2 Vectors and Projectiles

2-1 Vectors and Scalars

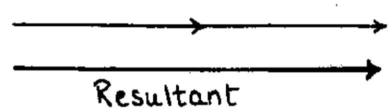
Vocabulary **Vector:** A quantity with magnitude (size) and direction.

Some examples of vectors are displacement, velocity, acceleration, and force.

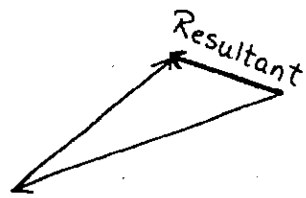
Vocabulary **Scalar:** A quantity with magnitude only.

Some examples of scalars are distance, speed, mass, time, and volume.

Vectors are represented by arrows. They can be added by placing the arrows head to tail. The arrow that extends from the tail of the first vector to the head of the last vector is called the **resultant.** It indicates both the magnitude and direction of the vector sum.

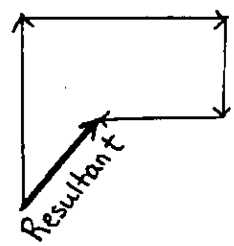


Remember, vectors don't always have to be in a straight line but may be oriented at angles to each other, such as

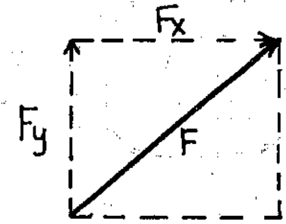


Resultant vectors can be determined by a number of different methods. Here you will solve vector addition exercises both **graphically** and with **vector components.**

Graphical addition of vectors: Using a ruler, draw all vectors to scale and connect them head to tail. The resultant is the vector that connects the tail of the first vector with the head of the last. (Hint: Using graph paper makes this method even easier!)



Vector Components: Because a vector has both magnitude and direction, you can separate it into horizontal (or x) and vertical (or y) components. To do this, draw a rectangle with horizontal and vertical sides and a diagonal equal to the vector. Draw arrow heads on one horizontal and one vertical side to make the original vector the resultant of the horizontal and vertical components.

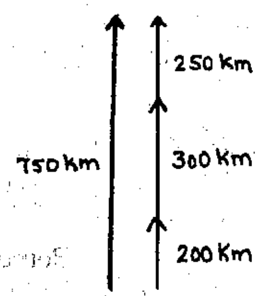


After you have drawn the components, you can find their lengths by using simple trigonometry. If you are not familiar with trigonometry or need a quick refresher, refer to Appendix A.

Solved Examples

Example 1: Every March, the swallows return to San Juan Capistrano, California after their winter in the south. If the swallows fly due north and cover 200 km on the first day, 300 km on the second day, and 250 km on the third day, draw a vector diagram of their trip and find their total displacement for the three-day journey.

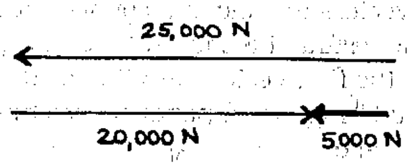
Solution: Because the swallows continue to fly in the same direction throughout the entire trip, these vectors simply add together. This can be shown by placing the displacement vectors head to tail.



200 km + 300 km + 250 km = **750 km north**

Example 2: In the record books, there are men who claim that they have such strong teeth that they can even use them to move cars, trains, and helicopters. Joe Ponder of Love Valley, North Carolina is one such man. Suppose a car pulling forward with a force of 20 000 N was pulled back by a rope that Joe held in his teeth. Joe pulled the car with a force of 25 000 N. Draw a vector diagram of the situation and find the resultant force.

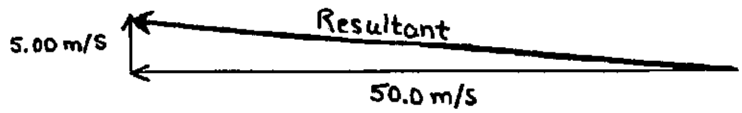
Solution: In this exercise, the vectors are pointing in opposite directions, so the situation would look like this.



25 000 N − 20 000 N = **5000 N in the direction Joe is pulling.** Strong teeth!

Example 3: If St. Louis Cardinals homerun king, Mark McGwire, hit a baseball due west with a speed of 50.0 m/s, and the ball encountered a wind that blew it north at 5.00 m/s, what was the resultant velocity of the baseball?

Solution: Begin by drawing a vector diagram of the situation.



