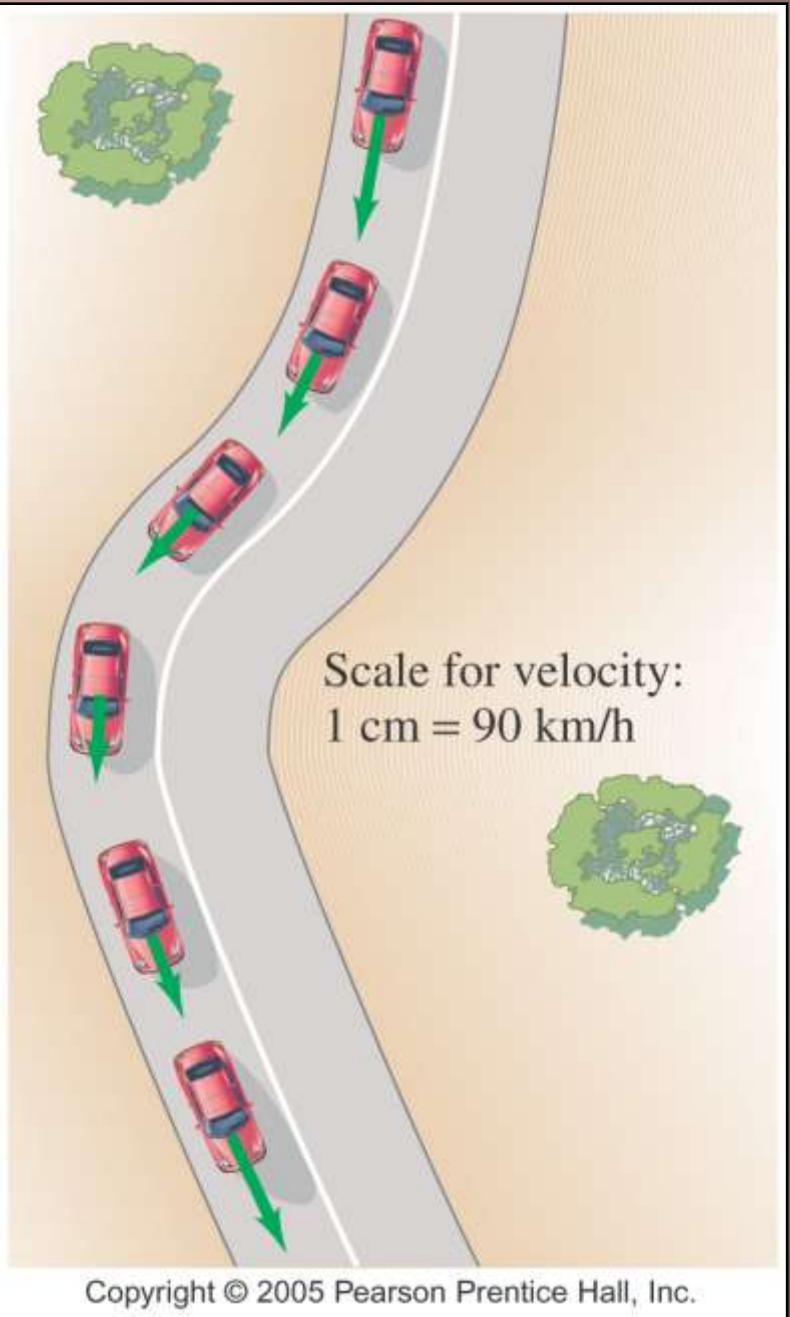
Chapter 3

Projectile Motion

3.1 Vectors and Scalars



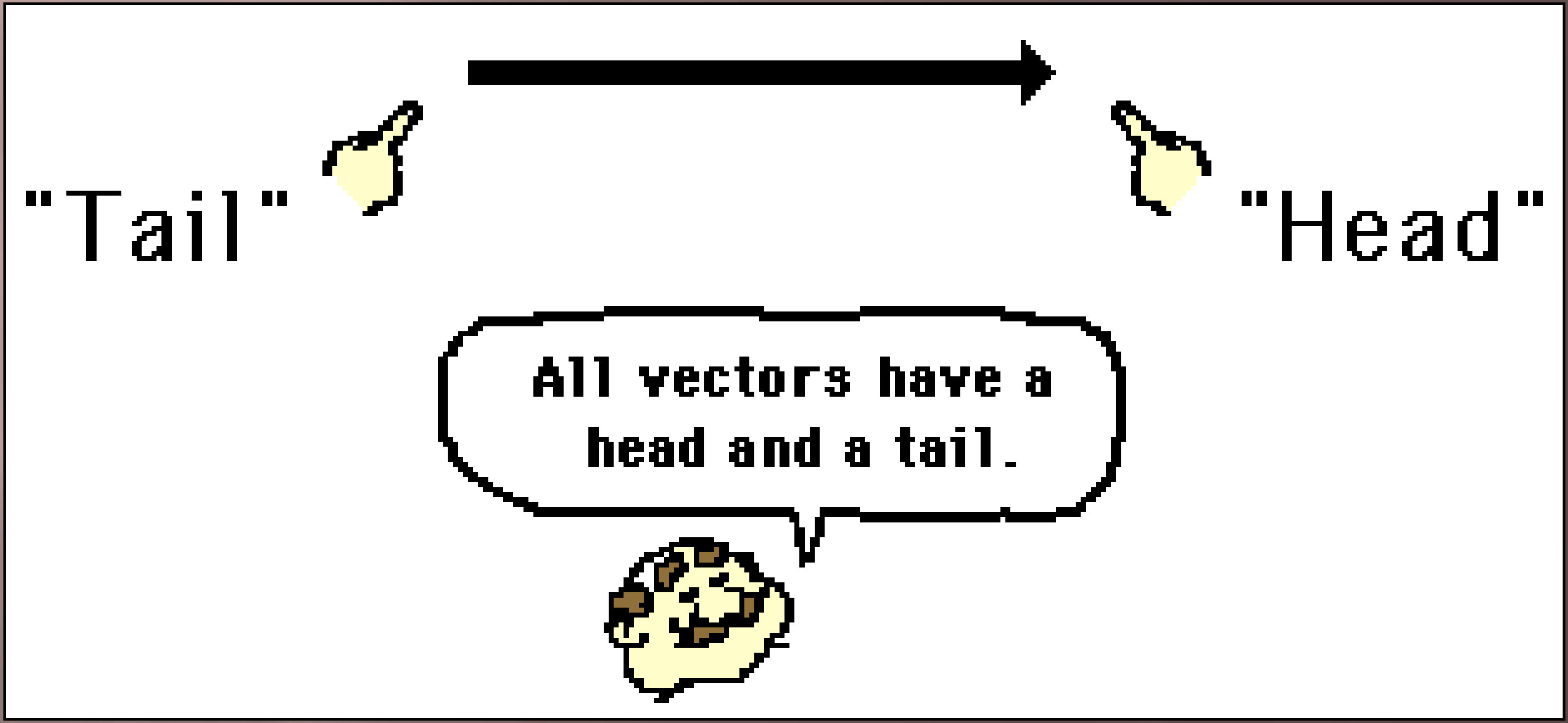
A vector has magnitude as well as direction.

