CHAPTER 30

Nuclear Physics and Radioactivity

CHAPTER 31

Nuclear Energy; Effects and Uses of Radiation

Structure and Properties of the Nucleus

The components of a nucleus are collectively described as **nucleons.**

- Nucleons consist of **protons,** positive charges where $+e = +1.60 \times 10^{-19}$ C with a mass of 1.6726×10^{-27} kg. Unlike larger atoms, the only nucleon in a hydrogen atom is a proton.
- Larger atoms also typically contain **neutrons,** neutral charges with a mass of 1.6726×10^{-27} kg.
- The nucleus arrangements are described as **nuclides.** An atom's **atomic number,** Z, is equal to the number of protons in its nucleus. An atom's **atomic mass number,** A, is the number of nucleons. The **neutron number,** N, is therefore $N = A - Z$.
- The content of a nuclide is expressed as ${}^{A}_{Z}X$ where X is the chemical symbol of an element.
- **Isotopes** of an atom have differing numbers of neutrons. The **natural abundance** of isotopes represent the range of their occurrence on earth.
- The radius of a nucleus is a function of atomic mass number, and it is approximated by $r \approx (1.2 \times 10^{-15}\ \text{m})(A^{1/3})$.
- **Unified atomic mass units** are a relative scale of nuclear mass, based on a neutral carbon atom as 12 u. Equivalently, $1\ \text{u} = 1.6605 \times 10^{-27}\ \text{kg} = 931.5\ \text{MeV}/c^2$.
- Like electrons, a nucleus has **nuclear spin,** I, which is a function of the number of nucleons, and it has values from the set of integers and half-integers.

Binding Energy and Nuclear Forces

For a stable nucleus, the net mass is less than the sum of the nucleons that comprise it.

- This difference in mass is equal to the **total binding energy** that must be added for a nucleus to separate into its component nucleons. This can also be expressed as **average energy per nucleon,** which is the ratio of total binding energy and the atomic mass number.
- The attractive **strong nuclear force** acts between nucleons and keeps the nucleus stable. It is a fundamentally different force from **long-range** electric and gravitational forces. It acts over distances at or below 10^{-15} m, and is thus called a **short-range** force. Both the strong nuclear force and **weak nuclear force,** another attractive force, keep the nucleons together.

Mass and Energy

Radioactivity, nuclear fission, nuclear fusion, and all other nuclear events confirm the idea that mass is a form of energy. If the event results in a net loss of mass, an equivalent amount of energy is released. If the event has a net gain in mass, equivalent energy must be put into the system. In both cases, the mass difference is related to the energy change according to Einstein's well-known formula, $E = mc^2$, where c is the speed of light in a vacuum.

Radioactivity; Alpha, Beta, and Gamma Decay

Radioactivity describes radiation emitted without an external stimulus as a consequence of the decay of an unstable nucleus. This is common for certain isotopes. Radioactive decay can produce three forms of radiation: positively charged alpha α particles, negatively charged beta β particles or neutral gamma γ rays.

- Alpha particles consist of helium nuclei, ${}^4_2He$, and when emitted, the **parent nucleus** which emitted the alpha particle becomes a **daughter nucleus** containing two fewer protons and two fewer neutrons. This process is called **transmutation,** because it changes the parent nucleus into a different element. The short range nuclear forces cannot keep the nucleons in a large nucleus together, due to the repulsive force between protons. The instability of a nucleus is quantified in the **disintegration energy** released, $Q = (M_p - M_d - m_\alpha)c^2$, where the subscripts of mass refer to the parent nucleus, daughter nucleus, and alpha particle.
- Beta particles consist of electrons produced in the nucleus. Their discharge is accompanied by an **antineutrino,** a neutral charged particle with an apparent rest mass of zero. The daughter nucleus has one less neutron, one more proton, and an increase in charge of +1. Unstable nuclei with fewer neutrons than protons emit **positrons** and **neutrinos** instead of electrons and antineutrinos. Positrons have the mass of electrons and the charge of protons. A positron is referred to as the **antiparticle** of an electron. Alternately, a nucleus can absorb an orbital electron in a process called **electron capture,** causing a nuclear proton to become a neutron, while the electron disappears and a neutrino is

released. The weak nuclear force, which affects neutrinos, is important in all forms of beta decay.

