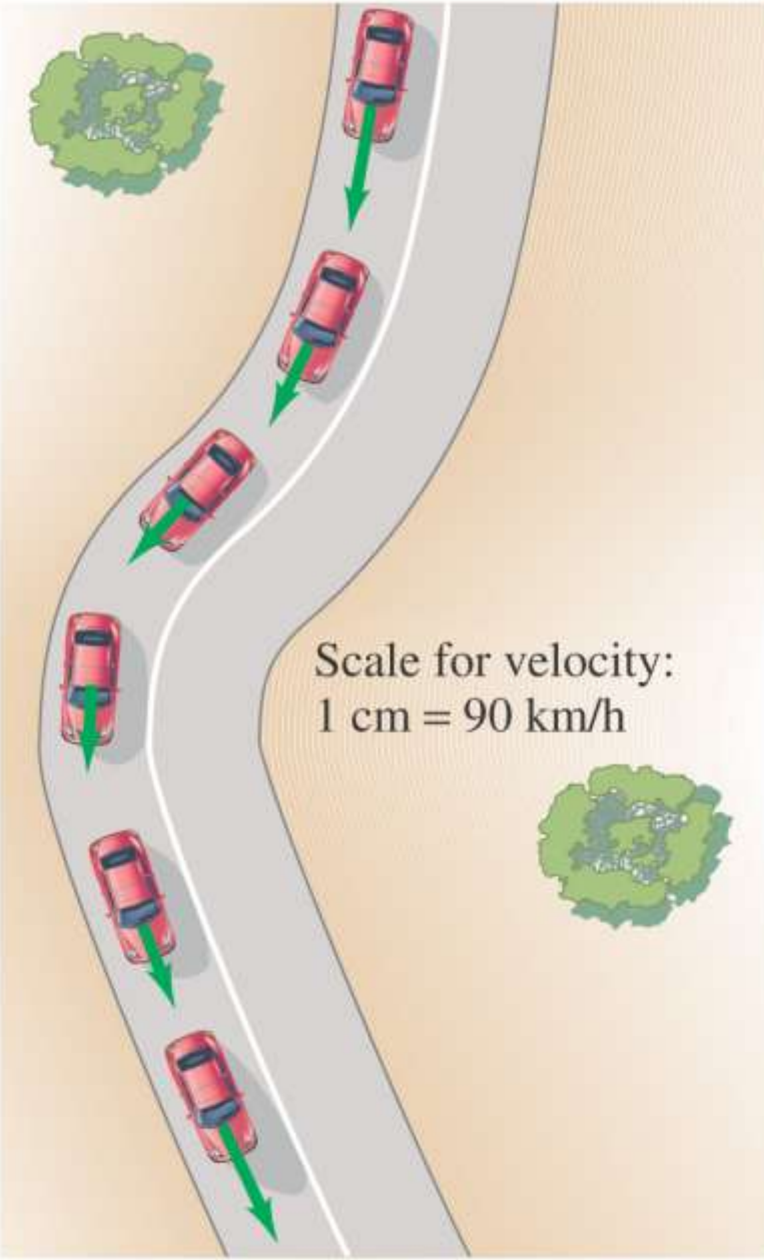
Chapter 3

Kinematics in Two Dimensions; Vectors

Units of Chapter 3

- **Vectors and Scalars**
- **Addition of Vectors – Graphical Methods**
- **Subtraction of Vectors, and Multiplication of a Vector by a Scalar**
- **Adding Vectors by Components**
- **Projectile Motion**
- **Solving Problems Involving Projectile Motion**
- **Projectile Motion Is Parabolic**
- **Relative Velocity**

3-1 Vectors and Scalars



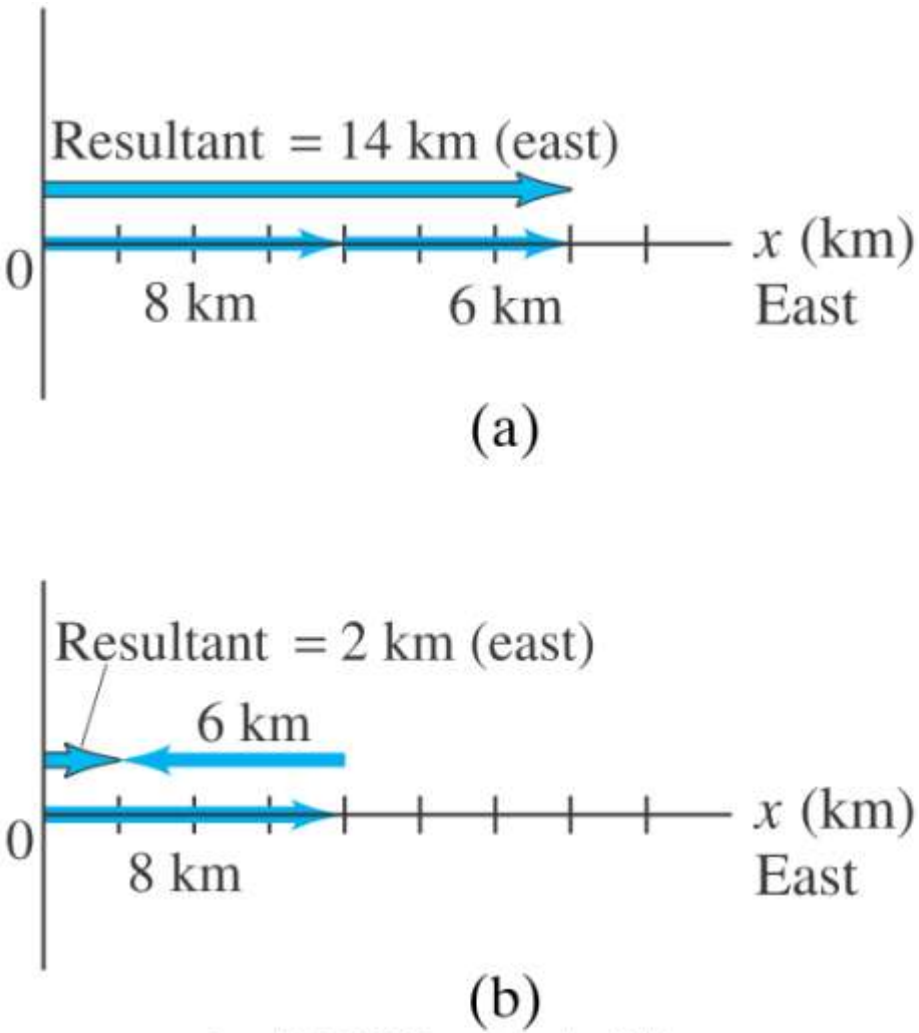
A **vector** has **magnitude** as well as **direction**.

Some vector quantities: **displacement**, **velocity**, **force**, **momentum**

A **scalar** has only a **magnitude**.

Some scalar quantities: **mass**, **time**, **temperature**

3-2 Addition of Vectors – Graphical Methods



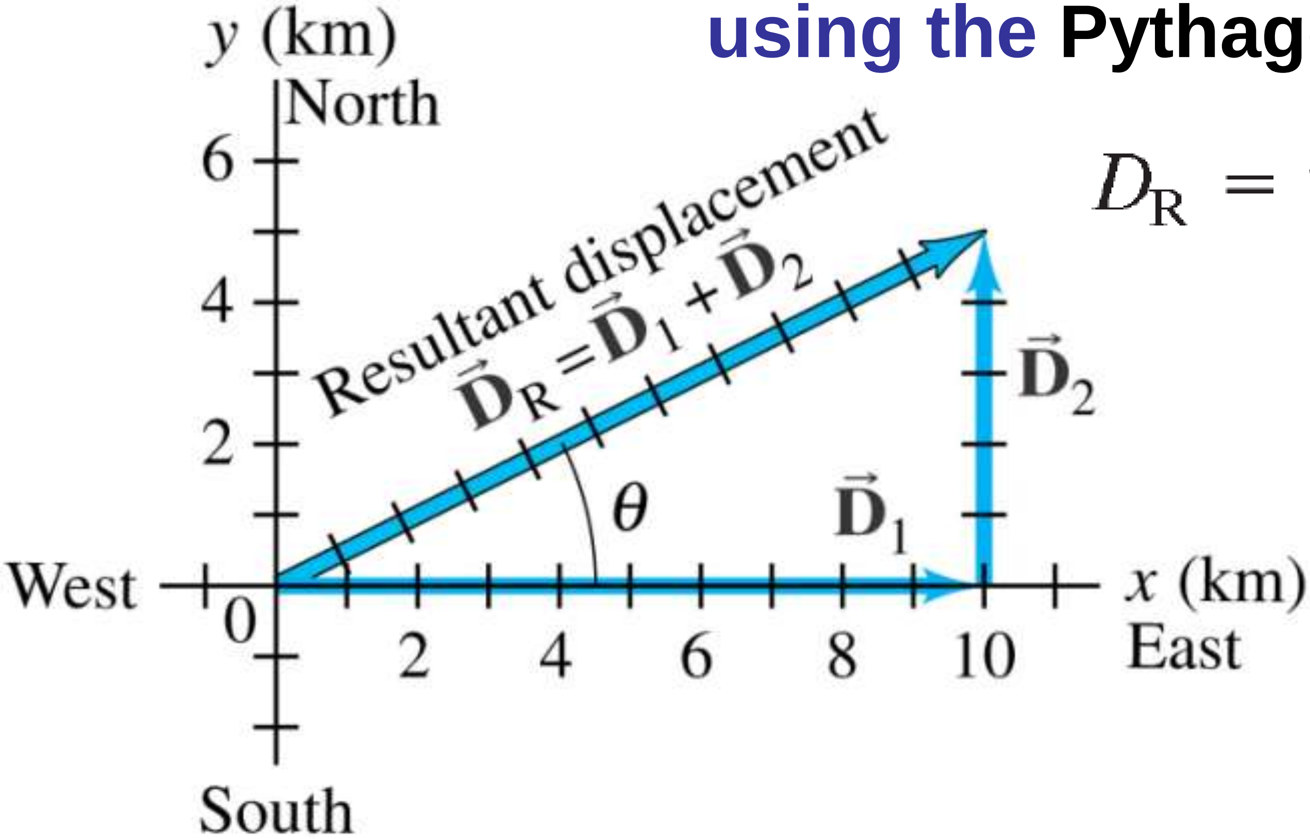
For vectors in one dimension, simple **addition** and **subtraction** are all that is needed.

You do need to be careful about the **signs**, as the figure indicates.

3-2 Addition of Vectors – Graphical Methods

If the motion is in two dimensions, the situation is somewhat more complicated.