Some vector quantities: displacement, velocity, force, momentum

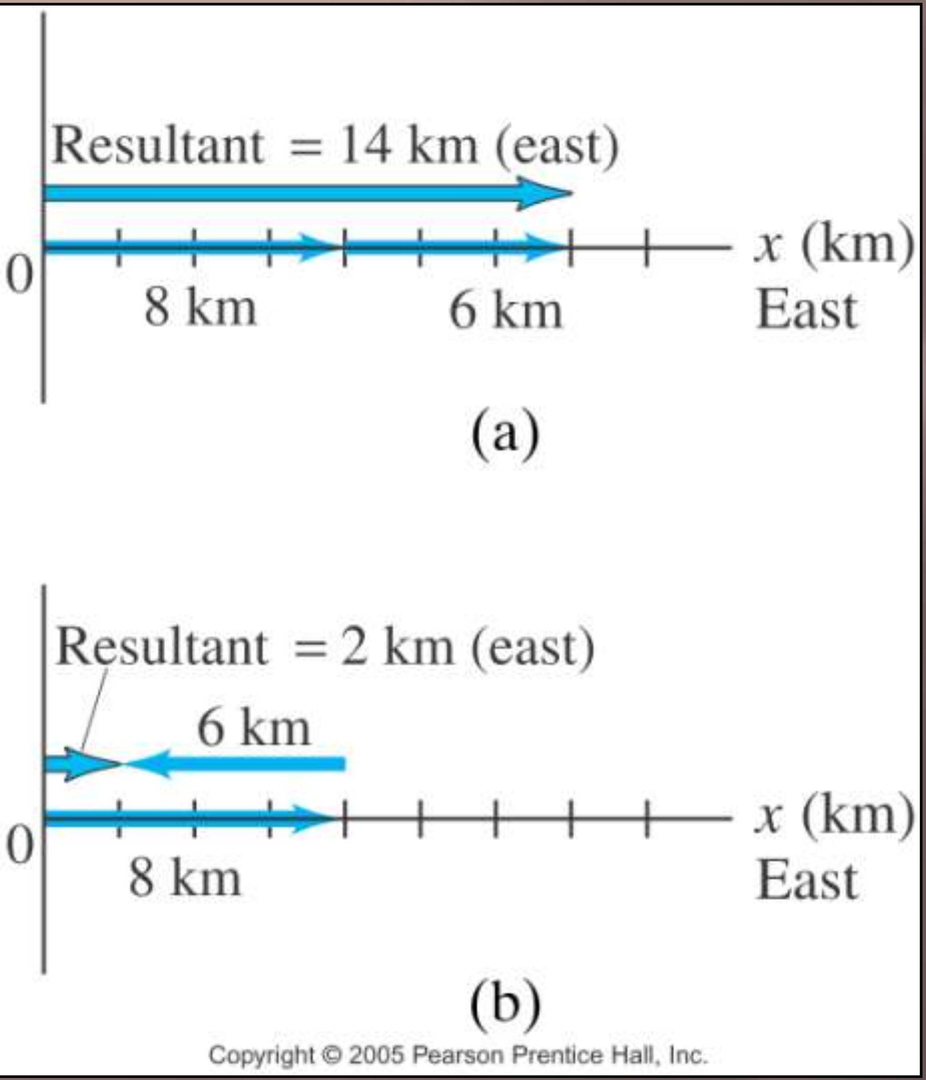
A scalar has only a magnitude.