- Gamma rays are composed of high energy photons emitted when a nucleus jumps to a lower energy state. Alternately, an excited nucleus can jump to its ground state by releasing an orbital electron without releasing photons.
- In each of these decay processes, the **conservation of nucleon number** is exhibited.

Calculations Involving Decay Rates and Half-Life

The constituent radioactive nuclei of radioactive isotopes decay in a manner closely modeled by the **radioactive decay law,** given by the exponential expression $N = N_0 e^{-\lambda t}$, where N is the number of radioactive nuclei left after the time t that has elapsed since an initial number of radioactive nuclei, N_0, were measured. The proportionality constant, λ, is called the **decay constant,** whose value is characteristic of the isotope. The rate of decay, called the **activity,** is given by the expression $\Delta N/\Delta t = (\Delta N/\Delta t)_0 e^{-\lambda t}$. More generally, isotopes are identified by their **half-life,** $T_{1/2}$, which represents the time necessary for the quantity of radioactive nuclei in a given sample to be halved. The decay constant is inversely proportional to the half-life, such that $T_{1/2} = 0.693/\lambda$.

Nuclear Reactions and the Transmutation of Elements

The process of transmuting one element to another can occur following α or β decay, or by the bombardment of a nucleus with a nucleon, γ ray, or another nucleus, which is called a **nuclear reaction.**

- Nuclear reactions are expressed in the form $a + X \rightarrow Y + b$, where a is the projectile, X is the target nucleus, Y is the product nucleus, and b is the product particle.
- The **reaction energy** is a function of their masses, such that $Q = (M_a + M_X - M_b - M_Y)c^2$, or alternately, the change in kinetic energy $Q = \text{KE}_b + \text{KE}_Y - \text{KE}_a - \text{KE}_X$.
- Exothermic reactions describe those with positive values for reaction energy, and endothermic reactions with negative values for reaction energy.
- In the 1930s Enrico Fermi proposed using neutrons as efficient projectiles (due to their neutral charge) to produce new isotopes and new elements heavier than uranium.

Nuclear Fission; Nuclear Reactors; Fusion

An examination of Fermi's work showed that a uranium atom struck by neutrons occasionally divided into nearly equal products, a process called **nuclear fission.**

- During this process, visualized as the **liquid drop model,** there is an intermediate stage known as the **compound nucleus** in which the engorged nucleus is in an excited state that causes it to split into **fission fragments.** The process also releases a large amount of energy relative to the atom's size.

- In a **self-sustaining chain reaction,** the released neutrons from a fission reaction in turn create other fission reactions and create large quantities of energy.
- This process can be controlled in a **nuclear reactor,** in which a **moderator** is used to slow down neutrons released by fission reactions to the speeds that make subsequent fission reactions more probable. Common moderators include deuterium in the form of **heavy water** or graphite. Artificially **enriched** uranium has a higher percentage of fissionable uranium. The minimum mass of reactor fuel necessary for a self-sustaining nuclear reaction, called the **critical mass,** is a function of the type of fuel and the moderator. For a self-sustaining chain reaction, at least one neutron produced from each fission in turn produces another fission reaction; that is, for the **multiplication factor,** f, $f \geq 1$.
- **Nuclear fusion** describes the creation of heavier nuclei by combining previously isolated neutrons and protons or by combining existing lighter nuclei. This latter process occurs because the binding energy per nucleon increases as the atomic mass rises (up to about $A = 60$). Thus energy will be released when these nuclei combine.

For Additional Review

Consider whether the nucleon number is conserved in nuclear fission reactions as it is in radioactive decay.

Multiple-Choice Questions

1. The number of neutrons in ${}^{17}_{8}O$ is
   (A) 1
   (B) 8
   (C) 9
   (D) 17
   (E) 25

2. What is the approximate diameter of an oxygen nucleus that has two more protons than neutrons?
   (A) 1.2×10^{-15} m
   (B) 2.4×10^{-15} m
   (C) 2.9×10^{-15} m
   (D) 3.1×10^{-15} m
   (E) 5.2×10^{-15} m

3. The mass of a proton and electron is given by 1.007825 u and the mass of a neutron is 1.008665 u. What is the total binding energy of a ${}^{12}_{6}C$ nucleus if the conversion from atomic mass units to energy is given by 931.5 MeV/u?
   (A) 0.01299 MeV
   (B) 92.16 MeV
   (C) 982.1 MeV
   (D) 7621 MeV
   (E) 11260 MeV