Solve using the Pythagorean theorem:

$$a^2 + b^2 = c^2$$
$$(50.0 \text{ m/s})^2 + (5.00 \text{ m/s})^2 = c^2$$
$$c = \sqrt{2500 \text{ m}^2/\text{s}^2 + 25.0 \text{ m}^2/\text{s}^2} = \textbf{50.2 m/s toward the northwest}$$

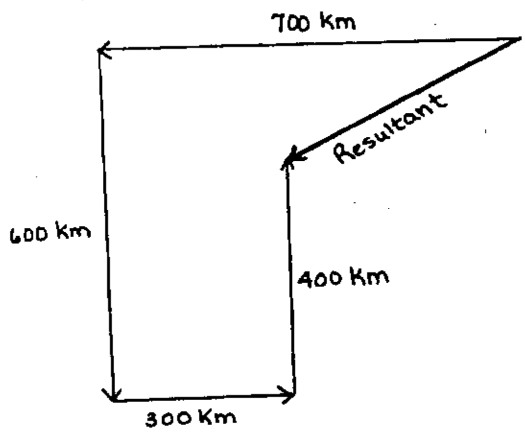
For those of you who understand trigonometry, you can find the exact angle at which the ball travels by saying:

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{5.00 \text{ m/s}}{50.0 \text{ m/s}} = 0.100 \qquad \text{so } \tan^{-1} 0.100 = \mathbf{5.71°}$$

However, don't worry. If you are not familiar with trigonometry, you can simply write the answer as 50.2 m/s to the north of west. For a brief review of trigonometry, see Appendix A.

Example 4: The Maton family begins a vacation trip by driving 700 km west. Then the family drives 600 km south, 300 km east, and 400 north. Where will the Matons end up in relation to their starting point? Solve graphically.

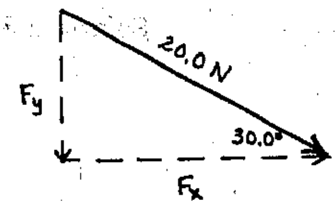
Solution: First, draw the appropriate diagram to scale using a relationship such as 1 cm = 1 km, and you will see a space remaining between where the Matons began their trip and where they ended. Because you are solving this exercise graphically, measure with a ruler the length of the remaining space and convert your measurement back into km. This is the resultant displacement. (Hint: You may find it easier to use graph paper for your drawing so that you can have 1 km equal to a certain number of squares.)



Answer is **450 km**, as measured with a ruler.

Example 5: Ralph is mowing the back yard with a push mower that he pushes downward with a force of 20.0 N at an angle of 30.0° to the horizontal. What are the horizontal and vertical components of the force exerted by Ralph?

Solution: Begin solving by drawing a diagram of the situation, labeling the horizontal and vertical components of the force.



Horizontal component: The hypotenuse in this exercise is the 20.0-N force. The horizontal component is the one going in the x direction. This is the side adjacent to the 30.0° angle so you use the equation for the cosine of an angle.

$$\cos\theta = \frac{F_x}{F} \qquad F_x = F\cos\theta = (20.0\text{ N})\cos 30.0° = \mathbf{17.3\ N}$$

Vertical component: Again, the 20.0-N force is the hypotenuse of the triangle. The vertical component is the one going in the y direction. This is the side opposite the 30.0° angle so you use the equation for the sine of an angle.

$$\sin\theta = \frac{F_y}{F} \qquad F_y = F\sin\theta = (20.0\text{ N})\sin 30.0° = \mathbf{10.0\ N}$$

Practice Exercises

Exercise 1: Some Antarctic explorers heading due south toward the pole travel 50. km during the first day. A sudden snow storm slows their progress and they move only 30. km on the second day. With plenty of rest they travel the final 65 km the last day and reach the pole. What was the explorers' displacement?

Answer: ____________________

Exercise 2: Erica and Tory are out fishing on the lake on a hot summer day when they both decide to go for a swim. Erica dives off the front of the boat with a force of 45 N, while Tory dives off the back with a force of 60. N. a) Draw a vector diagram of the situation. b) Find the resultant force on the boat.

Answer: b. ____________________

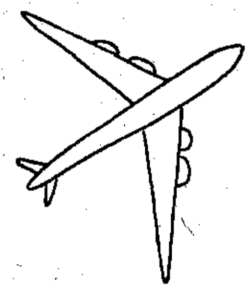
Exercise 3: Young thoroughbreds are sometimes reluctant to enter the starting gate for their first race. Astro Turf is one such horse, and it takes two strong men to get him set for the race. Derek pulls Astro Turf's bridle from the front with a force of 200. N and Dan pushes him from behind with a force of 150. N, while the horse pushes back against the ground with a force of 300. N. a) Draw a vector diagram of the situation. b) What is the resultant force on Astro Turf?