Some scalar quantities: mass, time, temperature, speed, distance

3.2 Velocity Vectors



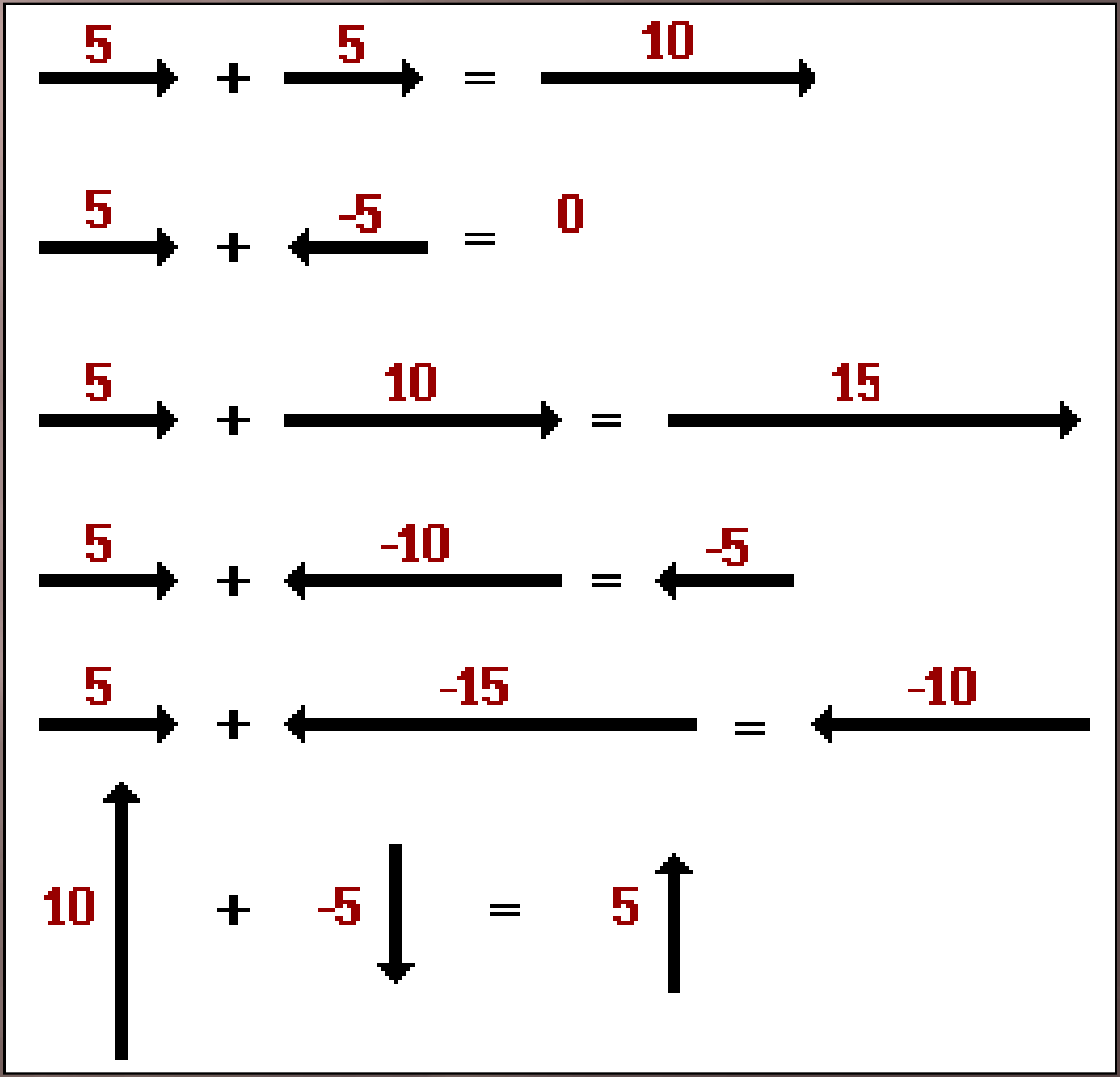
3.2 Velocity Vectors



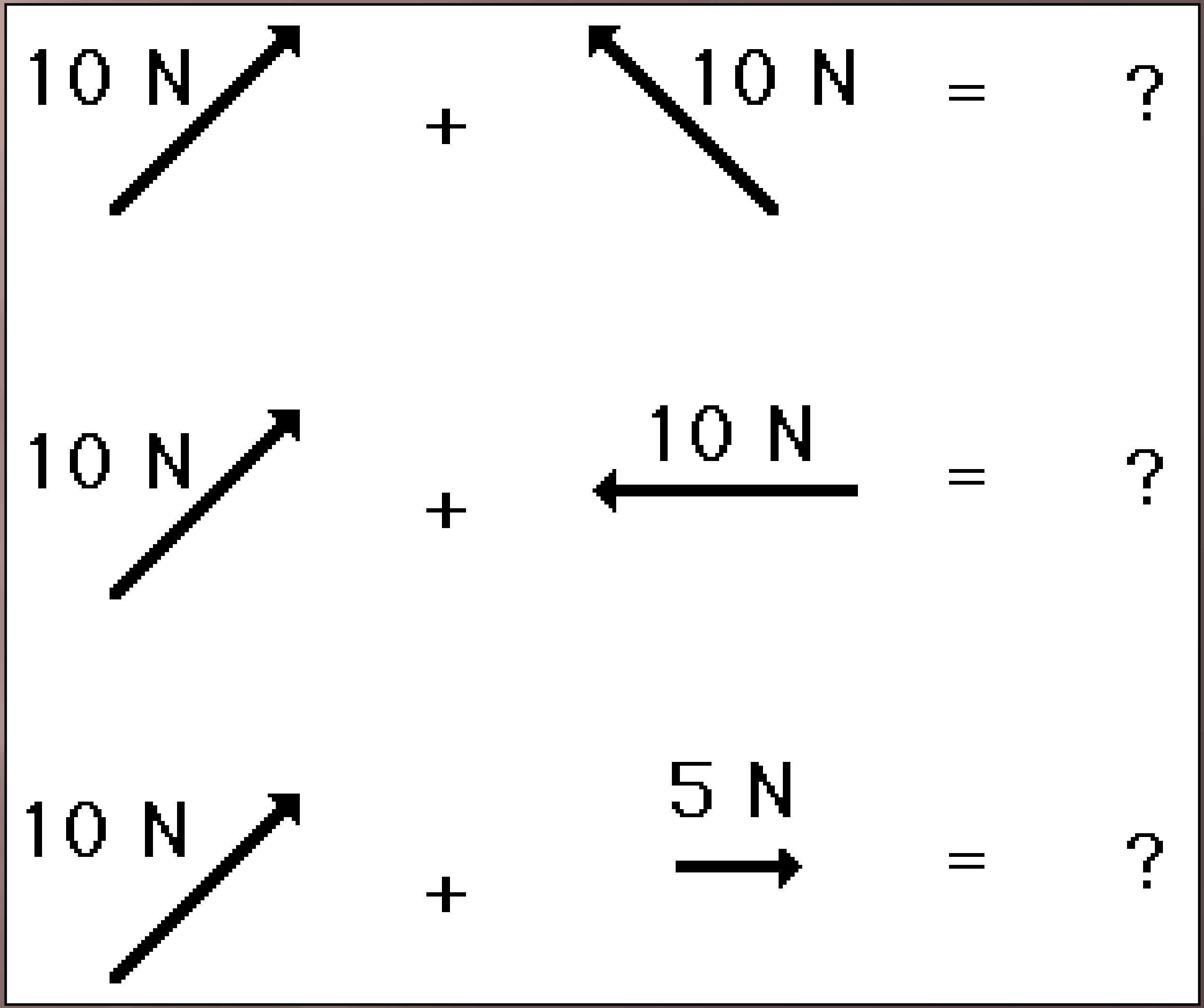
For vectors in one dimension, simple **addition** and **subtraction** are all that is needed.

You do need to be careful about the **signs**, as the figure indicates.

3.2 Velocity Vectors

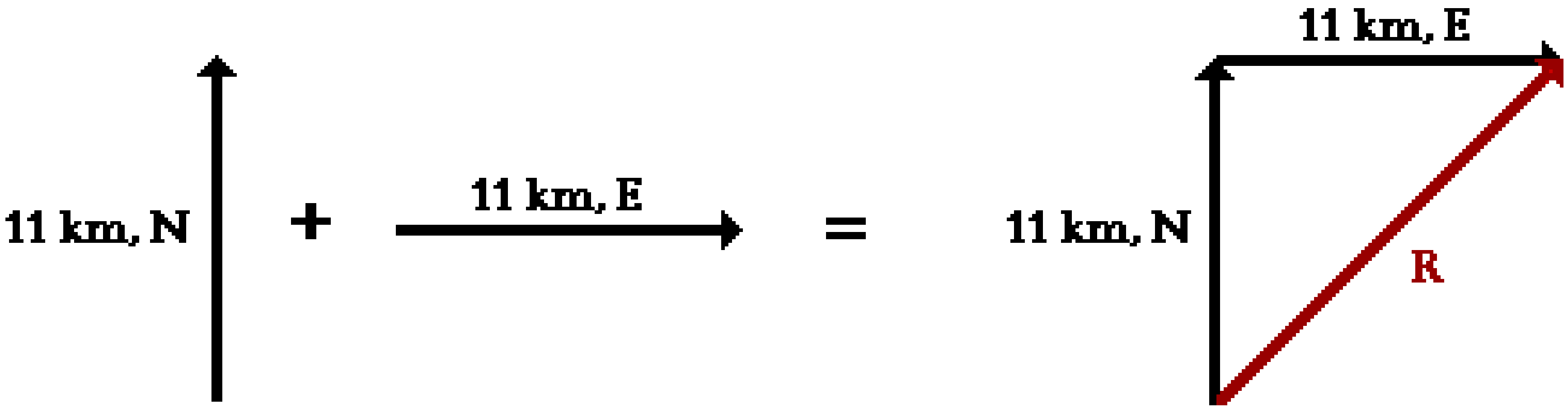


3.2 Velocity Vectors



Try to add up these vectors.