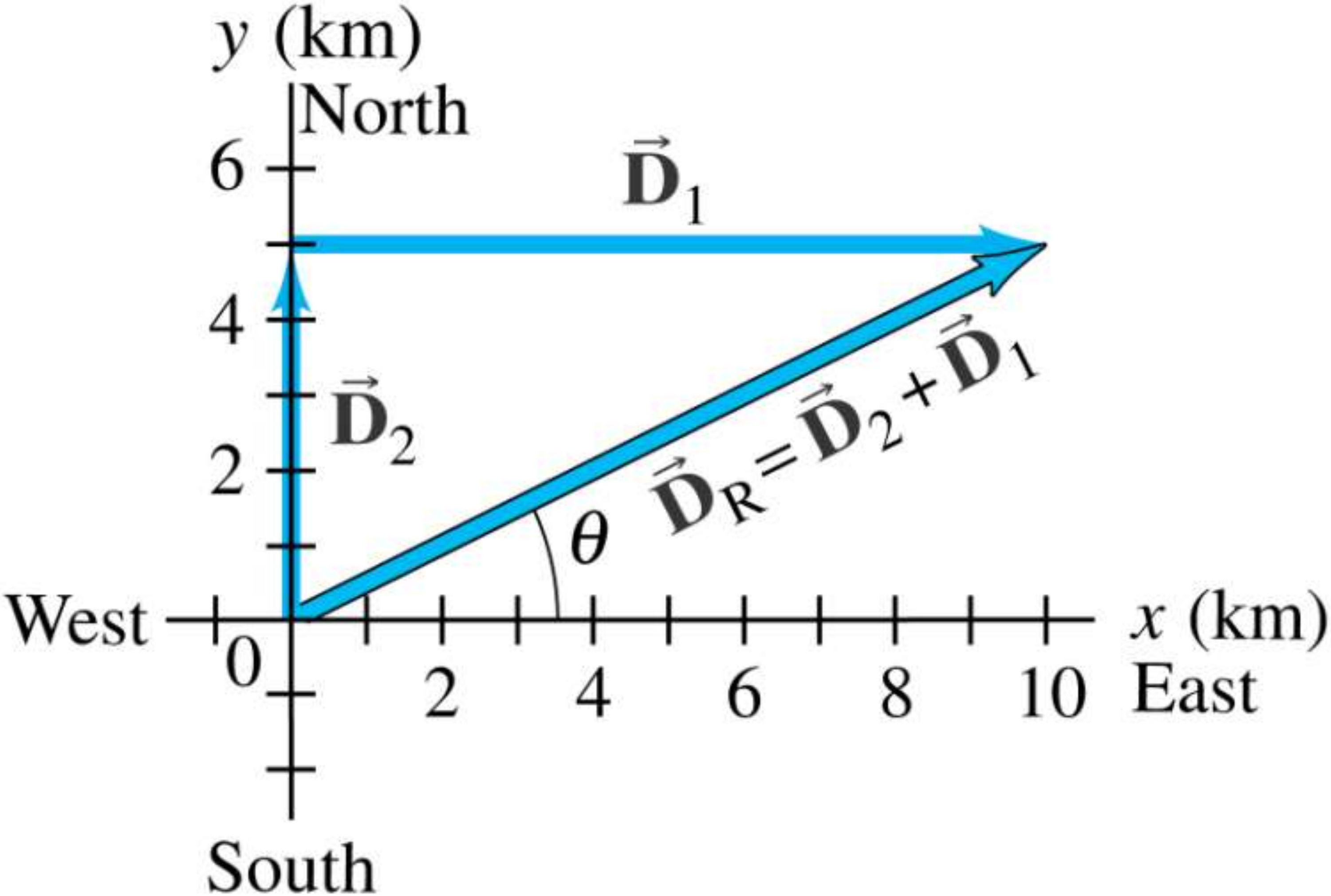
Here, the actual travel paths are at right angles to one another; we can find the displacement by using the Pythagorean Theorem.



$$D_R = \sqrt{D_1^2 + D_2^2}$$

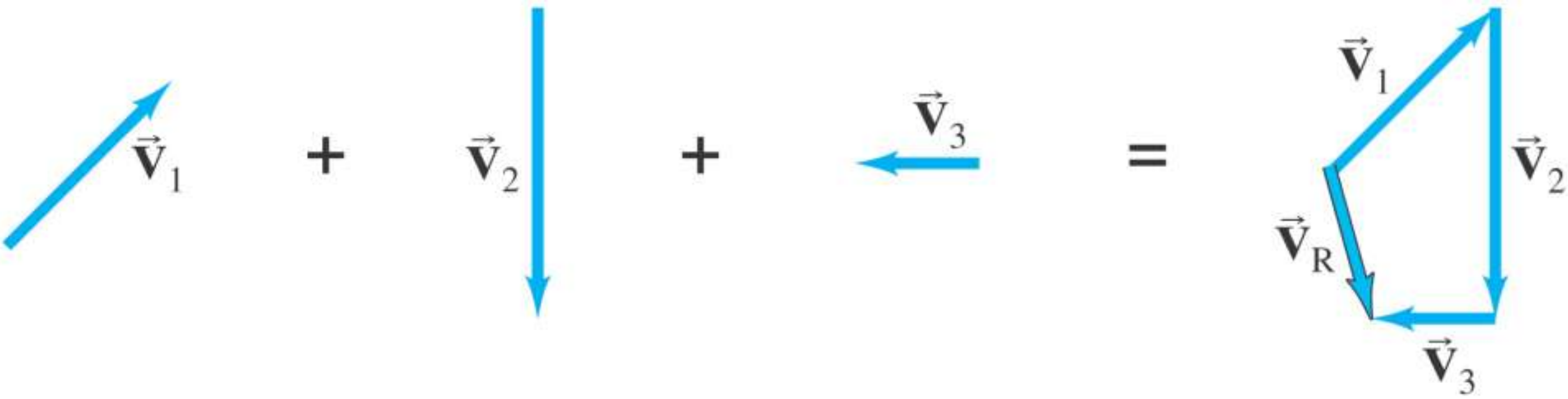
3-2 Addition of Vectors – Graphical Methods

Adding the vectors in the **opposite** order gives the same result: $\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_1$



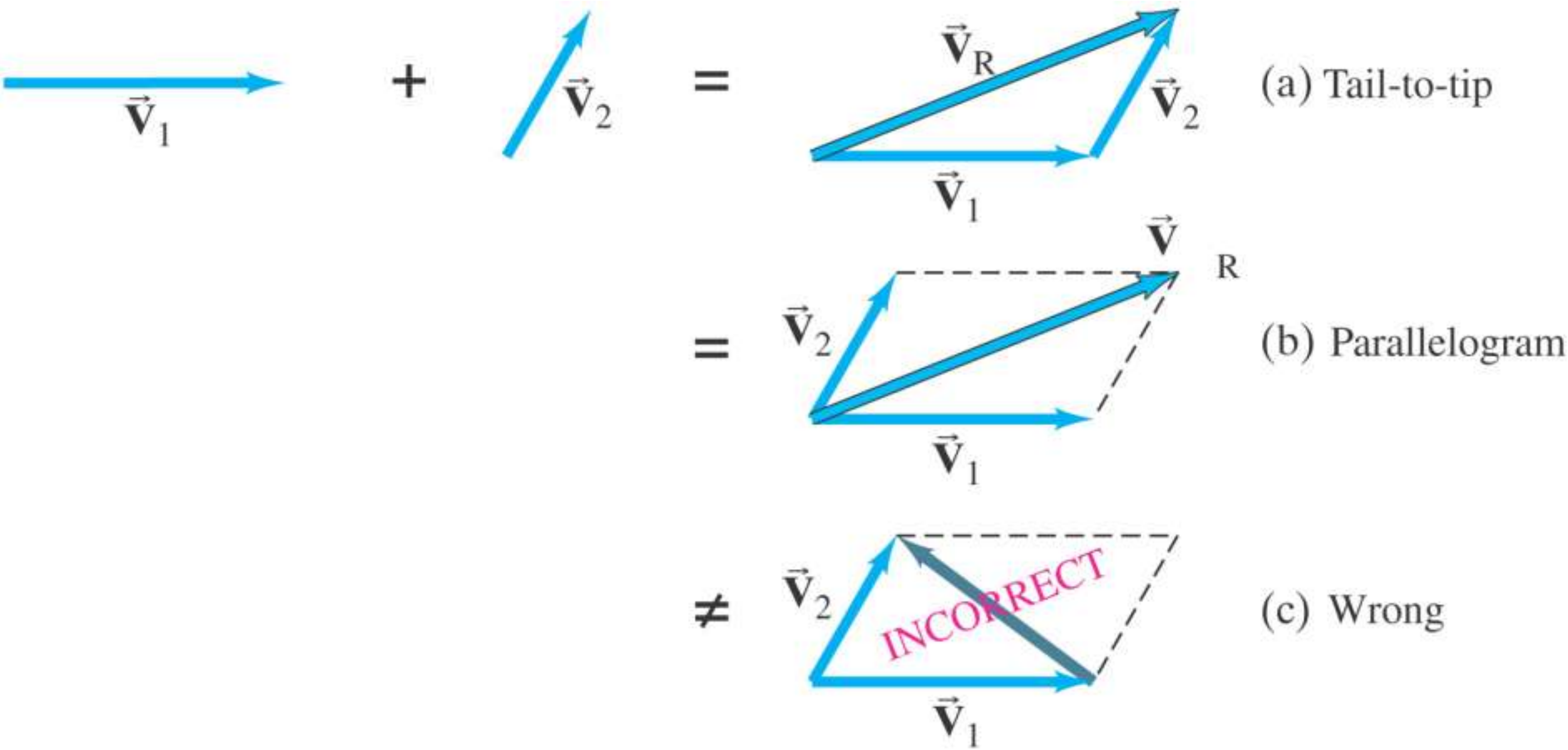
3-2 Addition of Vectors – Graphical Methods

Even if the vectors are not at right angles, they can be added graphically by using the "tail-to-tip" method.

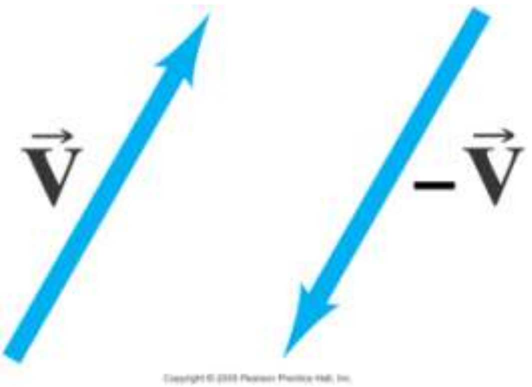


3-2 Addition of Vectors – Graphical Methods

The **parallelogram** method may also be used; here again the vectors must be "tail-to-tip."



3-3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar

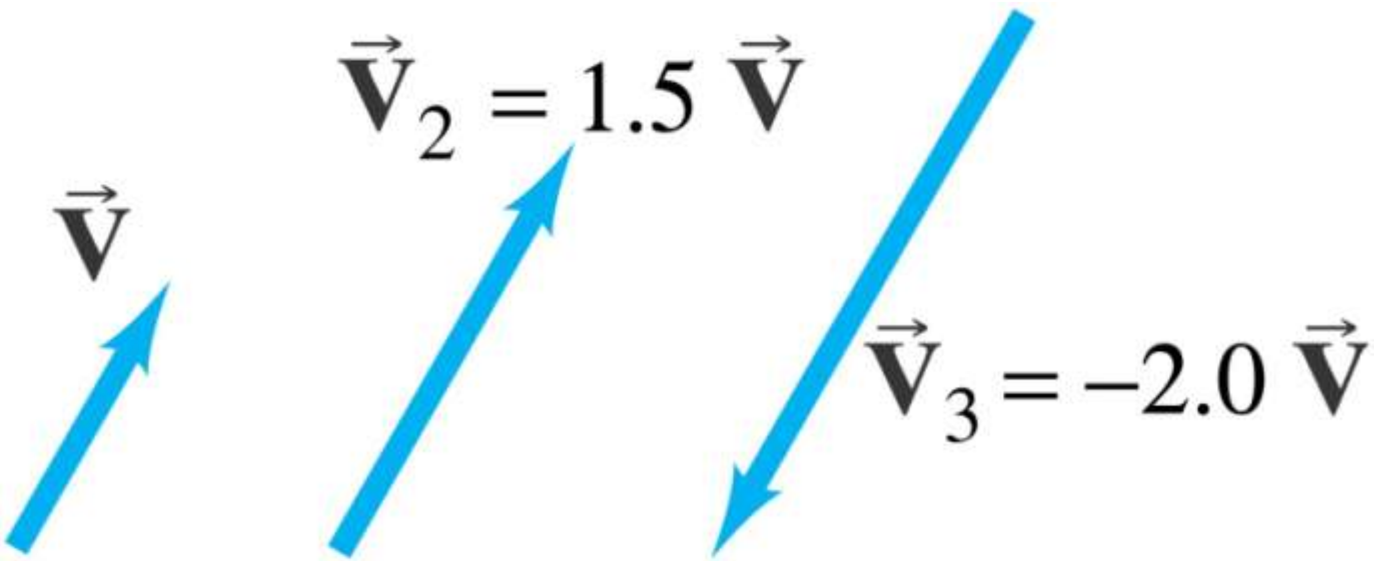


In order to **subtract** vectors, we define the **negative** of a vector, which has the same magnitude but points in the **opposite** direction.

