocity to average speed?
   (A) 0
   (B) 1
   (C) 4/3
   (D) 120
   (E) Undefined

4. What is the final velocity of an object constantly accelerating from rest at 14.5 m/s^2 over 35 meters?
   (A) 3.1 m/s
   (B) 9.8 m/s
   (C) 17 m/s
   (D) 27 m/s
   (E) 32 m/s

5. What is the average velocity of an object that travels for 20 seconds with a starting velocity of 7 m/s with a constant acceleration of 25 m/s^2?
   (A) 165 m/s
   (B) 257 m/s
   (C) 500 m/s
   (D) 507 m/s
   (E) 514 m/s

6. How long does it take a car accelerating constantly at 4 m/s^2 from rest to travel 200 meters?
   (A) 4 seconds
   (B) 7 seconds
   (C) 10 seconds
   (D) 14 seconds
   (E) 33 seconds

7. What is the minimum positive initial velocity necessary for an object to be launched from ground level so that it reaches a height of 80 meters?
   (A) 5 m/s
   (B) 9.80 m/s
   (C) 12.5 m/s
   (D) 40 m/s
   (E) 80 m/s

8. Assuming negligible air resistance, how long will it take for a sphere to fall 36.0 meters if thrown downward at 12.0 m/s?
   (A) 1.75 s
   (B) 3.40 s
   (C) 5.15 s
   (D) 9.80 s
   (E) 15.0 s

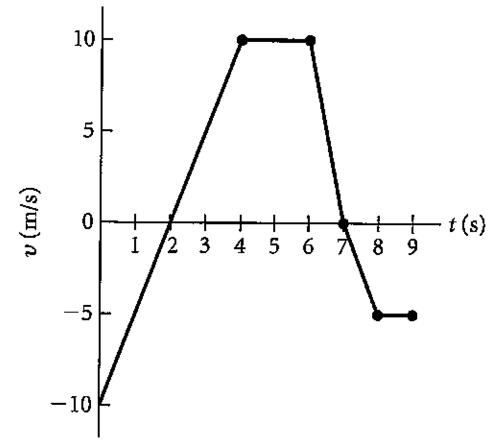
9. What is the magnitude of velocity of an object after free falling from rest after 3 minutes?
   (A) 9.8 m/s
   (B) 32 m/s
   (C) 87 m/s
   (D) 180 m/s
   (E) 1800 m/s

10. What is the magnitude of acceleration after 1.5 seconds of a sphere thrown upward from a height of 13 meters at 20 m/s²?
   (A) 5.3 m/s^2
   (B) 9.8 m/s^2
   (C) 15 m/s^2
   (D) 32 m/s^2
   (E) 390 m/s^2

Free-Response Questions

1. The influence of gravity on the motion of objects is predictable under ideal conditions.
   (a) At what velocity would a ball have to be thrown vertically from ground level so it remains aloft for exactly 3 minutes?
   (b) How could a precise stopwatch and a ball be used to measure the depth of a canyon?
   (c) How many seconds apart should two balls be released so that a ball being dropped from a height of H reaches the ground at the same time as a ball dropped from a height of 5 H?

2. Use the velocity vs. time graph below to answer the following questions.



   (a) Find the displacement between $t = 0$ s and $t = 3$ s.
   (b) Find the displacement between $t = 3$ s and $t = 7$ s.
   (c) Find the displacement between $t = 0$ s and $t = 9$ s.
   (d) Add to the velocity vs. time graph from $t = 9$ to $t = 13$ so that there is no net displacement from $t = 0$ to $t = 13$.

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. **(D) is correct.** A vector expresses both magnitude and direction. Displacement is a vector, whereas distance traveled is a scalar. Both instantaneous velocity and average acceleration are vectors.

2. **(D) is correct.** The distance traveled is 20 km + 15 km = 35 km, whereas the displacement is given by 20 km − 15 km = 5 km. The ratio of distance traveled to displacement 35 km/5 km = 7.