Answer: b. ____________________

Exercise 4: Shareen finds that when she drives her motorboat upstream she can travel with a speed of only 8 m/s, while she moves with a speed of 12 m/s when she heads downstream. What is the current of the river on which Shareen is traveling?

Answer: ____________________

Exercise 5: Rochelle is flying to New York for her big Broadway debut. If the plane heads out of Los Angeles with a velocity of 220. m/s in a northeast direction, relative to the ground, and encounters a wind blowing head-on at 45 m/s, what is the resultant velocity of the plane, relative to the ground?



Answer: ______________________

Exercise 6: While Dexter is on a camping trip with his boy scout troop, the scout leader hands each boy a compass and map. The directions on Dexter's map read as follows: "Walk 500.0 m north, 200.0 m east, 300.0 m south, and 400.0 m west." If he follows the map, what is Dexter's displacement? Solve graphically.

Answer: ______________________

Exercise 7: Amit flies due east from San Francisco to Washington, D.C., a displacement of 5600. km. He then flies from Washington to Boston, a displacement of 900. km at an angle of 55.0° east of north. What is Amit's total displacement?

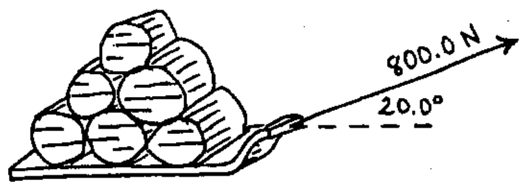
Answer: ______________________

Exercise 8: Marcie shovels snow after a storm by exerting a force of 30.0 N on her shovel at an angle of 60.0° to the vertical. What are the horizontal and vertical components of the force exerted by Marcie?

Answer: ____________________

Answer: ____________________

Exercise 9: Ivan pulls a sled loaded with logs to his cabin in the woods. If Ivan pulls with a force of 800. N in a direction 20.0° above the horizontal, what are the horizontal and vertical components of the force exerted by Ivan?



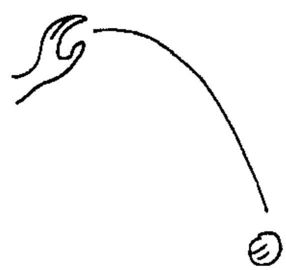
Answer: ____________________

Answer: ____________________

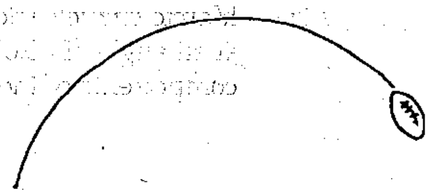
2-2 Projectile Motion

Vocabulary **Projectile:** An object that moves through space acted upon only by the earth's gravity.

A projectile may start at a given height and move toward the ground in an arc. For example, picture the path a rock makes when it is tossed straight out from a cliff.



A projectile may also start at a given level and then move upward and downward again as does a football that has been thrown.



Regardless of its path, a projectile will always follow these rules:

1. Projectiles always maintain a constant horizontal velocity (neglecting air resistance).

2. Projectiles always experience a constant vertical acceleration of 10.0 m/s^2 downward (neglecting air resistance).

3. Horizontal and vertical motion are completely independent of each other. Therefore, the velocity of a projectile can be separated into horizontal and vertical components.

4. For a projectile beginning and ending at the same height, the time it takes to rise to its highest point equals the time it takes to fall from the highest point back to the original position.

5. Objects dropped from a moving vehicle have the same velocity as the moving vehicle.

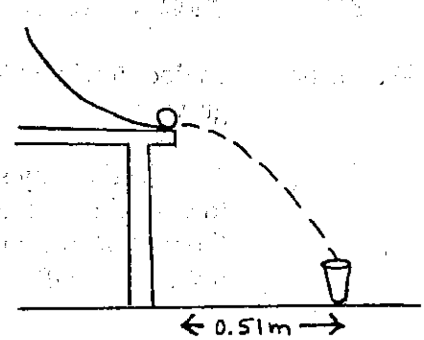
In order to solve projectile exercises, you *must* consider horizontal and vertical motion separately. All of the equations for linear motion in Chapter 1 can be used for projectile motion as well. You don't need to learn any new equations!