Then we add the negative vector:

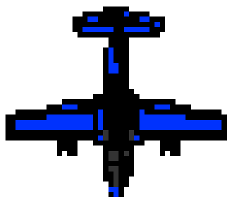
3-3 Subtraction of Vectors, and Multiplication of a Vector by a Scalar

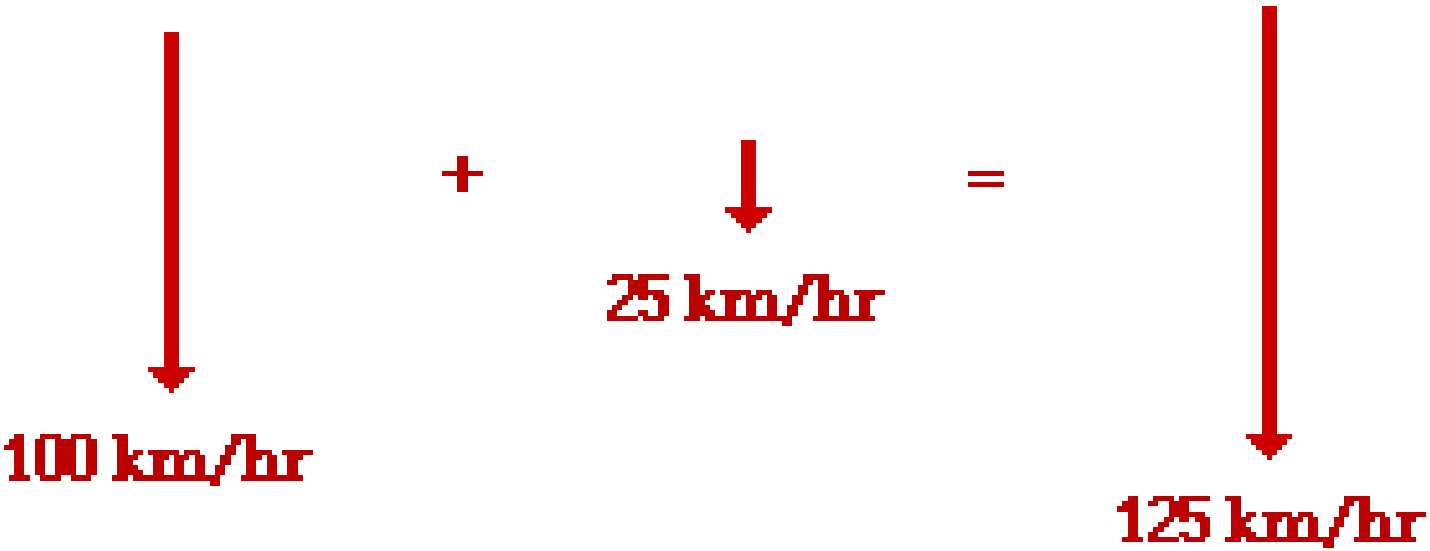
A vector V can be multiplied by a scalar *c*; the result is a vector *c*V that has the same direction but a magnitude *cV*. If *c* is negative, the resultant vector points in the opposite direction.



3-4 Adding Vectors by Components

Plane Velocity + Wind Velocity = Resultant Velocity

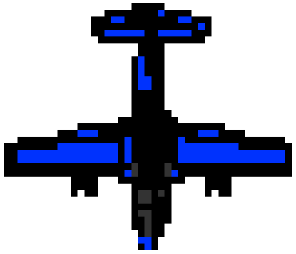


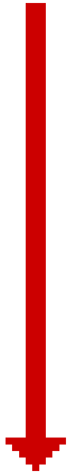
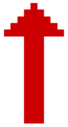
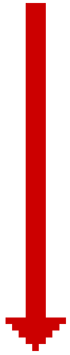


The plane travels with a velocity relative to the ground which is the vector sum of the plane's velocity (relative to the air) plus the wind velocity.

3-4 Adding Vectors by Components

Plane Velocity + Wind Velocity = Resultant Velocity



 +  = 

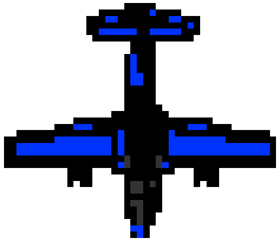
25 km/hr

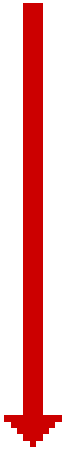
100 km/hr

75 km/hr

3-4 Adding Vectors by Components

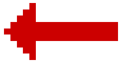
Plane Velocity + Wind Velocity = Resultant Velocity





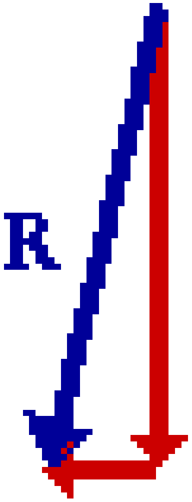
100 km/hr

+

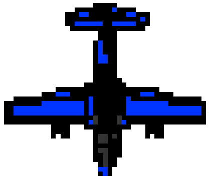
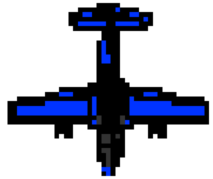
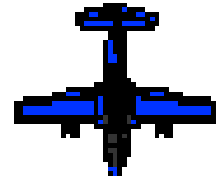


25 km/hr

=



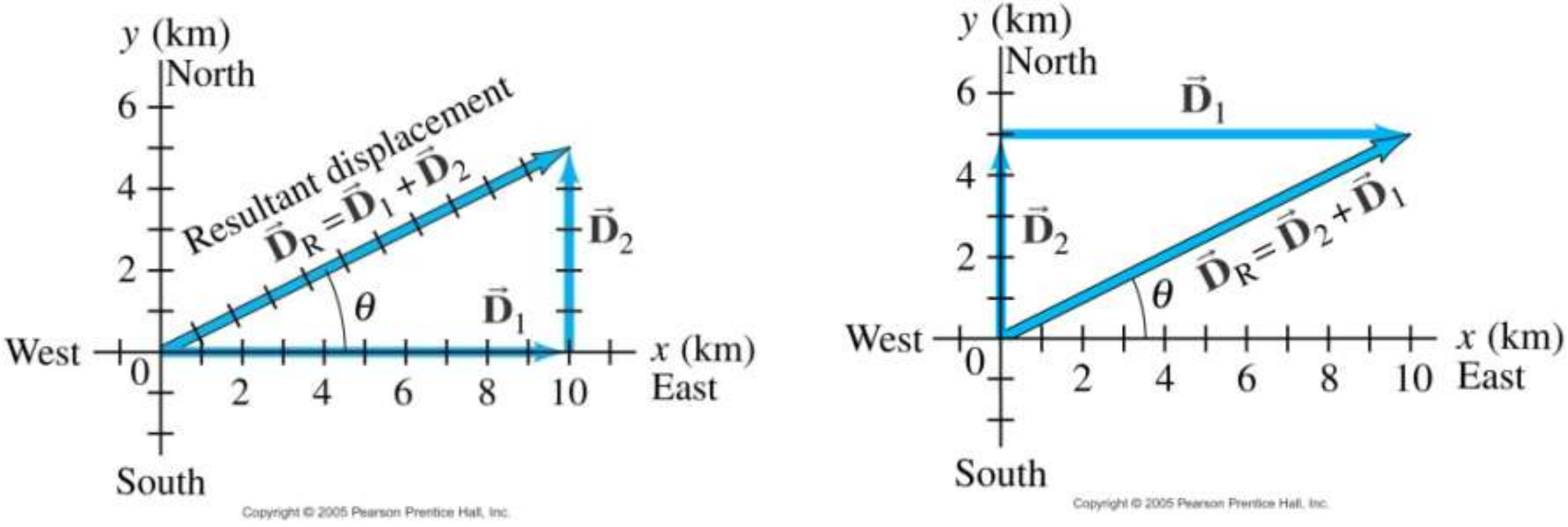
R = 103.1 km/hr

Tailwind	Headwind	Crosswind
		

3-4 Adding Vectors by Components

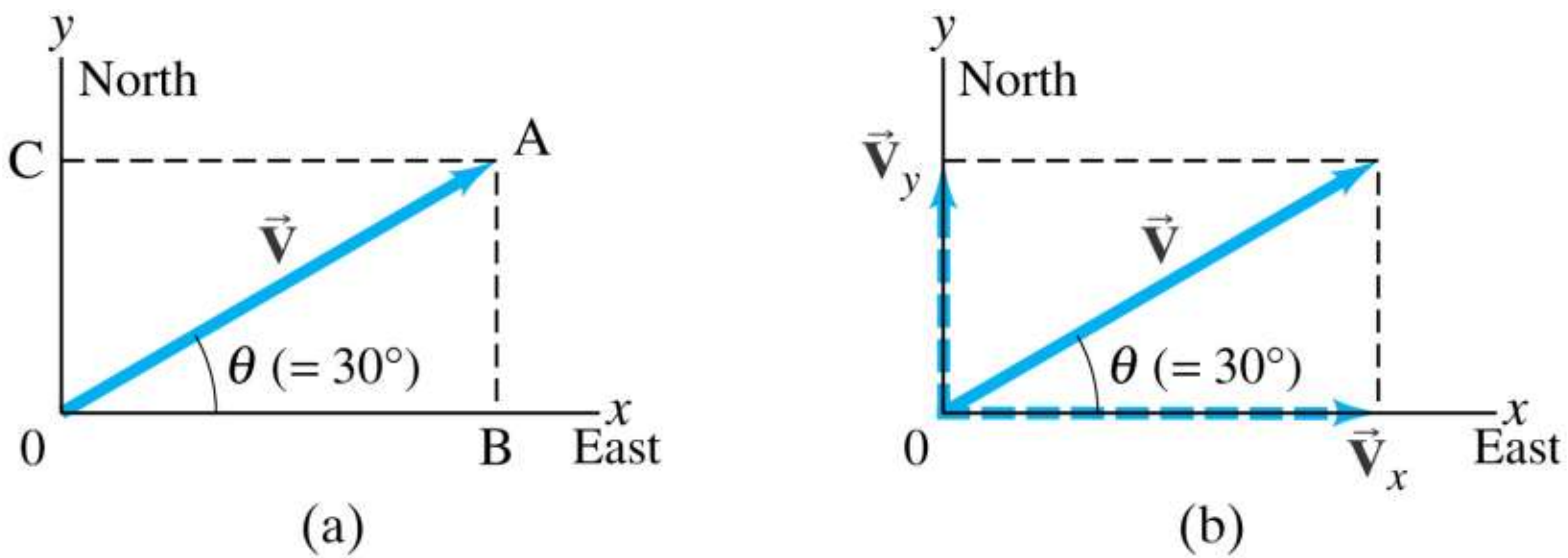
Adding the vectors in the opposite order gives the same result:

$$\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_1$$

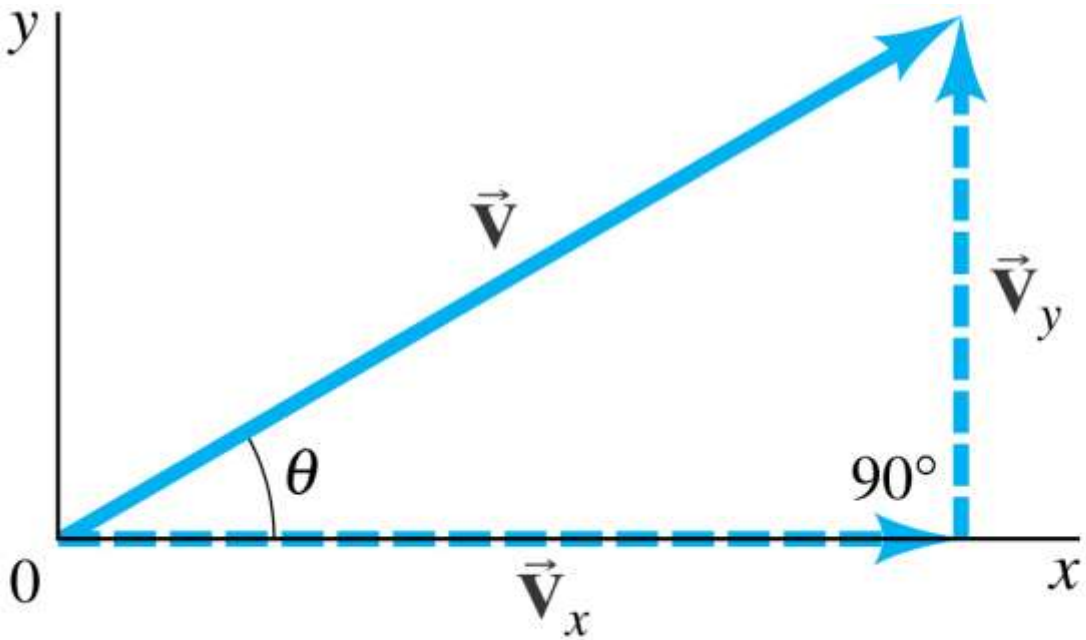


3-4 Adding Vectors by Components

Any vector can be expressed as the **sum** of two other vectors, which are called its **components**. Usually the other vectors are chosen so that they are **perpendicular** to each other.