3.2 Velocity Vectors



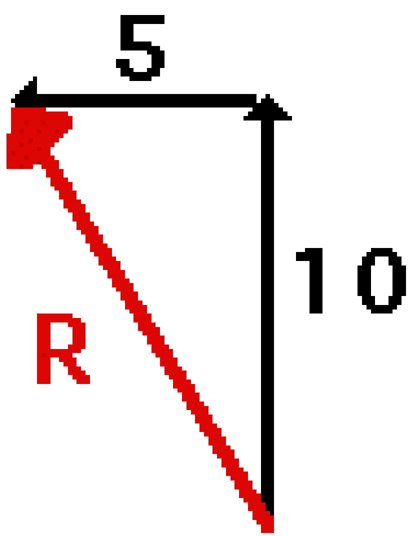
$$11^2 + 11^2 = R^2$$

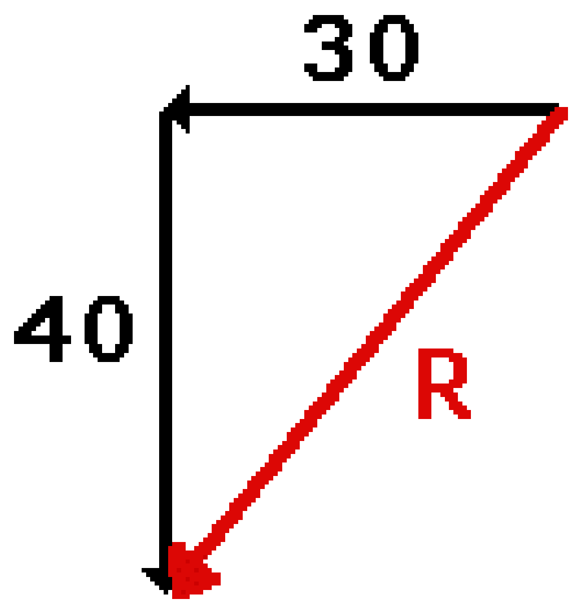
$$242 = R^2$$

$$15.6 = R$$

3.2 Velocity Vectors

Practice A	Practice B
10 km, North	30 km, West
+	+
5 km, West	40 km, South

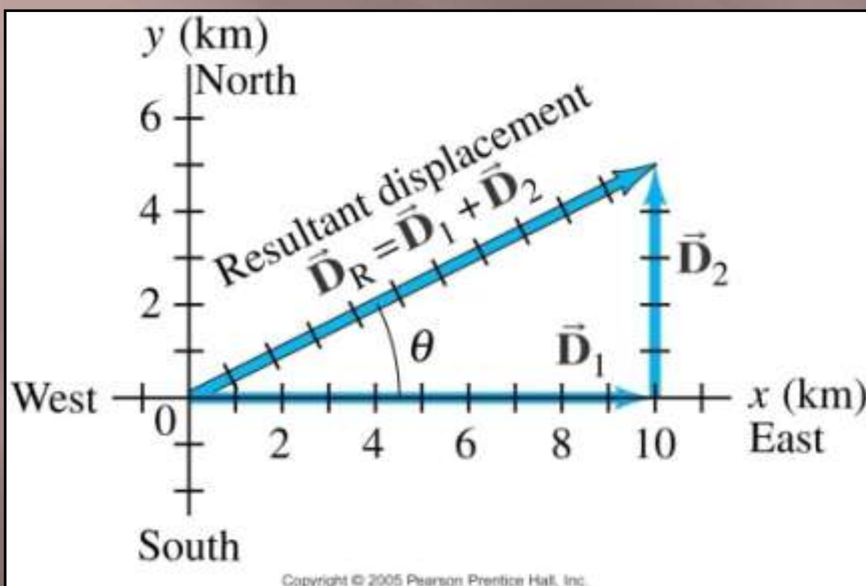


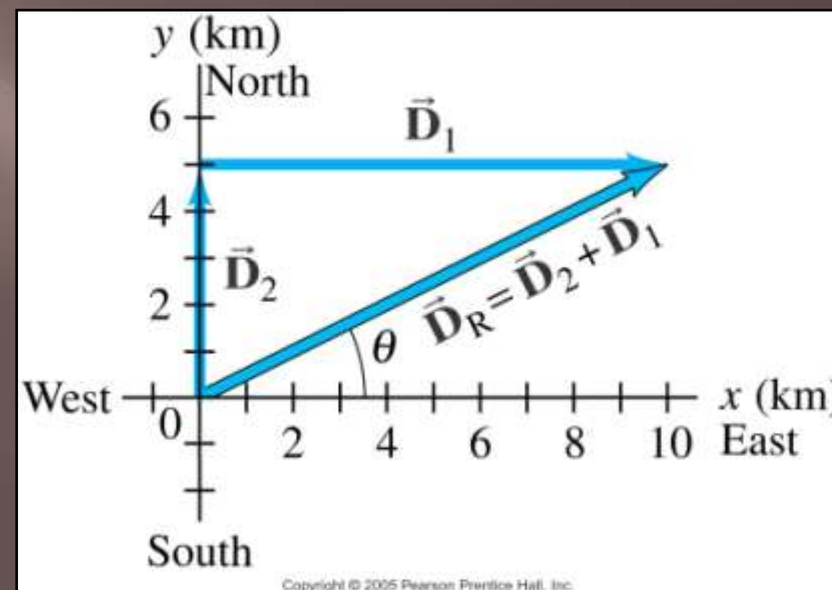


3.2 Velocity Vectors

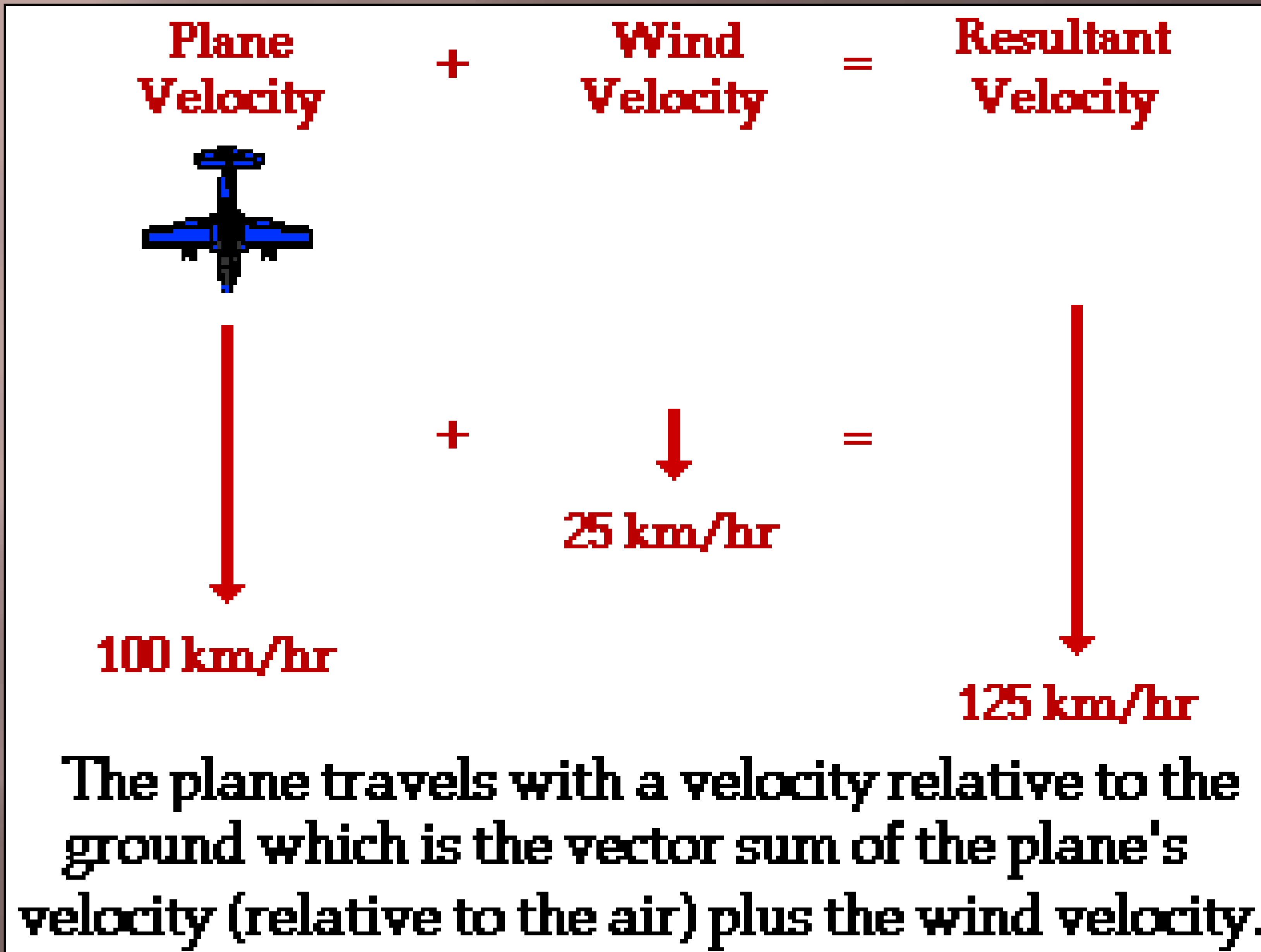
Adding the vectors in the opposite order gives the same result:

$$\vec{\mathbf{V}}_1 + \vec{\mathbf{V}}_2 = \vec{\mathbf{V}}_2 + \vec{\mathbf{V}}_1$$