To simplify calculations, the term for initial vertical velocity, v_{yo}, will be left out of all equations in which an object is projected horizontally. For example, $\Delta d_y = v_{yo}\Delta t + \left(\frac{1}{2}\right)g\Delta t^2$ will be written as $\Delta d_y = \left(\frac{1}{2}\right)g\Delta t^2$.

Solved Examples

Example 6: In her physics lab, Melanie rolls a 10-g marble down a ramp and off the table with a horizontal velocity of 1.2 m/s. The marble falls in a cup placed 0.51 m from the table's edge. How high is the table?

Solution: The first thing you should notice about projectile exercises is that you do not need to consider the mass of the object projected. Remember, if you ignore air resistance, all bodies fall at exactly the same rate regardless of their mass.



Before you can find the height of the table, you must first determine how long the marble is in the air. The horizontal distance traveled equals the constant horizontal velocity times the travel time.

Given: $\Delta d_x = 0.51\text{ m}$
$v_x = 1.2\text{ m/s}$

Unknown: $\Delta t = ?$
Original equation: $v_x = \frac{\Delta d_x}{\Delta t}$

Solve: $\Delta t = \frac{\Delta d_x}{v_x} = \frac{0.51\text{ m}}{1.2\text{ m/s}} = 0.43\text{ s}$

Now that you know the time the marble takes to fall, you can find the vertical distance it traveled.

Given: $g = 10.0\text{ m/s}^2$
$\Delta t = 0.43\text{ s}$

Unknown: $\Delta d_y = ?$
Original equation: $\Delta d_y = \left(\frac{1}{2}\right)g\Delta t^2$

Solve: $\Delta d_y = \left(\frac{1}{2}\right)(10.0\text{ m/s}^2)(0.43\text{ s})^2 = \mathbf{0.92\text{ m}}$

Example 7: Bert is standing on a ladder picking apples in his grandfather's orchard. As he pulls each apple off the tree, he tosses it into a basket that sits on the ground 3.0 m below at a horizontal distance of 2.0 m from Bert. How fast must Bert throw the apples (horizontally) in order for them to land in the basket?

Solution: Before you can find the horizontal component of the velocity, you must first find the time that the apple is in the air.

Given: $\Delta d_y = 3.0\text{ m}$
$g = 10.0\text{ m/s}^2$

Unknown: $\Delta t = ?$
Original equation: $\Delta d_y = \left(\frac{1}{2}\right)g\Delta t^2$

Solve: $t = \sqrt{\frac{2\Delta d_y}{g}} = \sqrt{\frac{2(3.0\text{ m})}{10.0\text{ m/s}^2}} = \mathbf{0.77\text{ s}}$

Now that you know the time, you can use it to find the horizontal component of the velocity.

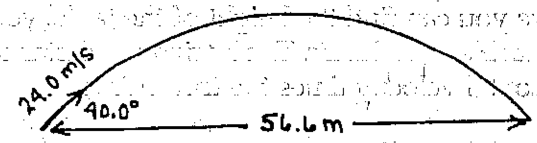
Given: $\Delta d_x = 2.0\text{ m}$
$\Delta t = 0.77\text{ s}$

Unknown: $v_x = ?$
Original equation: $\Delta d_x = v_x \Delta t$

Solve: $v_x = \frac{\Delta d_x}{\Delta t} = \frac{2.0\text{ m}}{0.77\text{ s}} = \mathbf{2.6\text{ m/s}}$

Example 8: Emanuel Zacchini, the famous human cannonball of the Ringling Bros. and Barnum & Bailey Circus, was fired out of a cannon with a speed of 24.0 m/s at an angle of 40.0° to the horizontal. If he landed in a net 56.6 m away at the same height from which he was fired, how long was Zacchini in the air?

Solution: Because Zacchini was in the air for the same amount of time vertically that he was horizontally, you can find his horizontal time and this will be the answer. First, you need the horizontal velocity component.



$$\cos\theta = \frac{v_x}{v} \qquad v_x = v\cos\theta = (24.0\ \text{m/s})\cos 40.0° = 18.4\ \text{m/s}$$

Now you have the horizontal velocity component and the horizontal displacement, so you can find the time.

Given: $v_x = 18.4$ m/s, $\Delta d_x = 56.6$ m

Unknown: $\Delta t = ?$

Original equation: $\Delta d_x = v_x \Delta t$

Solve: $\Delta t = \frac{\Delta d_x}{v_x} = \frac{56.6\ \text{m}}{18.4\ \text{m/s}} = \mathbf{3.08\ s}$

Example 9: On May 20, 1999, 37-year old Robbie Knievel, son of famed daredevil Evel Knievel, successfully jumped 69.5 m over a Grand Canyon gorge. Assuming that he started and landed at the same level and was airborne for 3.66 s, what height from his starting point did this daredevil achieve?