3-4 Adding Vectors by Components



$$\sin\theta = \frac{V_y}{V}$$

$$\cos\theta = \frac{V_x}{V}$$

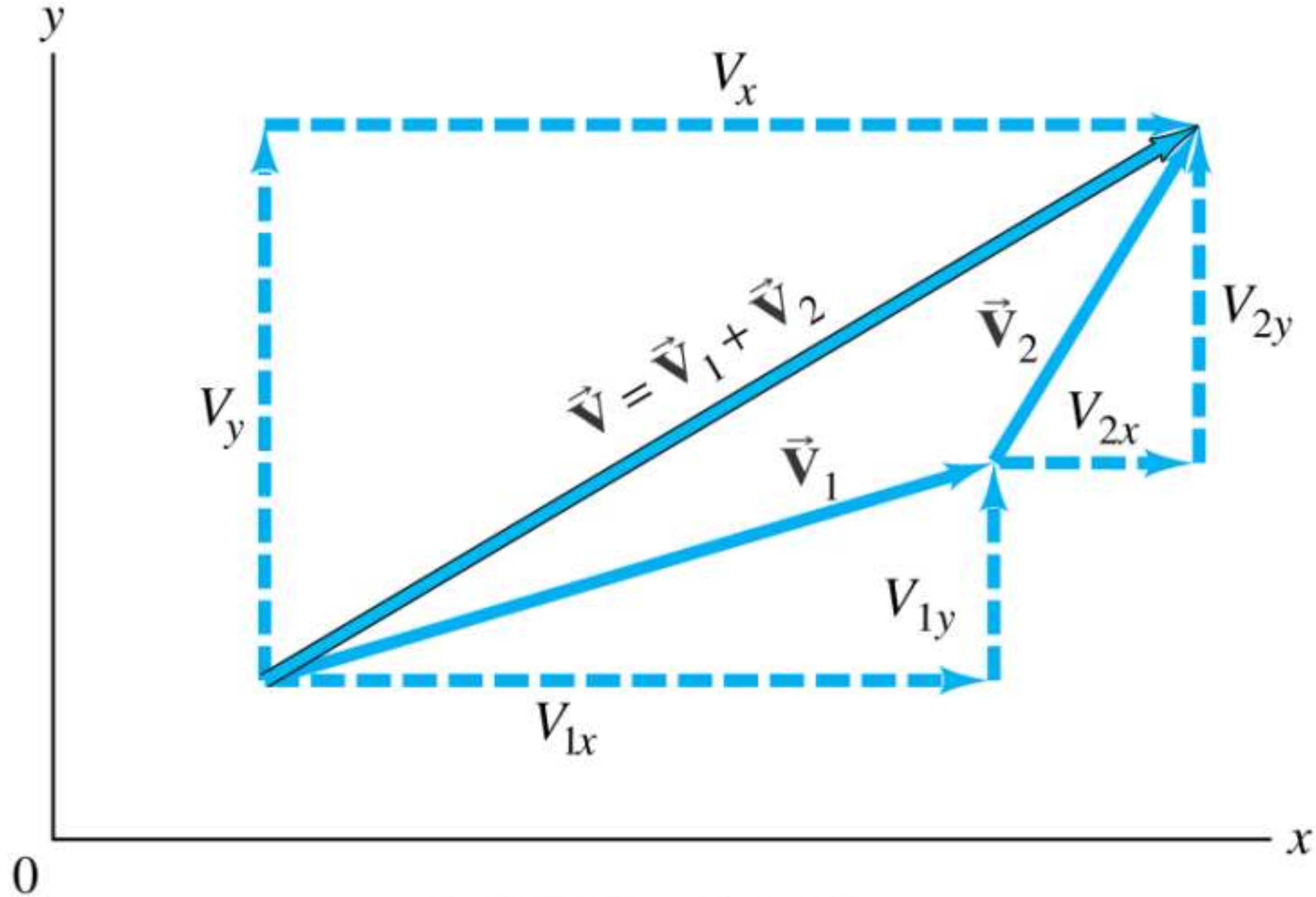
$$\tan\theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$

If the components are **perpendicular**, they can be found using **trigonometric** functions.

3-4 Adding Vectors by Components

The **components** are effectively one-dimensional, so they can be added **arithmetically**:



3-4 Adding Vectors by Components

Time for a Gizmo!

3-4 Adding Vectors by Components

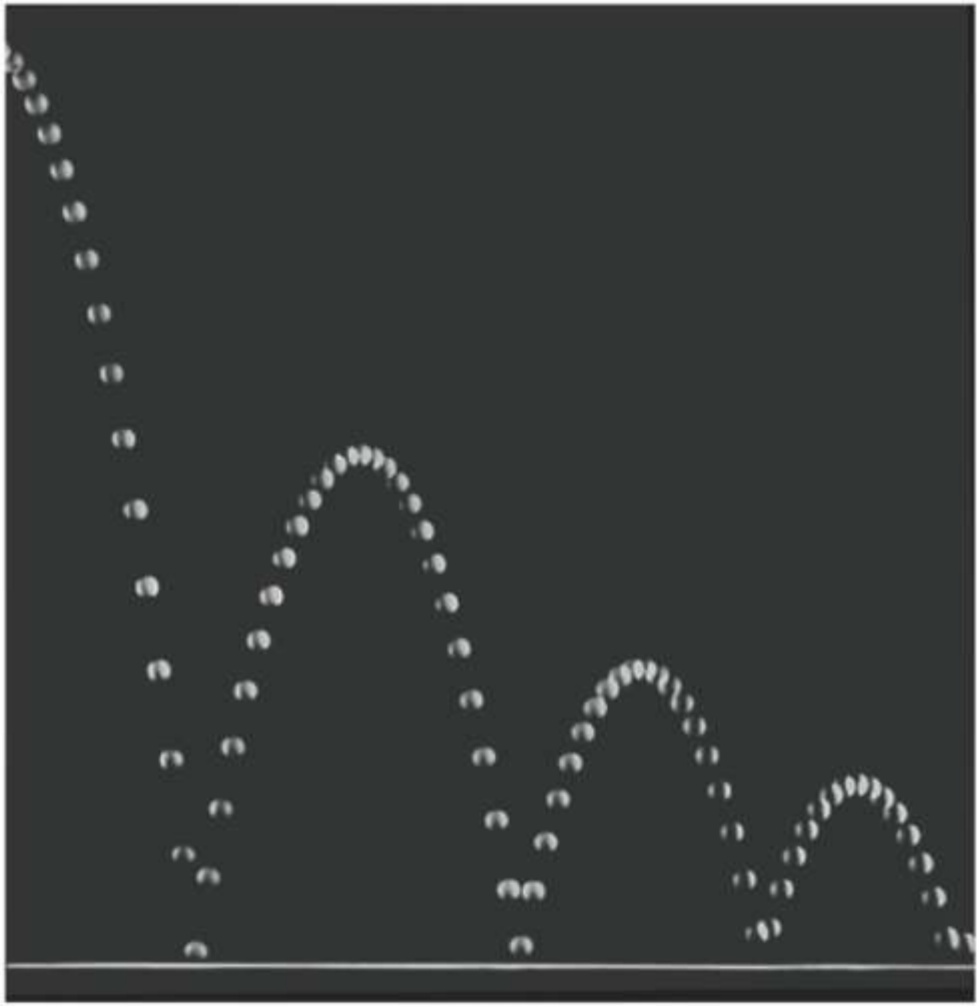
Adding vectors:

1. Draw a **diagram**; add the vectors **graphically.**
2. **Choose** *x* and *y* axes.
3. **Resolve** each vector into *x* and *y* **components**.
4. **Calculate** each component using sines and cosines.
5. **Add** the components in each direction.
6. To find the **length** and **direction** of the vector, use:

$$V = \sqrt{V_x^2 + V_y^2}$$

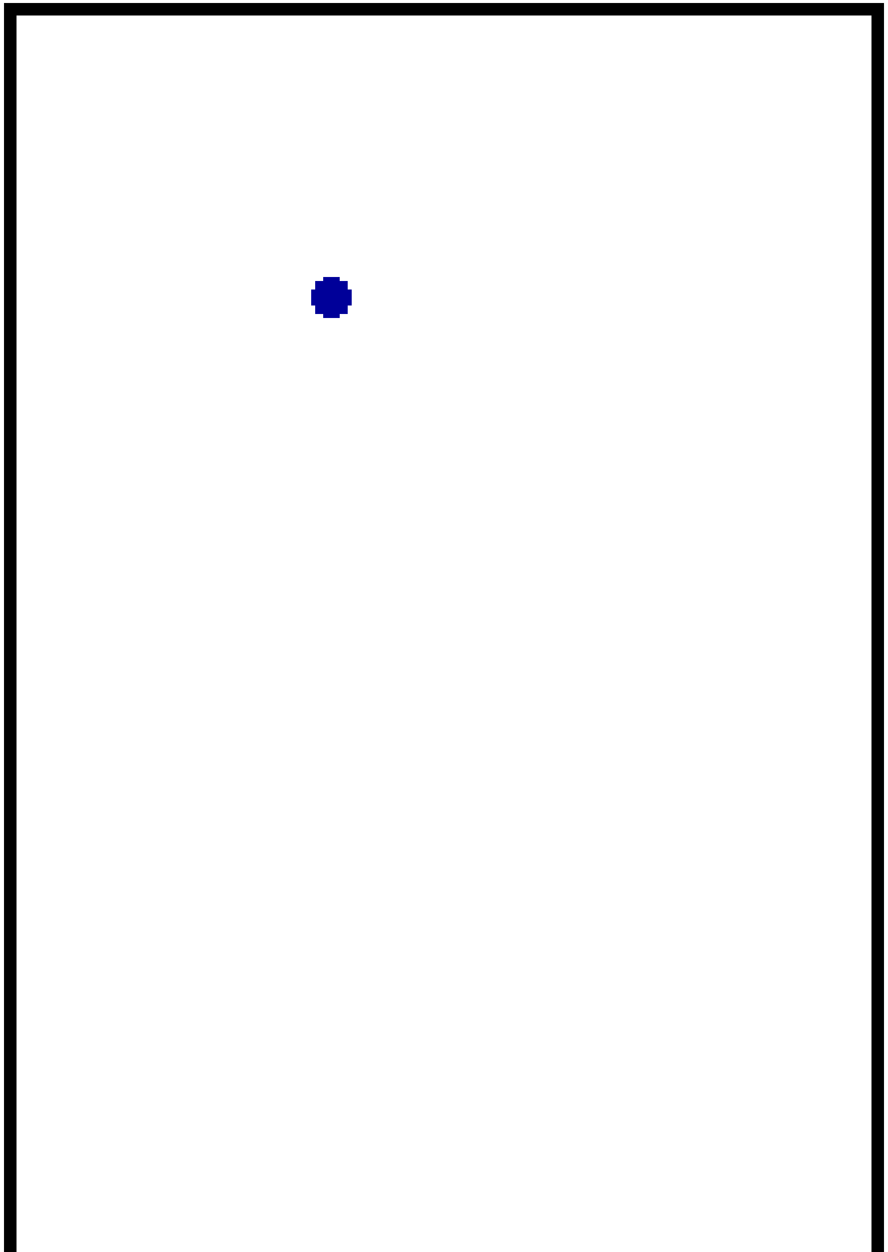
$$\tan\theta = \frac{V_y}{V_x}$$

3-5 Projectile Motion



A **projectile** is an object moving in two dimensions under the influence of Earth's **gravity**; its path is a **parabola**.