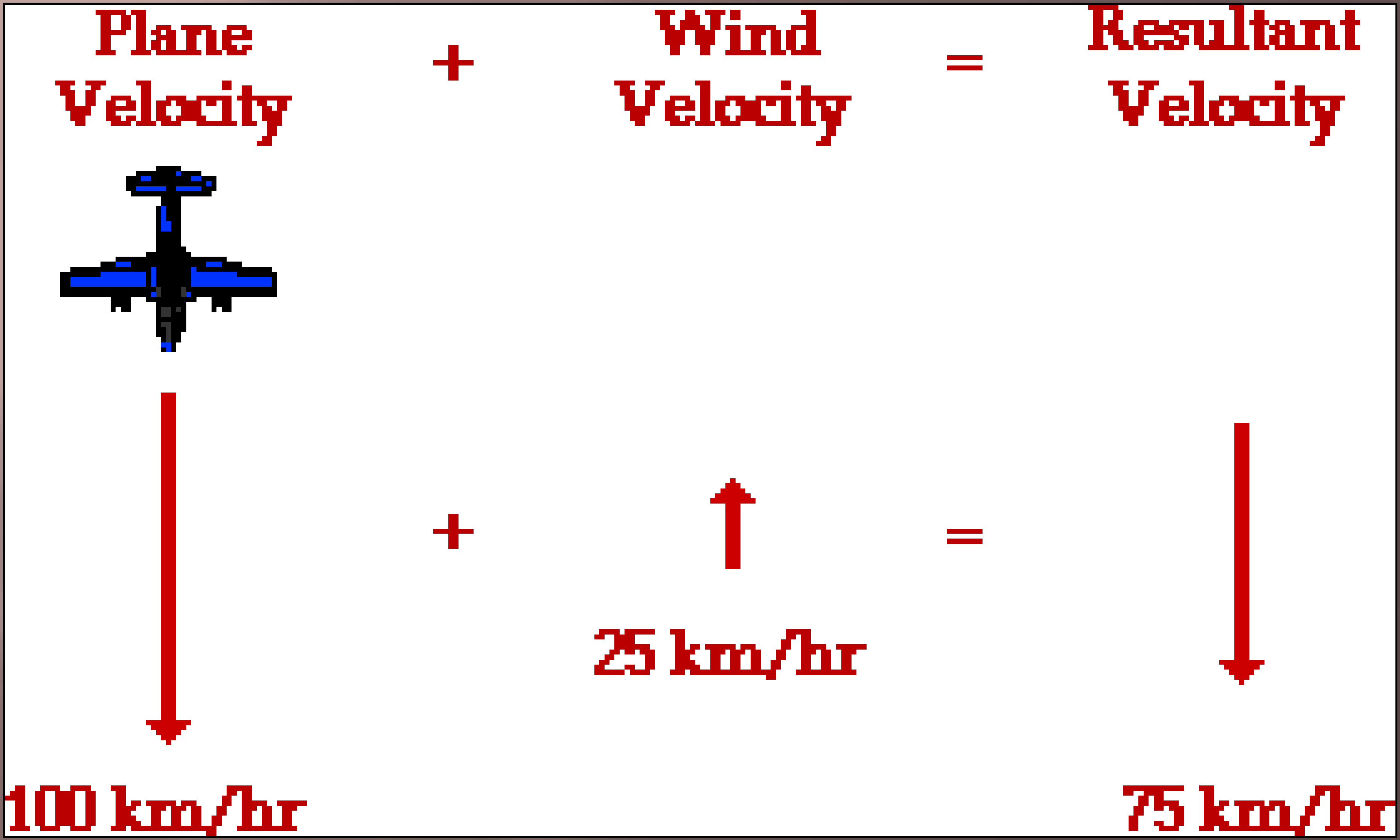




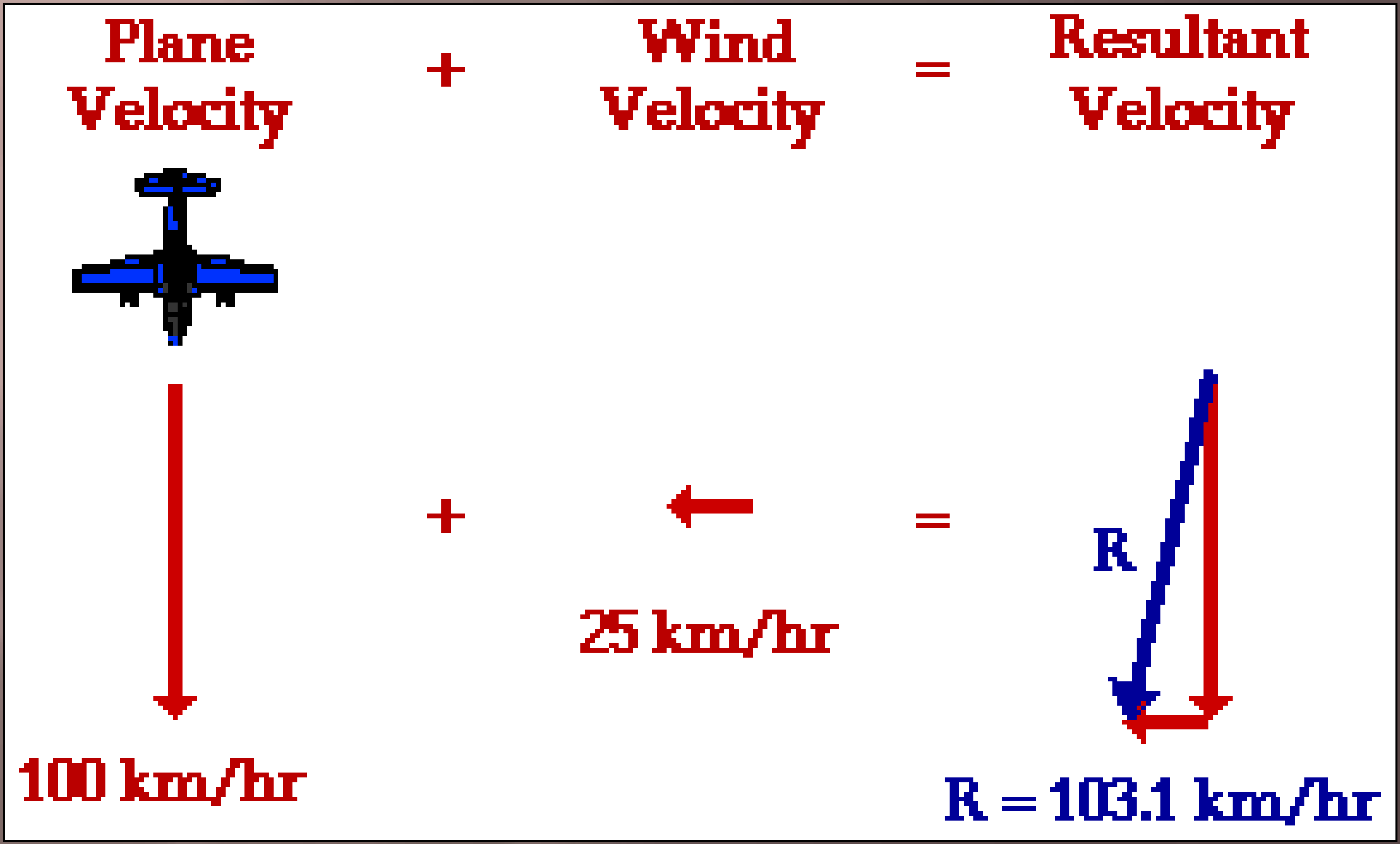
3.2 Velocity Vectors

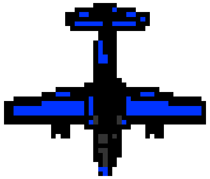
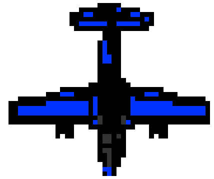
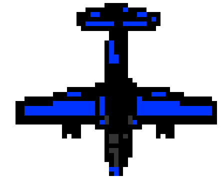