4. What is produced when ${}^{83}_{38}Sr$ emits an α particle?
   (A) ${}^{83}_{36}Kr$
   (B) ${}^{82}_{38}Sr$
   (C) ${}^{79}_{36}Kr$
   (D) ${}^{79}_{37}Rb$
   (E) ${}^{79}_{38}Sr$

5. For a nucleus that undergoes alpha decay, what is the disintegration energy if the mass of the alpha particle, daughter and parent are A, B, and C, respectively?
   (A) $(A + B + C)/c^2$
   (B) $(A + B + C)c^2$
   (C) $(A + B - C)c^2$
   (D) $(C + A - B)/c^2$
   (E) $(C - B - A)c^2$

6. When a gamma ray is emitted from the excited state of ${}^{12}_{6}C$, the resulting nucleus is
   (A) ${}^{12}_{6}C$
   (B) ${}^{12}_{5}B$
   (C) ${}^{13}_{6}C$
   (D) ${}^{8}_{4}Be$
   (E) ${}^{12}_{7}N$

7. When a neutron strikes an unknown nucleus, the observed products are ${}^{28}_{14}Si$ and ${}^{2}_{1}H$. What is the unknown nucleus?
   (A) ${}^{30}_{13}Al$
   (B) ${}^{30}_{14}Si$
   (C) ${}^{29}_{14}Si$
   (D) ${}^{30}_{15}P$
   (E) ${}^{29}_{15}P$

8. What is the reaction energy for a nuclear reaction in which the Q value calculated from the change in mass is -3.10 MeV? What type of reaction is this?
   (A) -3.10 MeV, exothermic
   (B) 3.10 MeV, exothermic
   (C) 3.10 MeV, endothermic
   (D) -3.10 MeV, endothermic
   (E) Not enough information is provided to determine the reaction energy and the type of reaction.

9. For a typical fission reaction, represented by $n + {}^{235}_{92}U \rightarrow X \rightarrow N_1 + N_2 +$ neutron(s), X represents
   (A) ${}^{234}_{92}U$
   (B) ${}^{236}_{92}U$
   (C) ${}^{235}_{93}U$
   (D) ${}^{236}_{93}U$
   (E) ${}^{235}_{91}U$

10. In a nuclear fission reaction, when the multiplication factor f is less than one,
   (A) a self-sustaining chain reaction cannot be maintained
   (B) on average, each released neutron goes on to cause at least one fission reaction
   (C) a moderator of heavy water or graphite must be used to control energy release
   (D) the fuel used has been enriched
   (E) all of the above are true

Free-Response Questions

1. Based on their definitions, trace the hypothetical results of the following three forms of decay.
   (a) Alpha decay for a ${}^{210}_{84}Po$
   (b) A form of beta decay for ${}^{35}_{17}Cl$
   (c) Gamma decay for ${}^{14}_{6}C$
   (d) What assumption must be made for gamma decay to occur?
   (e) How is gamma decay most different from alpha and beta decay?

2. Describe nuclear reactions for fissions according to the following multiplication factors.
   (a) $f < 1$
   (b) $f = 1$
   (c) $f > 1$
   (d) How would the value of a multiplication factor be relevant for the storage and testing of a destructive thermonuclear device?