3-5 Projectile Motion



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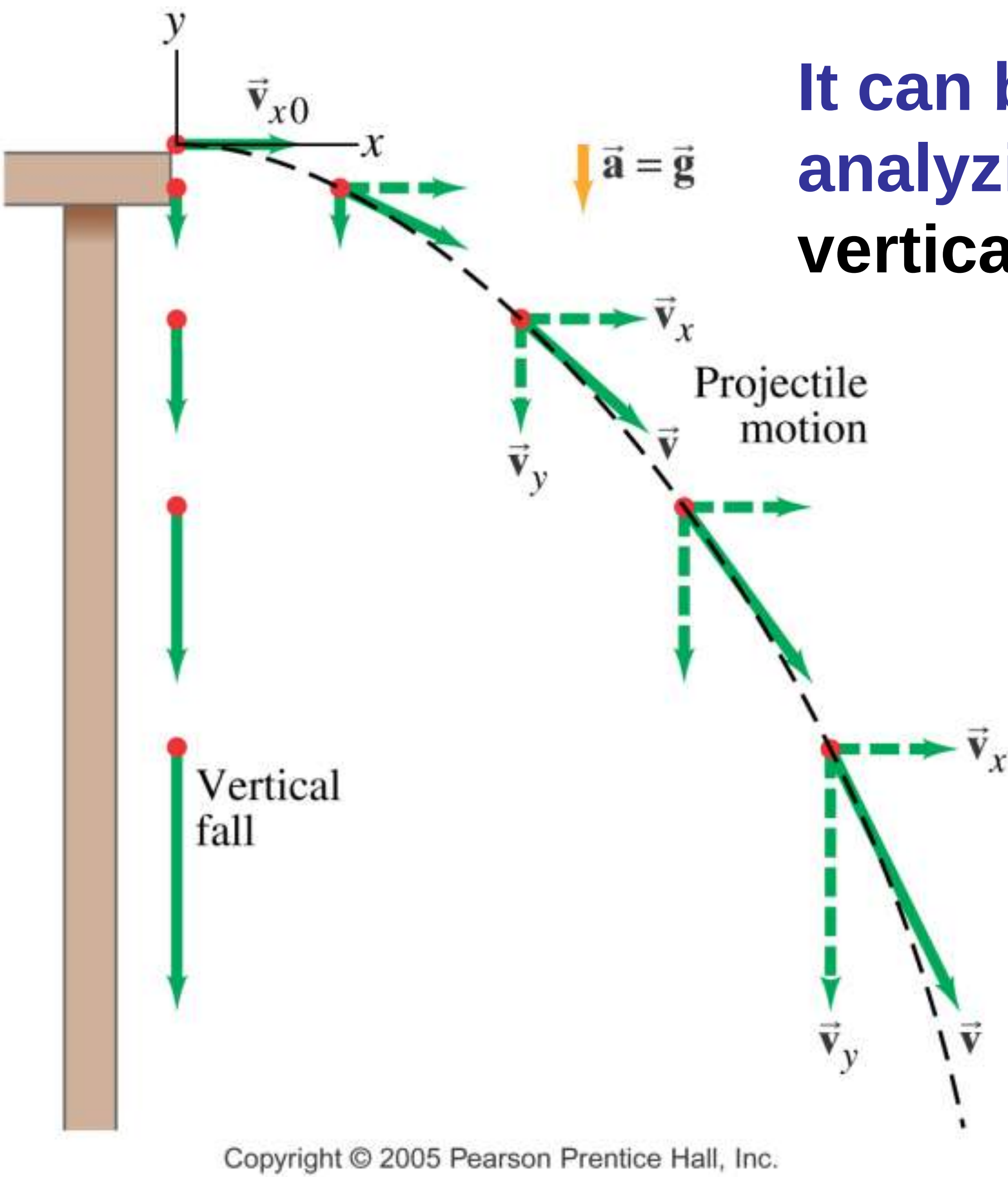
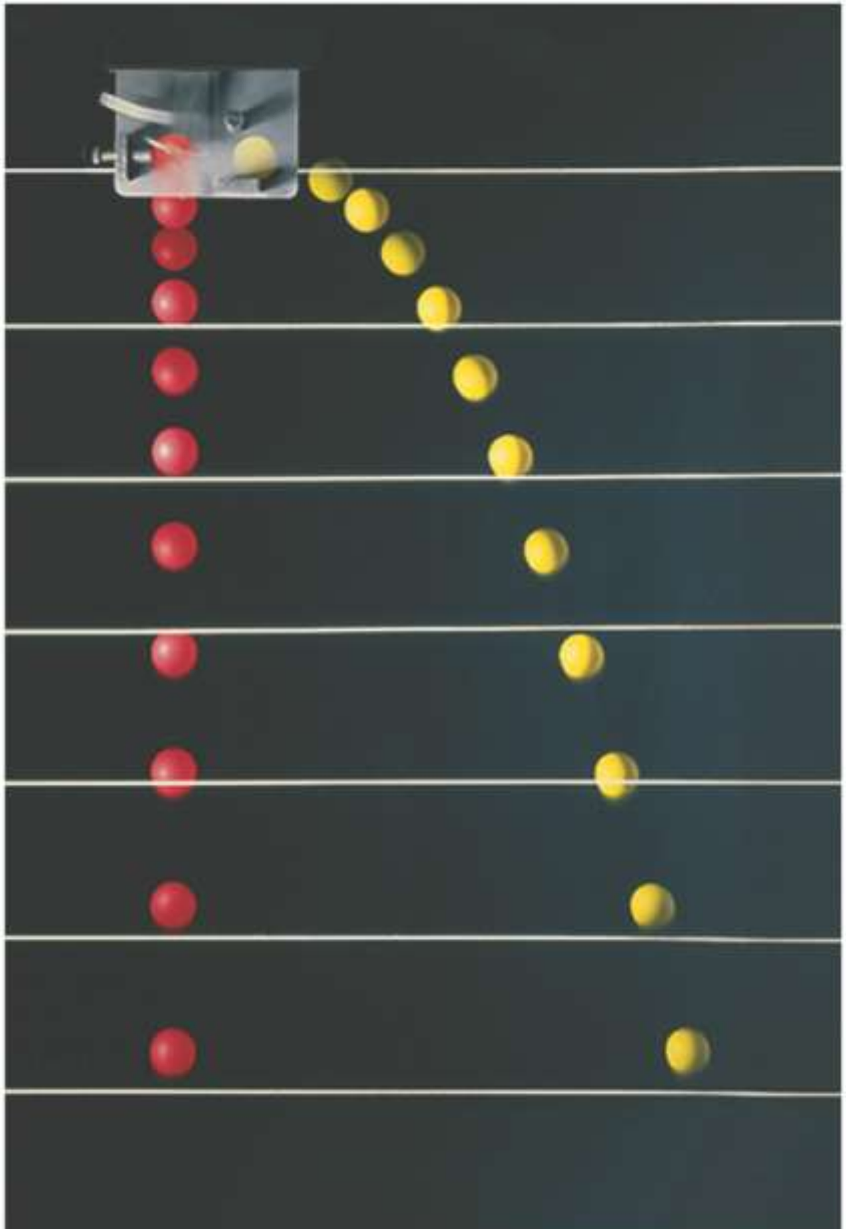
3-5 Projectile Motion



It can be understood by analyzing the **horizontal** and **vertical** motions separately.

3-5 Projectile Motion



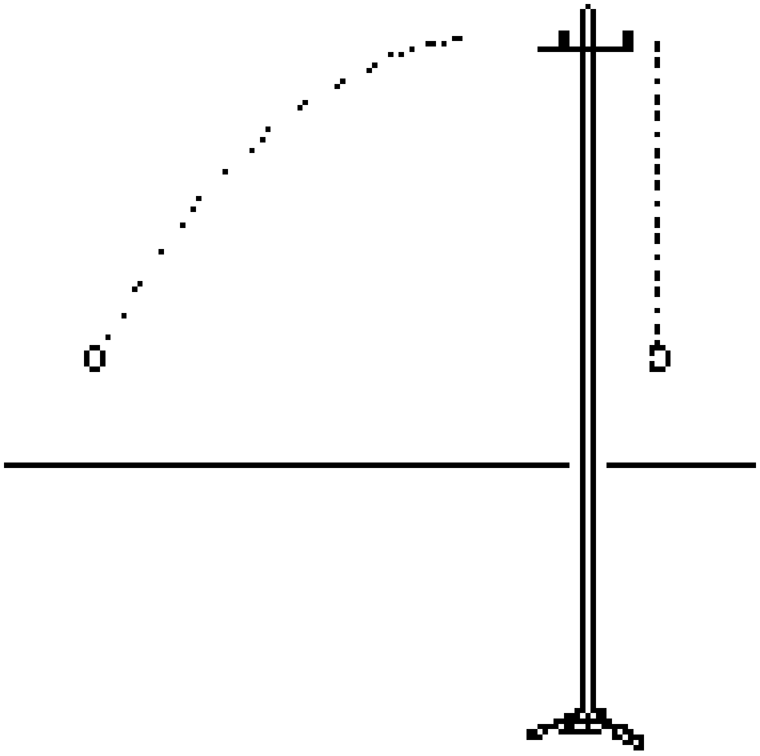
The speed in the *x*-direction is constant; in the *y*-direction the object moves with constant acceleration *g*.

This photograph shows two balls that start to fall at the same time. The one on the right has an initial speed in the *x*-direction. It can be seen that vertical positions of the two balls are identical at identical times, while the horizontal position of the yellow ball increases linearly.

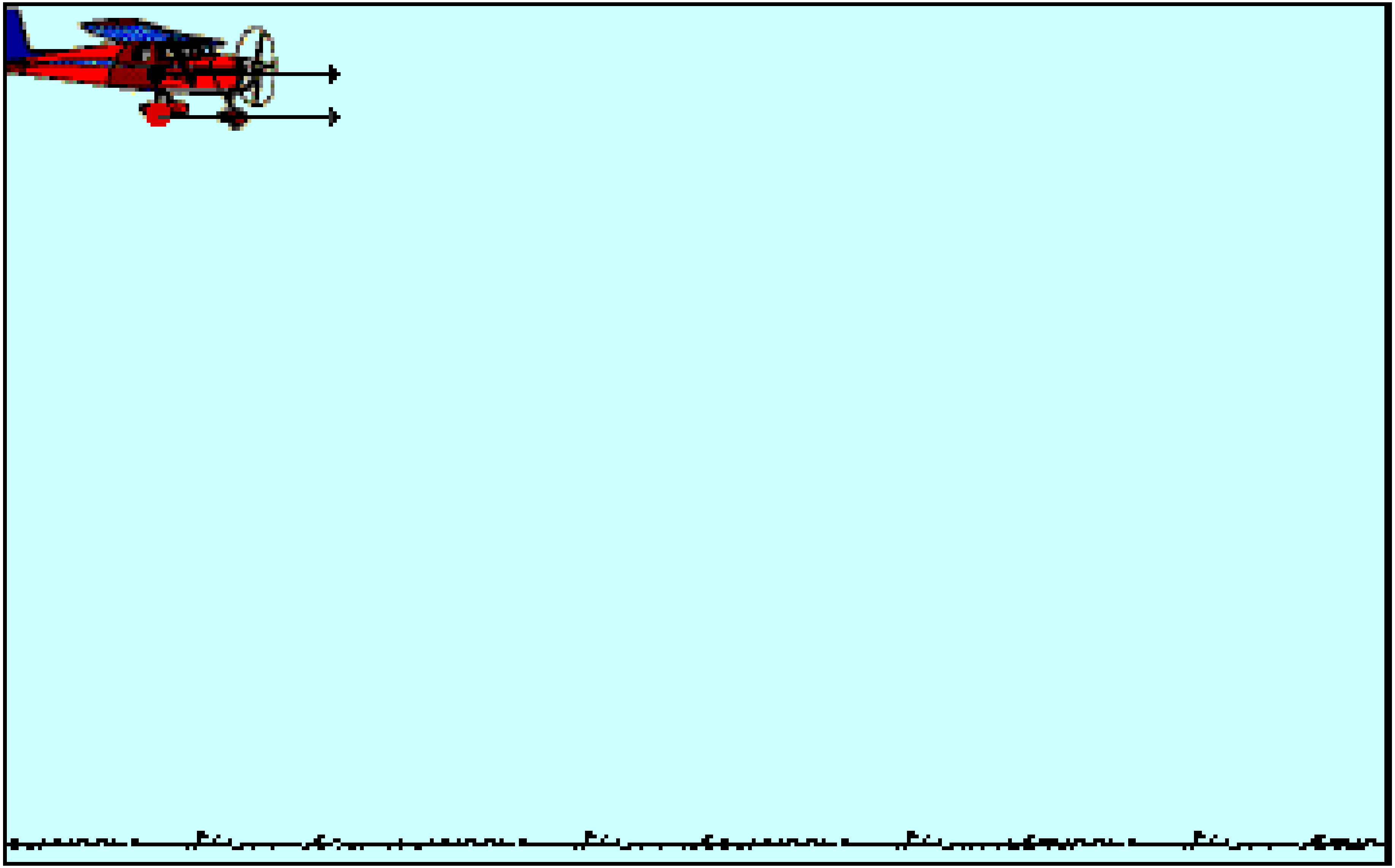
3-5 Projectile Motion

Galileo found…

An object projected horizontally will reach the ground in the same time as an object dropped.