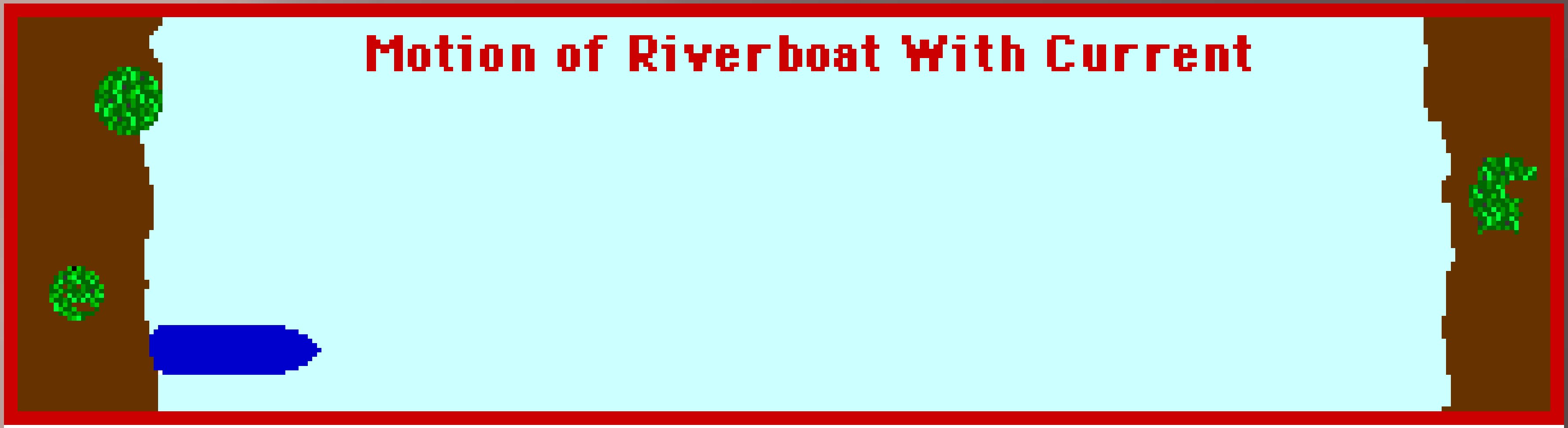
3.2 Velocity Vectors

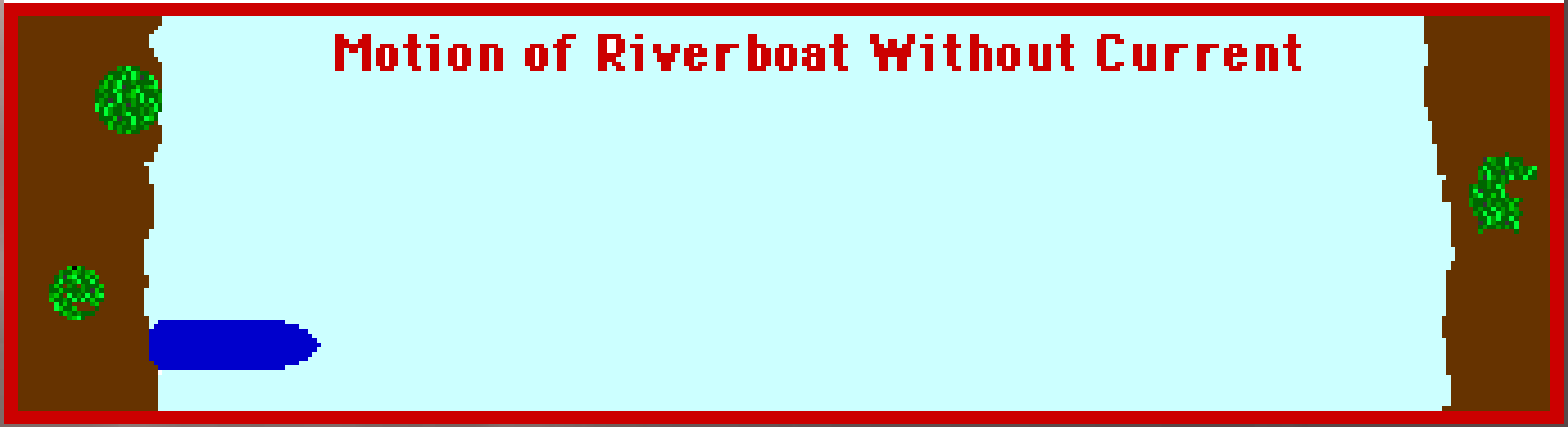


3.2 Velocity Vectors



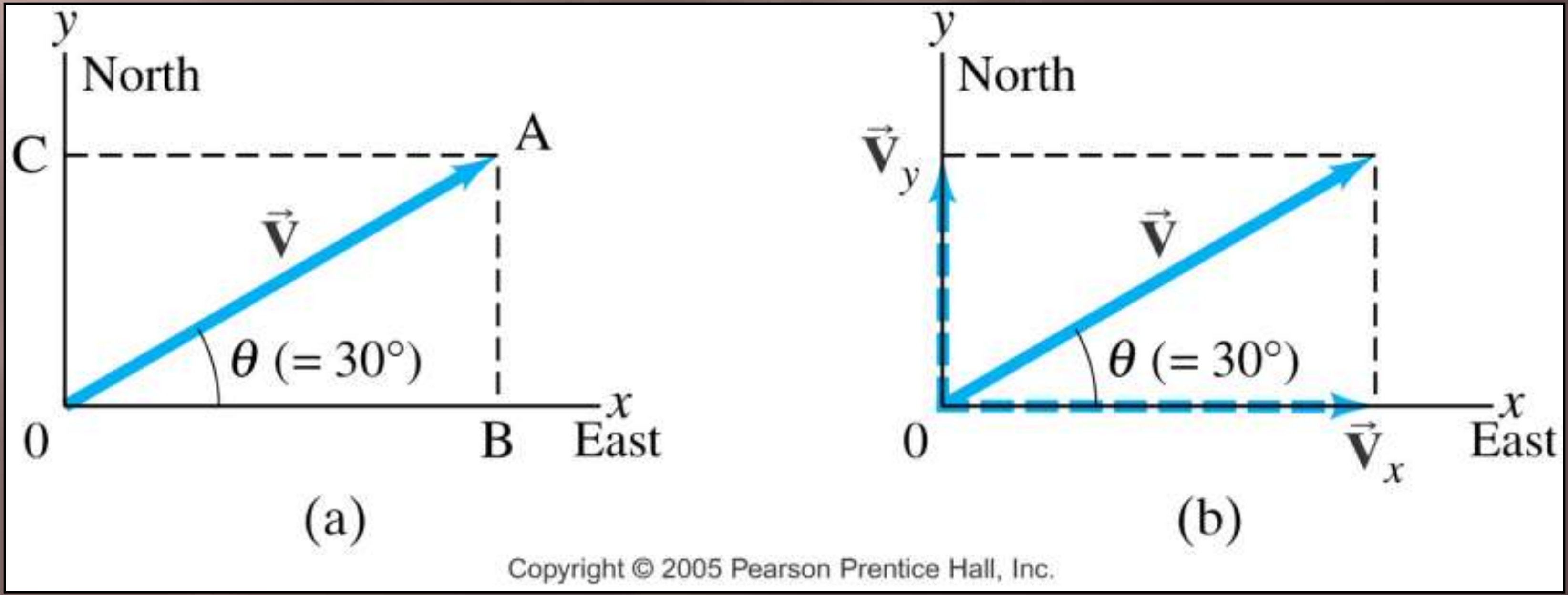
Tailwind	Headwind	Crosswind
		


Motion of Riverboat With Current


Motion of Riverboat Without Current

3.3 Components of Vectors

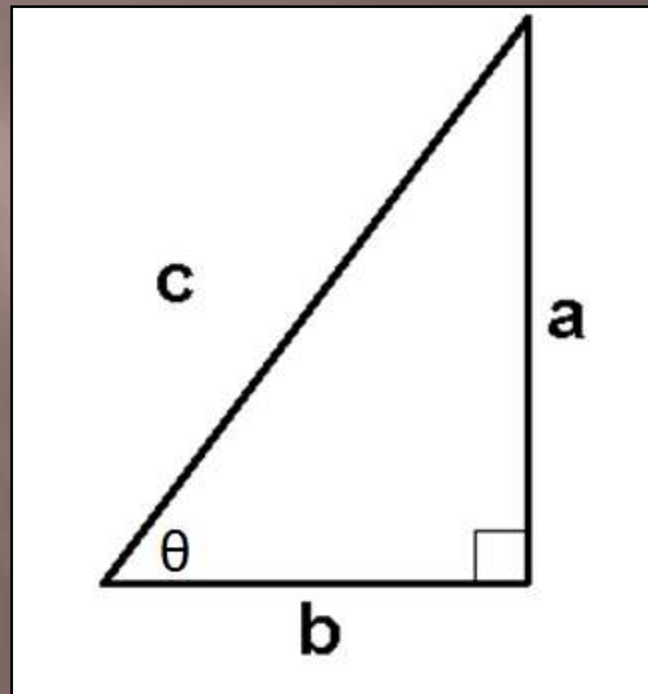
Any vector can be expressed as the **sum** of two other vectors, which are called its **components**. Usually the other vectors are chosen so that they are **perpendicular** to each other.