3. **(A) is correct.** The average velocity is given by the change in position over the change in time, $v = \Delta x/\Delta t = (60\text{ m} - 60\text{ m})/900\text{ s} = 0\text{ m/s}$, while average speed is given by distance traveled over change in time 120 m/900 s = 1.3 m/s—so the ratio is (0 m/s)/(1.3 m/s) = 0.

4. **(E) is correct.** The correct kinematics equation for the information given is $v^2 = v_0^2 + 2a(x - x_0)$, where $v_0 = 0$ m/s, $x_0 = 0$ m, $x = 35$ m and $a = 14.5\text{ m/s}^2$. $v^2 = 2(14.5\text{ m/s}^2)(35\text{ m}) = 1015\text{ m}^2/\text{s}^2$, so $v = \sqrt{1015\text{ m}^2/\text{s}^2} = 32\text{ m/s}$.

5. **(B) is correct.** Average velocity for a constantly accelerating object is the average of the starting velocity, which is given, and the final velocity, which is given by $v = v_0 + at = 7\text{ m/s} + (25\text{ m/s}^2)(20\text{ s}) = 507\text{ m/s}$. Therefore average velocity, $v_{avg} = (v + v_0)/2 = (507\text{ m/s} + 7\text{ m/s})/2 = 257\text{ m/s}$.

6. **(C) is correct.** The relevant kinematics relationship for the information given is $x = x_0 + v_0t + 1/2(at^2)$, where $x = 200$ m, $x_0 = 0$ m, $v_0 = 0$ m/s, $a = 4\text{ m/s}^2$, so $200 = 1/2(4\text{ m/s}^2)t^2$, $100 = t^2$, and $t = 10$ s.

7. **(D) is correct.** The minimum velocity necessary would be enough to overcome the acceleration due to gravity such that its maximum displacement is 80 meters; this would mean its velocity at that height would be zero. So $v^2 = v_0{}^2 + 2a(x - x_0) =$ where $v = 0$. $v_0 = \sqrt{[-2a(x - x_0)]}$ $= \sqrt{[19.6\text{ m/s}^2](80\text{ m})} = 40\text{ m/s}$.

8. **(A) is correct.** $x = x_0 + v_0t + 1/2(at^2)$ where $x_0 = 36.0$ meters, $x = 0$, $v_0 = -12.0$ m/s and $a = -9.80\text{ m/s}^2$. $0 = 36 + (-12\text{ m/s})t + 1/2(-9.8\text{ m/s}^2)\,t^2$. By the quadratic equation, where $a = -4.9$, $b = -12$, and $c = 36$,
$t = [12 + \sqrt{(144 - 4(-4.9)(36))}]/2(-4.9)$ or
$t = [12 - \sqrt{(144 - 4(-4.9)(36))}]/2(-4.9)$. Ignore the negative result, $t =$ 1.75 s.

9. **(E) is correct.** The velocity will be given by the equation $v = v_0 + at$, where $a = 9.8\text{ m/s}^2$ and $v_0 = 0$. (Direction is not sought in the problem, so the sign of acceleration is not relevant.) $v = 0 + (9.8\text{ m/s}^2)(180\text{ s}) = 1764\text{ m/s}$, or 1800 m/s to the correct number of significant figures.

10. **(B) is correct.** Acceleration due to gravity is a 9.8 m/s^2 downward for any free-falling body at any point in its path near the Earth's surface. The remaining information presented in the problem is not relevant.

Free-Response Questions

1. (a) The relationship between displacement, velocity, and acceleration is given by $x = x_0 + v_0t + 1/2(at^2)$, where $x = x_0 = 0$, $a = -g$, and $t = 180$ s. So $0 = 0 + v_0\,(180\text{ s}) + 1/2(-9.8\text{ m/s}^2)(180\text{ s})^2$ and $v_0 = 882$ m/s.