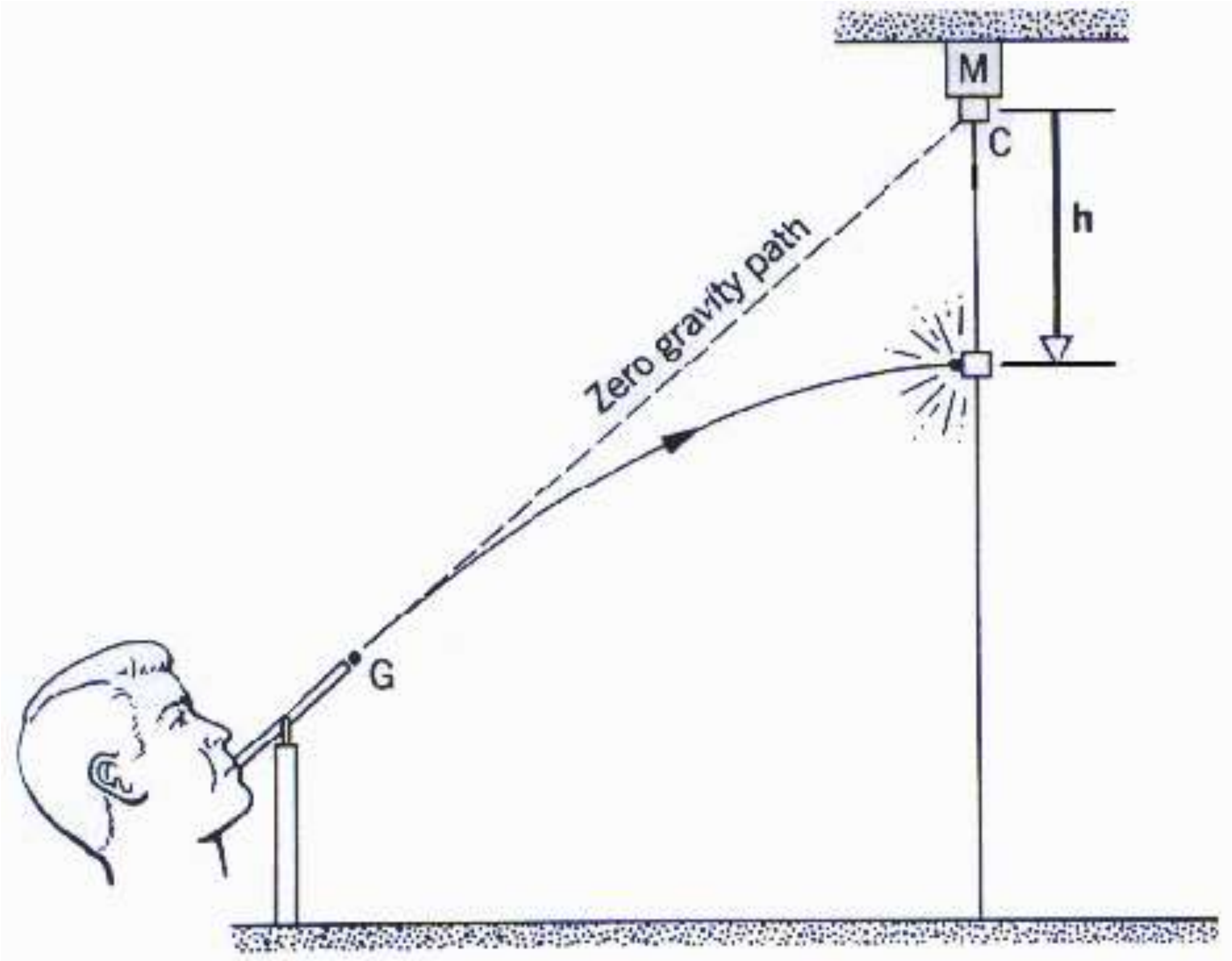


3-5 Projectile Motion



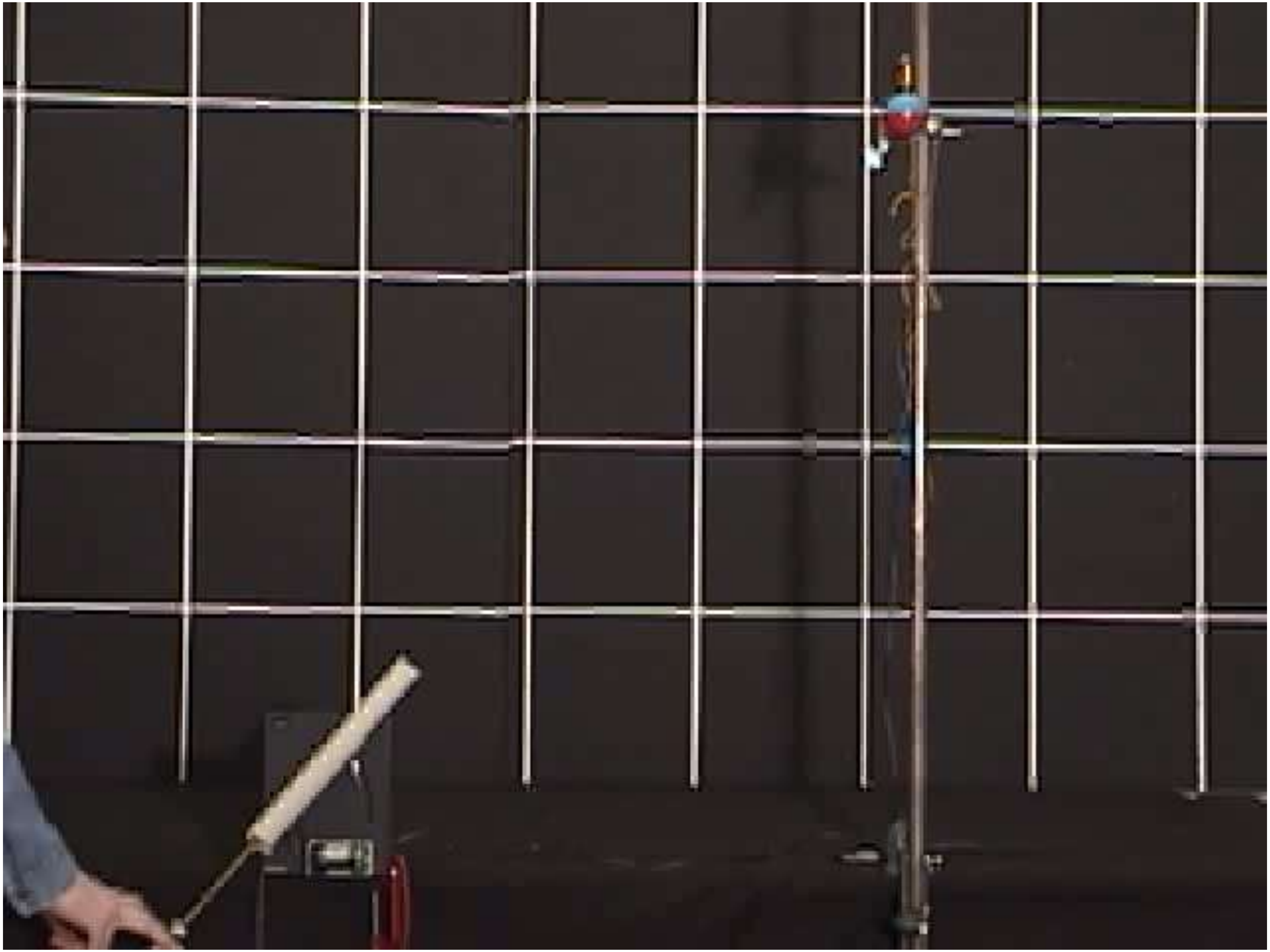
3-5 Projectile Motion

MONKEY HUNTER

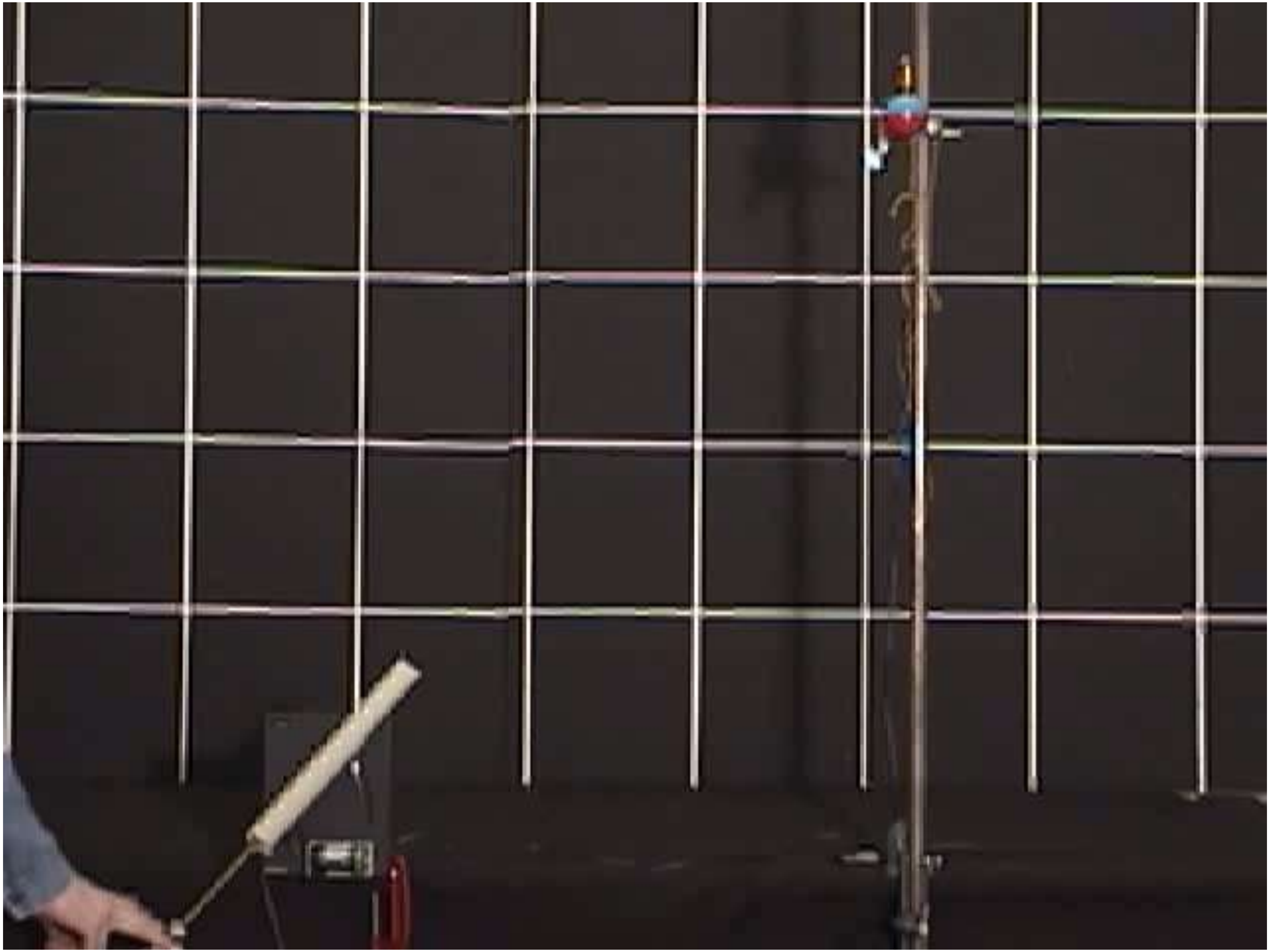


Monkey Hunter **Gizmo**

Monkey Hunter Video



Monkey Hunter Video Slow Motion

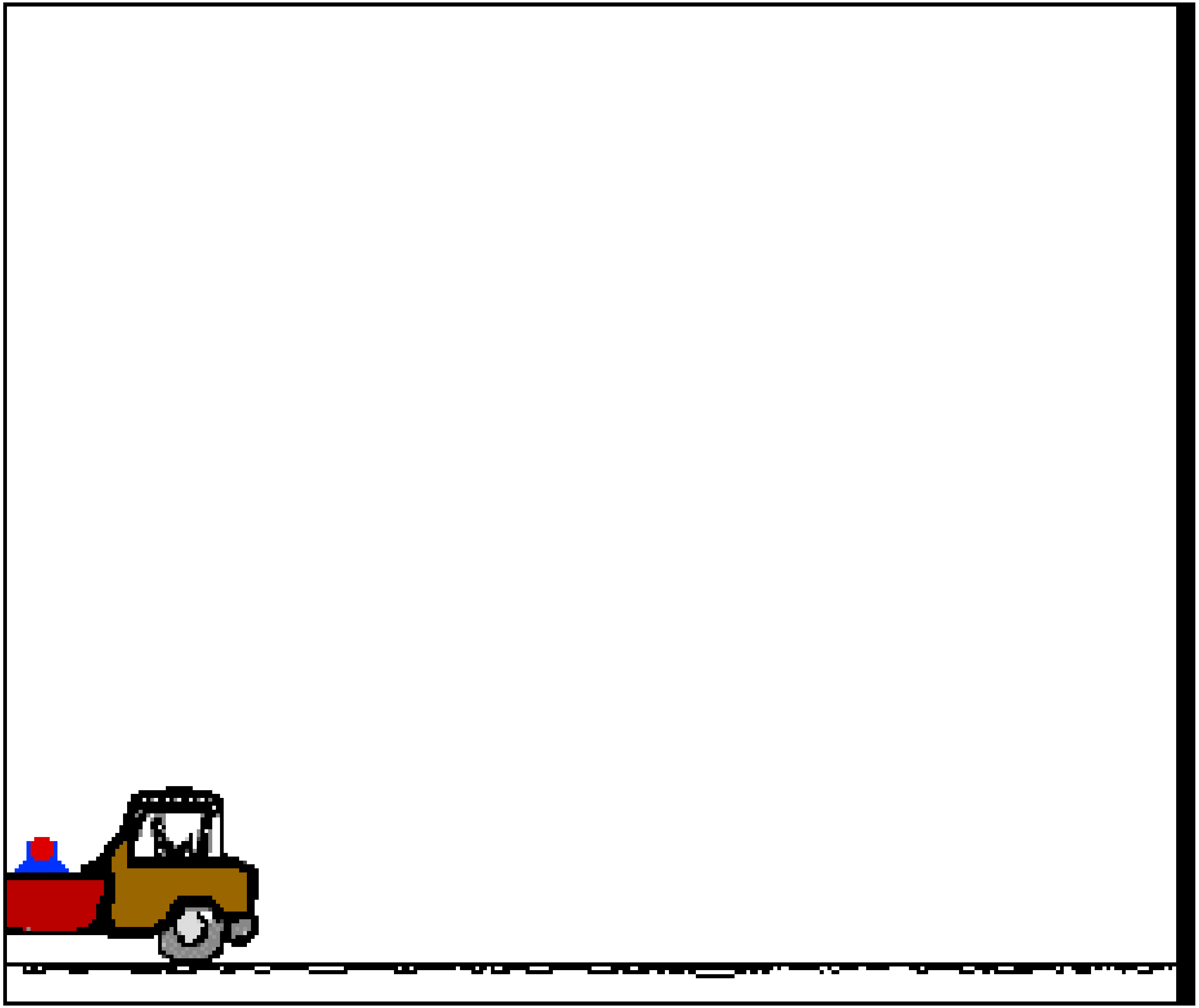


3-5 Projectile Motion

Galileo also found…

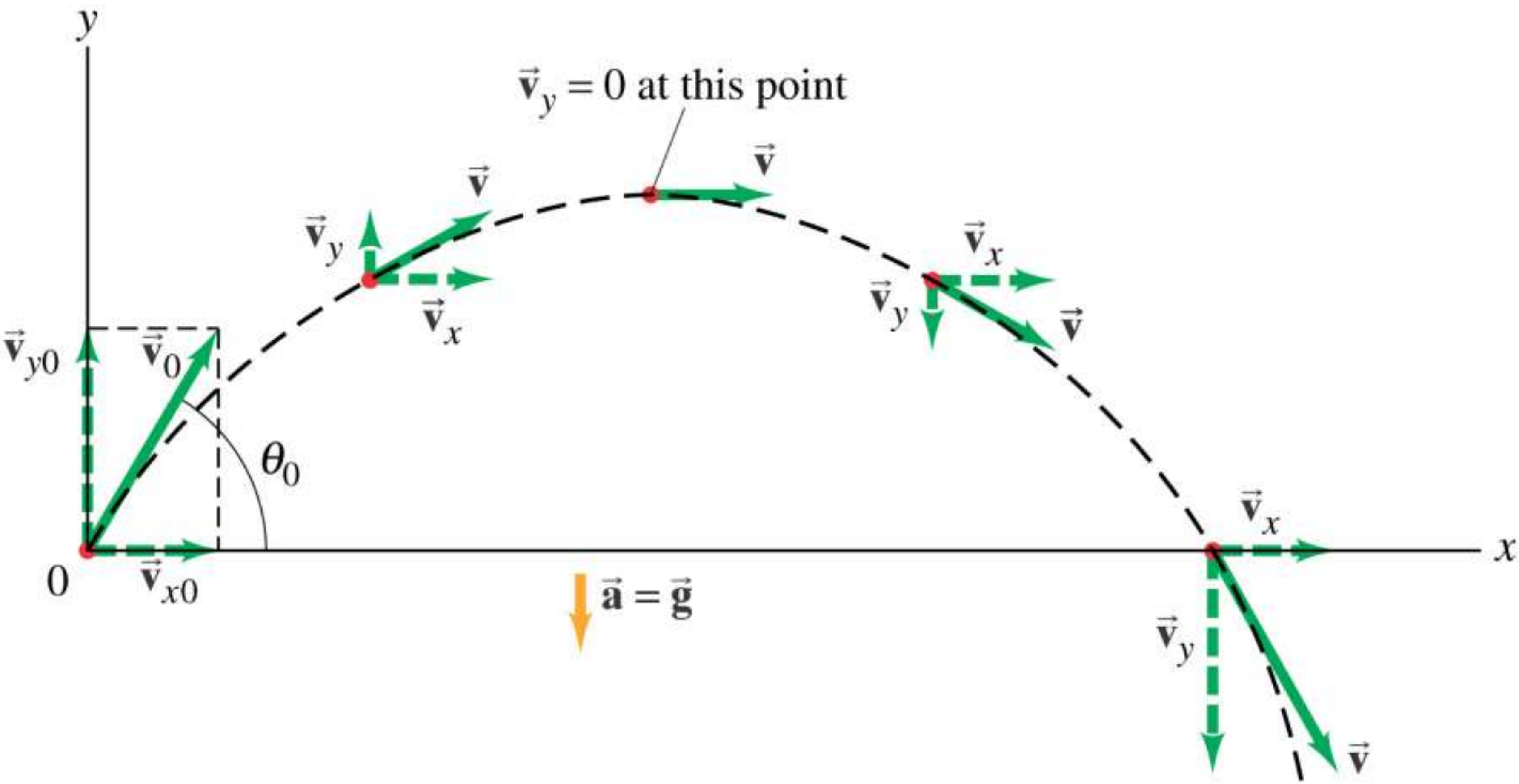