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. **(C) is correct.** The notation ${}^A_Z X$ describes a nucleus in which there are Z protons and A nucleons. Here there are 8 protons and 17 nucleons, so the number of neutrons are given by $N = Z - A = 17 - 8 = 9$.
2. **(C) is correct.** The approximate diameter of a nucleus is given by $r \approx (1.2 \times 10^{-15}\text{ m})(A^{1/3})$. The value of A is the number of protons (8) and neutrons (6) = 14, so $r \approx (1.2 \times 10^{-15}\text{ m})(14^{1/3})$.
3. **(B) is correct.** The total binding energy of a nucleus is given by the product of the difference in mass between a nucleus and its constituents and the equivalent conversion to energy. By definition, the atomic mass of ${}^{12}_{6}C$ is 12.00000. The combination of (6 protons)(1.007825 u/1 proton) + (6 neutrons)(1.008665 u/1 neutron) = 6.04695 u + 6.05199 u = 12.09894 u. Δm is therefore 0.09894 u and the total binding energy is (0.09894 u)(931.5 MeV/u) = 92.1626 MeV
4. **(C) is correct.** An alpha particle is a helium nucleus, ${}^4_2He$, so the decay is written as ${}^{83}_{38}Sr \rightarrow {}^A_Z X + {}^4_2He$, so $A = 79$ and $Z = 36$ for the reaction to balance. The nucleus must be krypton.
5. **(E) is correct.** The disintegration energy released is given by, $Q = (M_p - M_d - m_\alpha)c^2$, where the subscripts of mass refer to the parent nucleus, daughter nucleus and alpha particle. Based on the provided information, $Q = (C - B - A)c^2$.
6. **(A) is correct.** When a gamma ray is emitted, the decay is expressed as ${}^A_Z N^* \rightarrow {}^A_Z N + \gamma$. Unlike α and β decay, gamma decay does not change the atomic number, although the nucleus drops from an excited energy state to a lower energy state.
7. **(E) is correct.** In this reaction, $n + {}^A_Z X \rightarrow {}^{28}_{14}Si + {}^2_1H$. For the reaction to balance, the total nucleons on the left must be equal to the number on the right, $28 + 2 = 30$. So on the left $1 + A = 30$, so $A = 29$. Z is the total number of protons $Z = 14 + 1 = 15$, so the unknown element is a phosphorus nucleus.
8. **(D) is correct.** The value of reaction energy can be defined equivalently as mass and energy, $Q = (M_a + M_X - M_b - M_Y)c^2 = KE_b + KE_Y - KE_a - KE_X$. Therefore the value is -3.10 MeV. By definition, an endothermic reaction describes those in which $Q < 0$.
9. **(B) is correct.** When a neutron bombards a uranium nucleus, its intermediate stage is a compound nucleus, which has an extra neutron and is represented by ${}^{236}_{92}U$, since the atomic mass number—the upper index—represents the number of nucleons.
10. **(A) is correct.** When the multiplication factor is less than one, each neutron released in a fission reaction produces fewer than one subsequent fission reaction on average, and a self-sustaining chain reaction cannot be maintained.

Free-Response Questions

1. (a) Alpha decay occurs when a nucleus emits a ^4_2He particle. So for $^{210}_{84}Po$, if alpha decay were to occur, a ^4_2He particle is released, and by the conservation of nucleon number, the daughter nucleus is $^{206}_{82}Pb$.
   (b) In one type of beta decay, a nucleus emits a nonorbital electron and a form of neutrino. So for $^{35}_{17}Cl$, if beta decay were to occur, an electron is released, and by the conservation of nucleon number, the daughter nucleus is $^{35}_{18}Ar$.
   (c) Gamma decay occurs when an excited nucleus emits a photon. The daughter nucleus would remain $^{14}_{6}C$.
   (d) For gamma decay to occur, it is assumed the nucleus would have initially been in an excited state.
   (e) Gamma decay is most different from the others, because the daughter nucleus is of the same element, whereas transmutation occurs in alpha and beta decay.

   This response correctly hypothesizes the results of decays based on their definitions, accurately stating their products for parts a, b, and c, while utilizing the appropriate method (conservation of nucleon number) for calculating these results. The response to part d demonstrates an understanding of the prerequisite conditions for gamma decay, and the response to part e summarizes the primary difference between decay types.

2. The multiplication factor quantifies the average number of neutrons released in fission that initiate further fission.
   (a) For $f < 1$, the reactions decrease in number over time, and there will not be a self-sustaining chain reaction. A reactor in this state would be described as subcritical.
   (b) For $f = 1$, the reactions remain roughly constant in number over time and form a self-sustaining chain reaction. A reactor in this state would be described as critical.
   (c) For $f > 1$, the reactions increase in number over time going beyond the minimum number needed to be self-sustaining. A reactor in this state would be described as supercritical.
   (d) The value of multiplication factor should be $f < 1$ for the storage of a destructive thermonuclear device and $f > 1$ for its testing.

   This response correctly gauges the correlation between multiplication factor and the type of fission each condition describes in parts a, b, and c. It further states its definition of multiplication factor and the applicable titles that correspond to each situation. The response to part d applies these values and definitions to a realistic situation (for a researcher—not the student).