(b) A stopwatch can measure how long it takes a ball that is released from rest to hit the canyon floor. Using the relation $x = x_0 + v_0t + 1/2(at^2)$, where $v_0 = 0$, t is the time interval from release to impact, $a = -9.8\text{ m/s}^2$, $x = 0$, and x_0 represents the height from the canyon floor to the ledge above. Note that air resistance has been neglected here. Also, if you are positioned at the top of the canyon, you should watch (not listen) for the ball to hit the bottom, because the velocity of light is much faster than the velocity of sound.

(c) The length of time it will take for a ball released from height 5H to reach the ground can be found from $x = x_0 + v_0t + 1/2(at^2)$, where $x = 0$, $x_0 = 5\text{H}$, $v_0 = 0$, and $a = -9.8\text{ m/s}^2$. $0 = 5\text{H} + 1/2(-9.8t^2)$. So $t_{5\text{H}} = 1.01\sqrt{\text{H}}$ seconds. The length of time it will take for the second ball released from height H to reach the ground can be found from $x = x_0 + v_0t + 1/2(at^2)$, where $x = 0$, $x_0 = \text{H}$, $v_0 = 0$, and $a = -9.8\text{ m/s}^2$. $0 = \text{H} + 1/2(-9.8t^2)$. So $t_\text{H} = 0.45\sqrt{\text{H}}$ seconds. Therefore, the balls should be released $1.01\sqrt{\text{H}} - 0.45\sqrt{\text{H}} = 0.56\sqrt{\text{H}}$ seconds apart.

This response utilizes the same equation that underlies all three parts of the question: $x = x_0 + v_0t + 1/2(at^2)$. For (a), an exact value is obtained for the missing variable, while for (c), the equation is applied to two different situations to solve for an answer in variable terms. For (b), a theoretical application for the equation is presented.

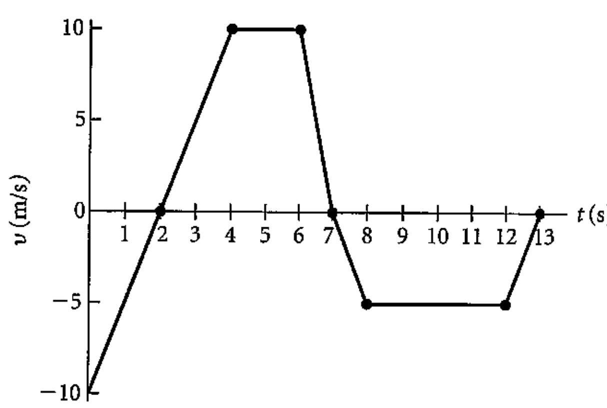
2. For a velocity vs. time graph, displacement is given by the area under the curve.

(a) Between $t = 0$ and $t = 3$, the area can be found by summing the areas of the two triangles, so the displacement is $-10\text{ m} + 2.5\text{ m} = -7.5\text{ m}$.

(b) Between $t = 3$ s and $t = 7$ s, the area can be found by summing the areas of the two triangles and two rectangles under the curve, so the displacement is $7.5\text{ m} + 20\text{ m} + 5\text{ m} = 32.5\text{ m}$.

(c) Between $t = 0$ s and $t = 9$ s, the area can be found by summing (a) + (b) + the area between $t = 7$ s and $t = 9$ s, so the displacement is $-7.5\text{ m} + 32.5\text{ m} + -7.5\text{ m} = 17.5\text{ m}$.

(d) A graph of displacement vs. time that fits the given criteria needs only to have an area of -17.5 m under the curve between $t = 9$ and $t = 13$:



This response uses the same method for parts a, b, and c: The area under a velocity vs. time curve corresponds to displacement. In responses to parts a and c, the student takes into account using the correct sign of displacement for the negative regions of area. The sketch for part d is one possible correct answer, and the response uses the information in part c to guide the solution.