If an object projected at an angle, the object will land with the same magnitude and angle as take off.

3-5 Projectile Motion



3-5 Projectile Motion

If an object is **launched** at an initial angle of θ_0 with the horizontal, the analysis is similar except that the initial velocity has a **vertical** component.



3-5 Projectile Motion

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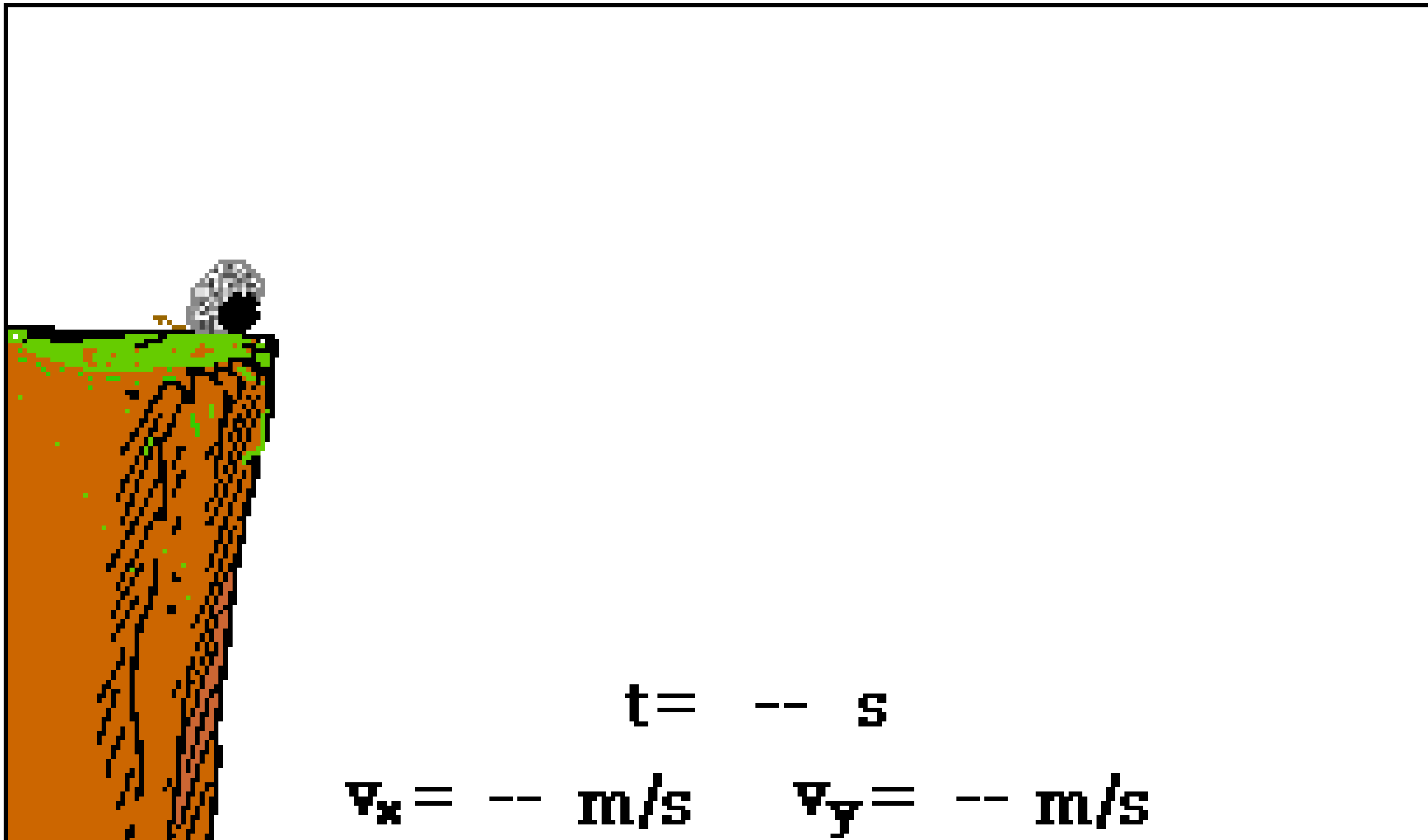
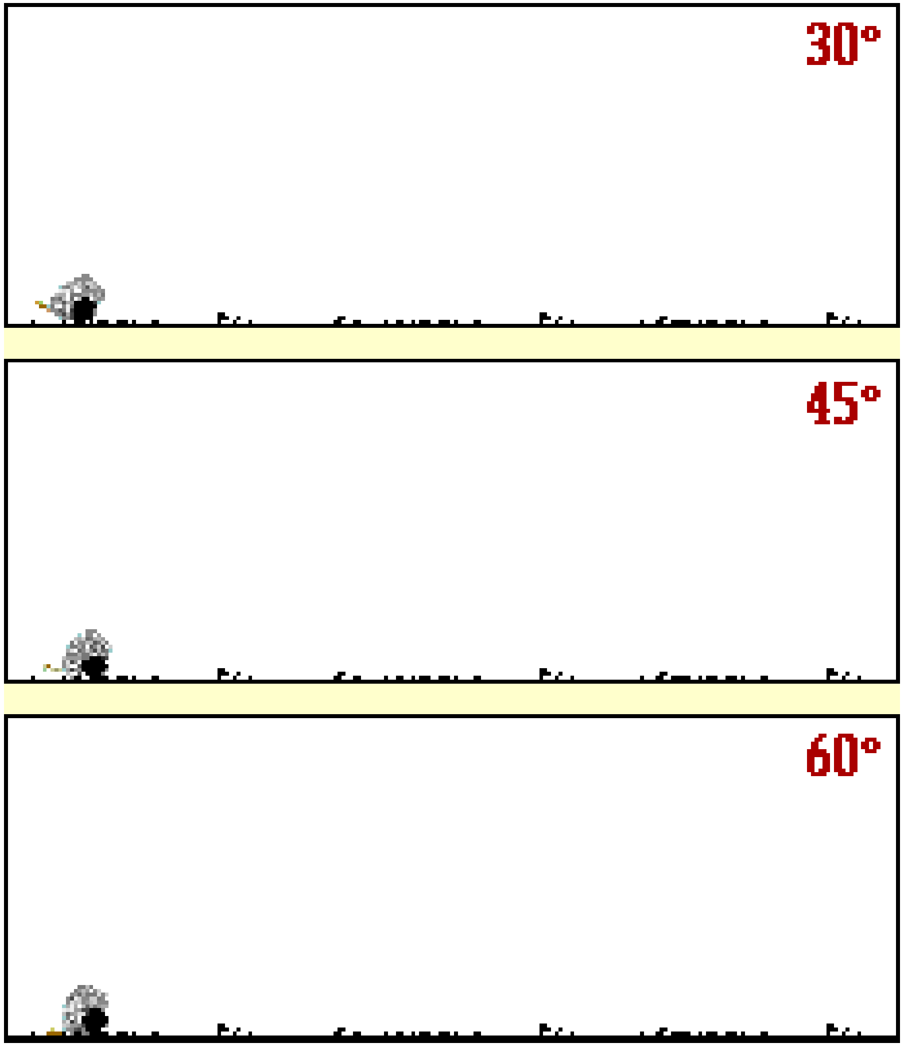