3.3 Components of Vectors

Time for a Gizmo!

Consider a Right Triangle.

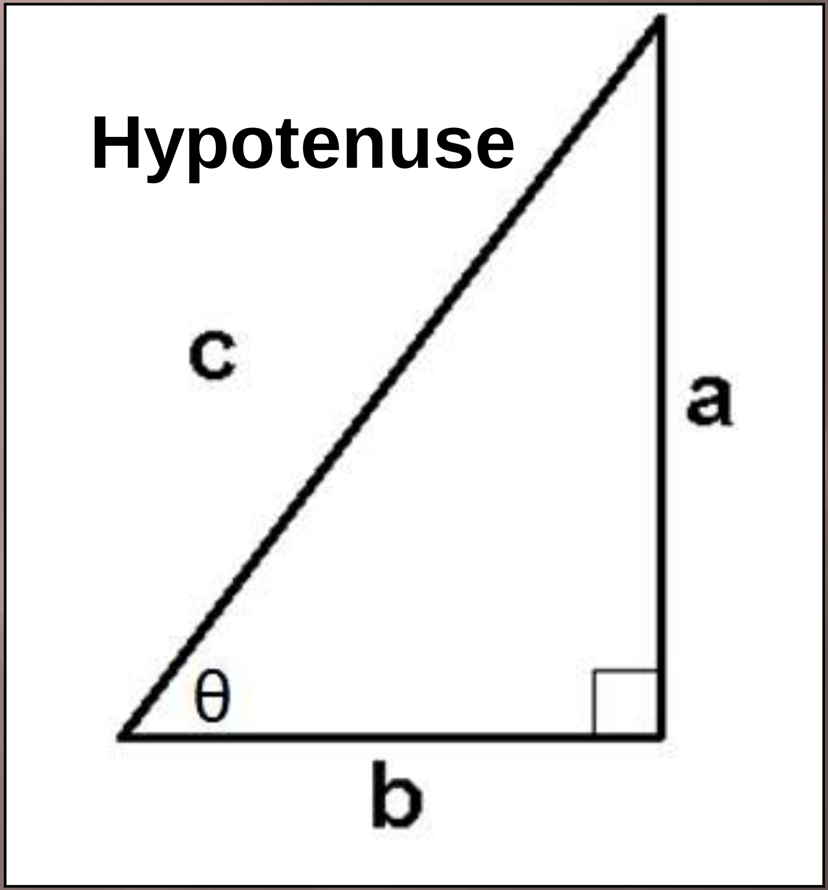


<u>Note</u> – a is the leg opposite θ

c is the leg opposite our right angle

b is the leg adjacent to θ

So that we have the following right triangle.



Opposite side from θ

Adjacent side from θ

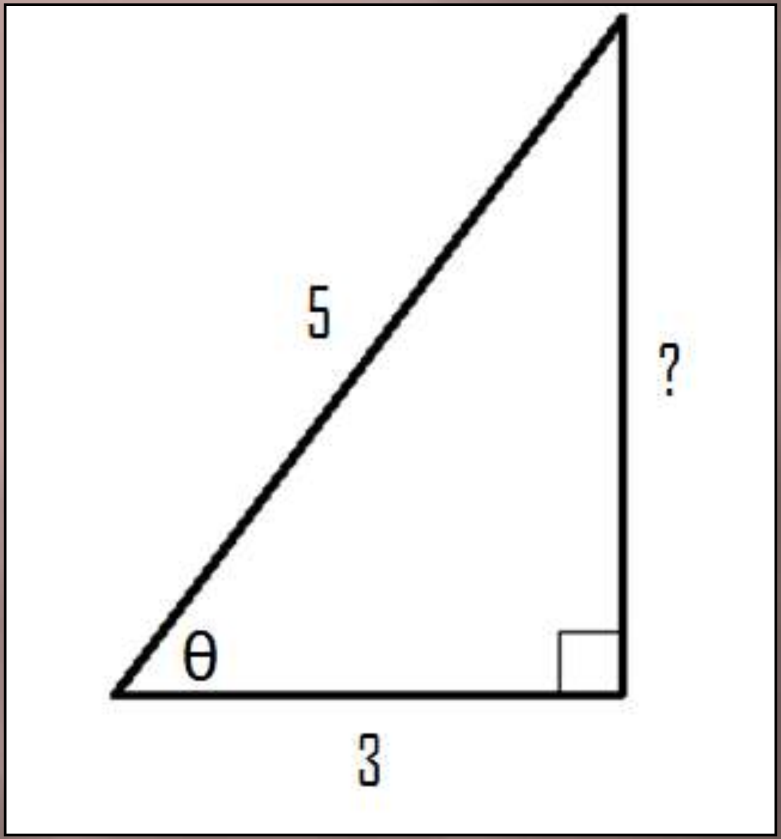
Three trigonometric ratios are defined as follows:

$$\sin\theta = \frac{o}{h}$$

$$\cos\theta = \frac{a}{h}$$

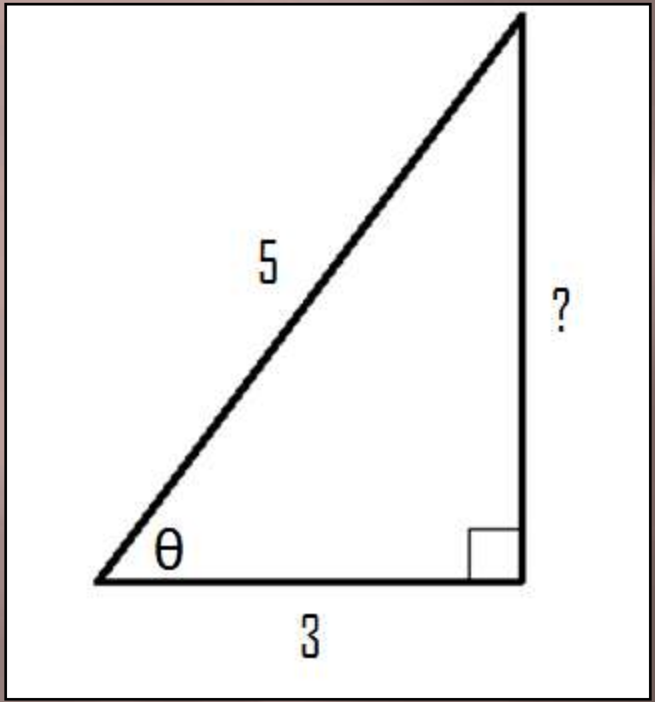
$$\tan\theta = \frac{o}{a}$$

What are the trigonometric ratios for θ?



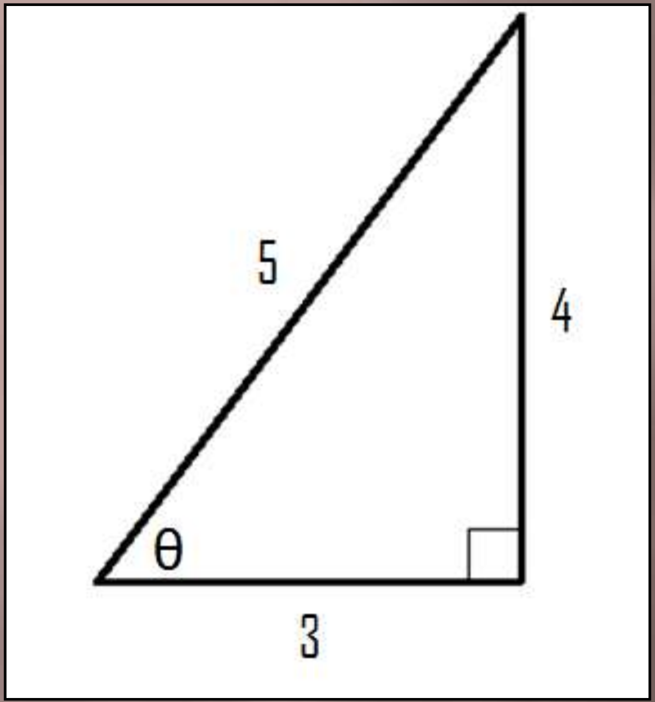
Note – We need the length of one of the legs of our right triangle.