Solution: Because 3.66 s is the time for the entire travel through the air, Robbie spent half of this time reaching the height of the jump. The motorcycle took 1.83 s to go up, and another 1.83 s to come down. To find the height the motorcycle achieved, look only at its downward motion as measured from the highest point.

Given: $\Delta t = 1.83$ s, $g = 10.0\ \text{m/s}^2$

Unknown: $\Delta d_y = ?$

Original equation: $\Delta d_y = \left(\frac{1}{2}\right) g\Delta t^2$

Solve: $\Delta d_y = \left(\frac{1}{2}\right) g\Delta t^2 = \left(\frac{1}{2}\right)(10.0\ \text{m/s}^2)(1.83\ \text{s})^2 = \mathbf{16.7\ m}$

Practice Exercises

Exercise 10: Billy-Joe stands on the Talahatchee Bridge kicking stones into the water below. a) If Billy-Joe kicks a stone with a horizontal velocity of 3.50 m/s, and it lands in the water a horizontal distance of 5.40 m from where Billy-Joe is standing, what is the height of the bridge? b) If the stone had been kicked harder, how would this affect the time it would take to fall?

Answer: a. ____________________

Answer: b. ____________________

Exercise 11: The movie "The Gods Must Be Crazy" begins with a pilot dropping a bottle out of an airplane. It is recovered by a surprised native below, who thinks it is a message from the gods. If the plane from which the bottle was dropped was flying at an altitude of 500. m, and the bottle lands 400. m horizontally from the initial dropping point, how fast was the plane flying when the bottle was released?

Answer: ____________________

Exercise 12: Tad drops a cherry pit out the car window 1.0 m above the ground while traveling down the road at 18 m/s. a) How far, horizontally, from the initial dropping point will the pit hit the ground? b) Draw a picture of the situation. c) If the car continues to travel at the same speed, where will the car be in relation to the pit when it lands?

Answer: **a.** ____________________

Answer: **c.** ____________________

Exercise 13: Ferdinand the frog is hopping from lily pad to lily pad in search of a good fly for lunch. If the lily pads are spaced 2.4 m apart, and Ferdinand jumps with a speed of 5.0 m/s, taking 0.60 s to go from lily pad to lily pad, at what angle must Ferdinand make each of his jumps?

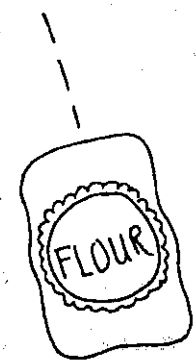
Answer: ____________________

Exercise 14: At her wedding, Jennifer lines up all the single females in a straight line away from her in preparation for the tossing of the bridal bouquet. She stands Kelly at 1.0 m, Kendra at 1.5 m, Mary at 2.0 m, Kristen at 2.5 m, and Lauren at 3.0 m. Jennifer turns around and tosses the bouquet behind her with a speed of 3.9 m/s at an angle of 50.0° to the horizontal, and it is caught at the same height 0.60 s later. a) Who catches the bridal bouquet? b) Who might have caught it if she had thrown it more slowly?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 15: At a meeting of physics teachers in Montana, the teachers were asked to calculate where a flour sack would land if dropped from a moving airplane. The plane would be moving horizontally at a constant speed of 60.0 m/s at an altitude of 300. m. a) If one of the physics teachers neglected air resistance while making his calculation, how far horizontally from the dropping point would he predict the landing? b) Draw a sketch that shows the path the flour sack would take as it falls to the ground (from the perspective of an observer on the ground and off to the side.)



Answer: **a.** ____________________

Exercise 16: Jack be nimble, Jack be quick, Jack jumped over the candlestick with a velocity of 5.0 m/s at an angle of 30.0° to the horizontal. Did Jack burn his feet on the 0.25-m-high candle?

Answer: ____________________

Additional Exercises

A-1: A flock of Canada geese is flying south for the winter. On the first day the geese fly due south a distance of 800. km. On the second day they fly back north 100. km and pause for a couple of days to graze on a sod farm. The last day the geese continue their journey due south, covering a distance of 750. km. a) Draw a vector diagram of the journey and find the total displacement of the geese during this time. b) How does this value differ from the total distance traveled?