3-5 Projectile Motion

Maximum Range is when an object is launched at an angle of 45 degrees.



3-5 Projectile Motion

Time for a Gizmo!

3-6 Solving Problems Involving Projectile Motion

Projectile motion is motion with constant acceleration in two dimensions, where the acceleration is *g* and is down.

TABLE 3–2 Kinematic Equations for Projectile Motion
(y positive upward; $a_x = 0$, $a_y = -g = -9.80\ \text{m/s}^2$)

Horizontal Motion ($a_x = 0$, v_x = constant)	V	ertical Motion† ($a_y = -g$ = constant)
$v_x = v_{x0}$	(Eq. 2–11a)	$v_y = v_{y0} - gt$
$x = x_0 + v_{x0}t$	(Eq. 2–11b)	$y = y_0 + v_{y0}t - \frac{1}{2}gt^2$
	(Eq. 2–11c)	$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

† If y is taken positive downward, the minus (−) signs in front of g become + signs.

3-6 Solving Problems Involving Projectile Motion

1. **Read** the problem carefully, and choose the object(s) you are going to analyze.

2. **Draw** a diagram.

3. **Choose** an origin and a coordinate system.

4. **Decide** on the **time** interval; this is the same in both directions, and includes only the time the object is moving with constant acceleration *g*.

5. **Examine** the *x* and *y* motions separately.

3-6 Solving Problems Involving Projectile Motion

6. **List** known and unknown quantities. Remember that v_x never changes, and that $v_y = 0$ at the highest point.

7. **Plan** how you will proceed. Use the appropriate equations; you may have to combine some of them.

3-7 Projectile Motion Is Parabolic

In order to demonstrate that projectile motion is **parabolic**, we need to write *y* as a function of *x*. When we do. we find that it has the form: $y = Ax - Bx^2$

This is indeed the **equation** for a parabola.

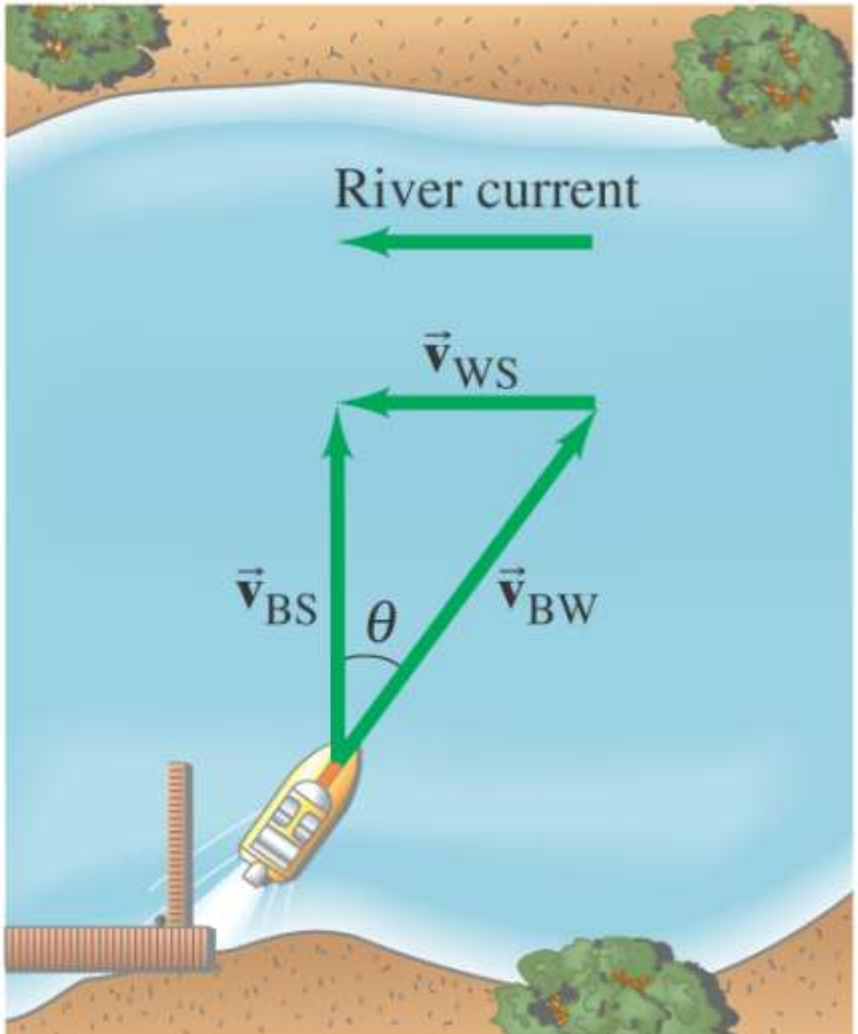
3-8 Relative Velocity

We already considered **relative speed** in one dimension; it is similar in two dimensions except that we must add and subtract velocities as **vectors**.

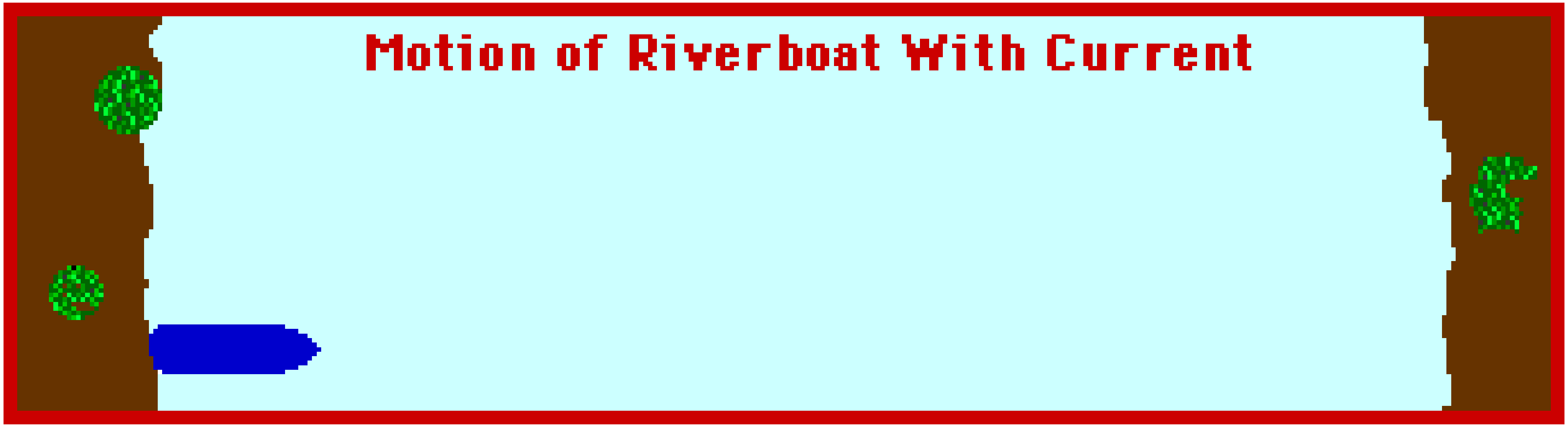
Each velocity is labeled first with the **object**, and second with the reference **frame** in which it has this velocity. Therefore, v_{WS} is the velocity of the water in the shore frame, v_{BS} is the velocity of the boat in the shore frame, and v_{BW} is the velocity of the boat in the water frame.

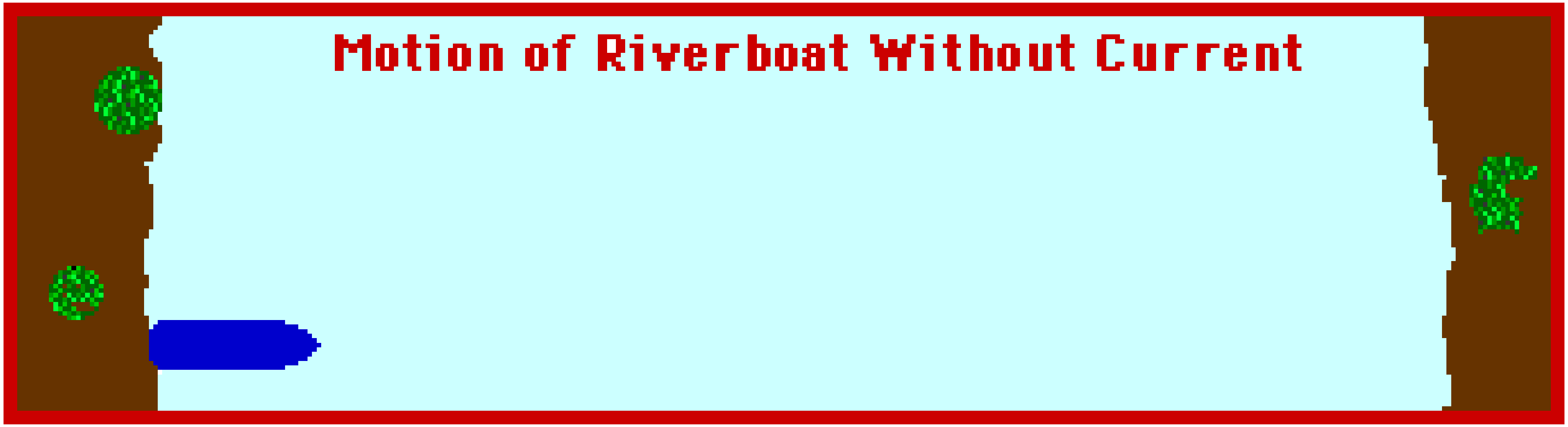
3-8 Relative Velocity

In this case, the relationship between the three velocities is:



$$\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS} \quad \textbf{(3-6)}$$


Motion of Riverboat With Current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