A-2: A seal swims toward an inlet with a speed of 5.0 m/s as a current of 1.0 m/s flows in the opposite direction. How long will it take the seal to swim 100. m?

A-3: In Moncton, New Brunswick, each high tide in the Bay of Fundy produces a large surge of water known as a tidal bore. If a riverbed fills with this flowing water that travels north with a speed of 1.0 m/s, what is the resultant velocity of a puffin who tries to swim east across the tidal bore with a speed of 4.0 m/s?

A-4: Lynn is driving home from work and finds that there is road construction being done on her favorite route, so she must take a detour. Lynn travels 5 km north, 6 km east, 3 km south, 4 km west, and 2 km south. a) Draw a vector diagram of the situation. b) What is her displacement? Solve graphically. c) What total distance has Lynn covered?

A-5: Avery sees a UFO out her bedroom window and calls to report it to the police. She says, "The UFO moved 20.0 m east, 10.0 m north, and 30.0 m west before it disappeared." What was the displacement of the UFO while Avery was watching? Solve graphically.

A-6: Eli finds a map for a buried treasure. It tells him to begin at the old oak and walk 21 paces due west, 41 paces at an angle 45° south of west, 69 paces due north, 20 paces due east, and 50 paces at an angle of 53° south of east. How far from the oak tree is the buried treasure? Solve graphically.

A-7: Dwight pulls his sister in her wagon with a force of 65 N at an angle of 50.0° to the vertical. What are the horizontal and vertical components of the force exerted by Dwight?

A-8: Esther dives off the 3-m springboard and initially bounces up with a velocity of 8.0 m/s at an angle of 80.° to the horizontal. What are the horizontal and vertical components of her velocity?

A-9: In many locations, old abandoned stone quarries have become filled with water once excavating has been completed. While standing on a 10.0-m-high quarry wall, Clarence tosses a piece of granite into the water below. If Clarence throws the rock horizontally with a velocity of 3.0 m/s, how far out from the edge of the cliff will it hit the water?

A-10: While skiing, Ellen encounters an unexpected icy bump, which she leaves horizontally at 12.0 m/s. How far out, horizontally, from her starting point will Ellen land if she drops a distance of 7.00 m in the fall?

A-11: The Essex county sheriff is trying to determine the speed of a car that slid off a small bridge on a snowy New England night and landed in a snow pile 4.00 m below the level of the road. The tire tracks in the snow show that the car landed 12.0 m measured horizontally from the bridge. How fast was the car going when it left the road?

A-12: Superman is said to be able to "leap tall buildings in a single bound." How high a building could Superman jump over if he were to leave the ground with a speed of 60.0 m/s at an angle of 75.0° to the horizontal?

A-13: Len is running to school and leaping over puddles as he goes. From the edge of a 1.5-m-long puddle, Len jumps 0.20 m high off the ground with a horizontal velocity component of 3.0 m/s in an attempt to clear it. Determine whether or not Len sits in school all day with wet socks on.

Challenge Exercises for Further Study

B-1: Veronica can swim 3.0 m/s in still water. While trying to swim directly across a river from west to east, Veronica is pulled by a current flowing southward at 2.0 m/s. a) What is the magnitude of Veronica's resultant velocity? b) If Veronica wants to end up exactly across stream from where she began, at what angle to the shore must she swim upstream?

B-2: Solve Practice Exercise A-6 using vector components.

B-3: Mubarak jumps and shoots a field goal from the far end of the court into the basket at the other end, a distance of 27.6 m. The ball is given an initial velocity of 17.1 m/s at an angle of 40.0° to the horizontal from a height of 2.00 m above the ground. What is its velocity as it hits the basket 3.00 m off the ground?

B-4: Drew claims that he can throw a dart at a dartboard from a distance of 2.0 m and hit the 5.0-cm-wide bulls-eye if he throws the dart horizontally with a speed of 15 m/s. He starts the throw at the same height as the top of the bulls-eye. See if Drew is able to hit the bulls-eye by calculating how far his shot falls from the bulls-eye's lower edge.

B-5: Caitlin is playing tennis against a wall. She hits the tennis ball from a height of 0.5 m above the ground with a velocity of 20.0 m/s at an angle of 15.0° to the horizontal toward the wall that is 6.00 m away. a) How far off the ground is the ball when it hits the wall? b) Is the ball still traveling up or is it on its way down when it hits the wall?

B-6: From Chapter 1, Exercise B-6, determine how far from the base of Niagara Falls Annie Taylor landed in her wooden barrel.