Use the Pythagorean Theorem . . .



$$a^2 + b^2 = c^2$$

For this triangle we get:



opp θ

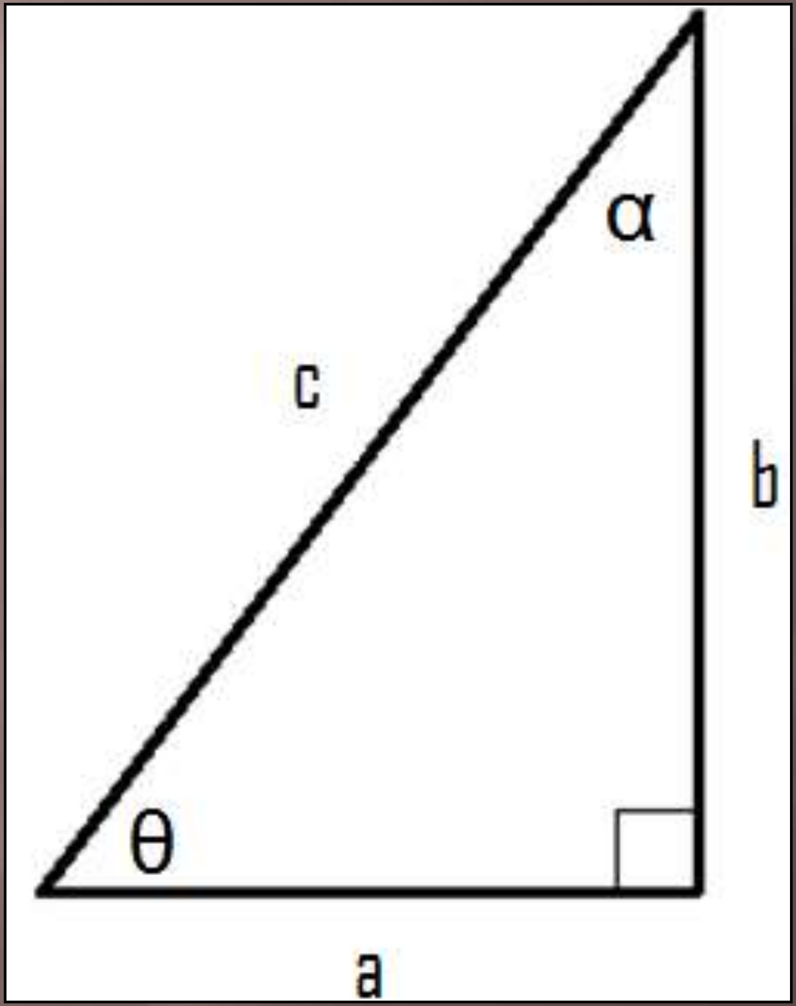
adj θ

$$\sin\theta = \frac{4}{5}$$

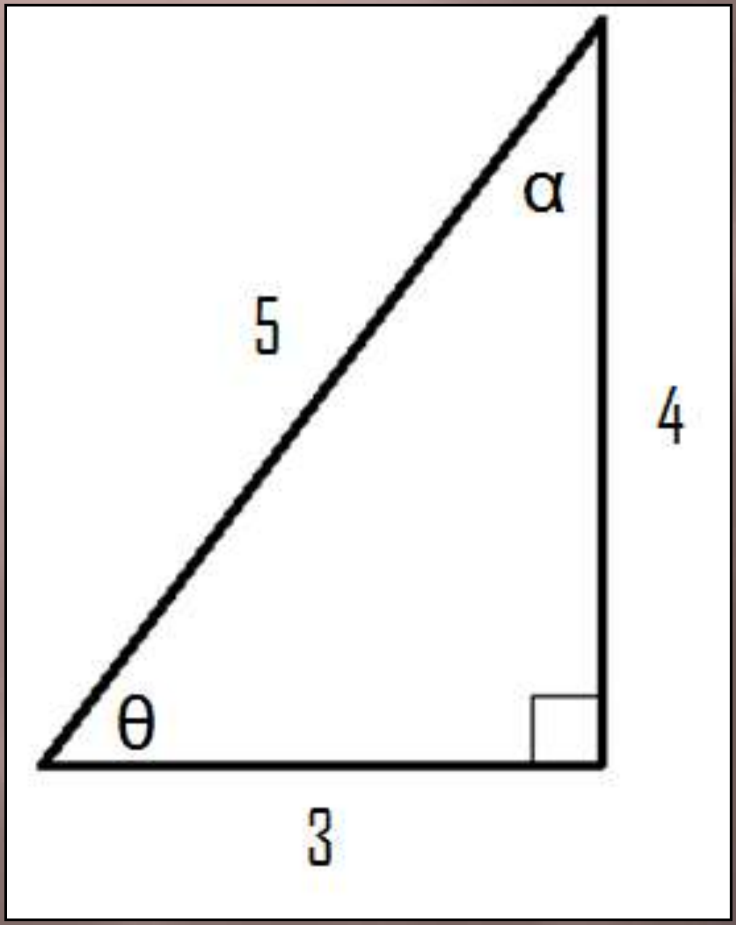
$$\cos\theta = \frac{3}{5}$$

$$\tan\theta = \frac{4}{3}$$

Notice we have another angle at α.



We can obtain the three trigonometric ratios for α,



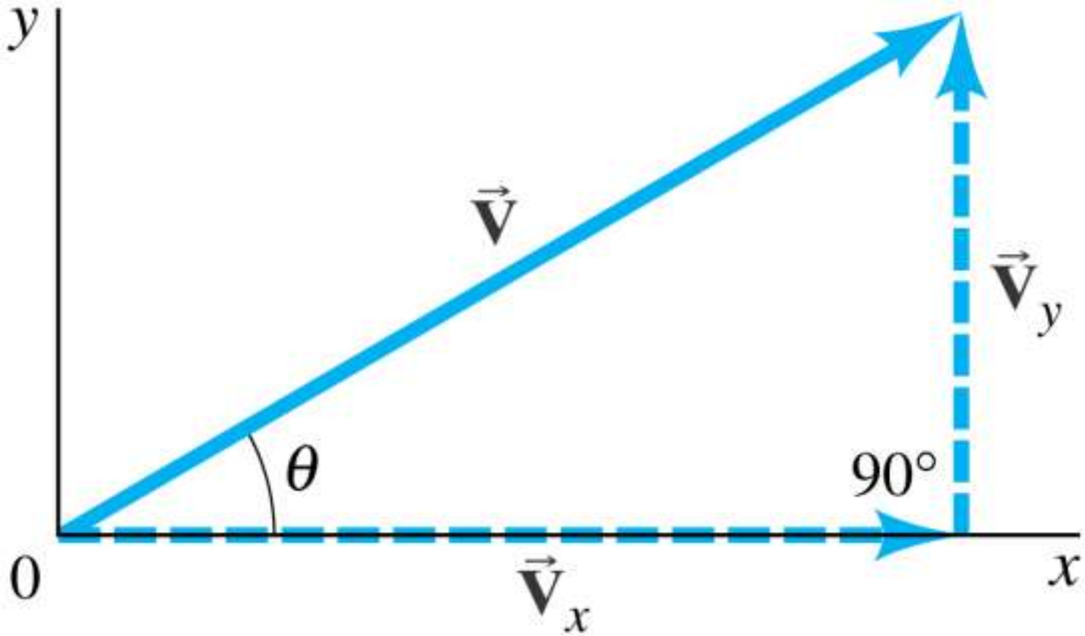
adj α

opp α

$$\sin\alpha = \frac{3}{5}$$

$$\cos\alpha = \frac{4}{5}$$

$$\tan\alpha = \frac{3}{4}$$



$$\sin\theta = \frac{V_y}{V}$$

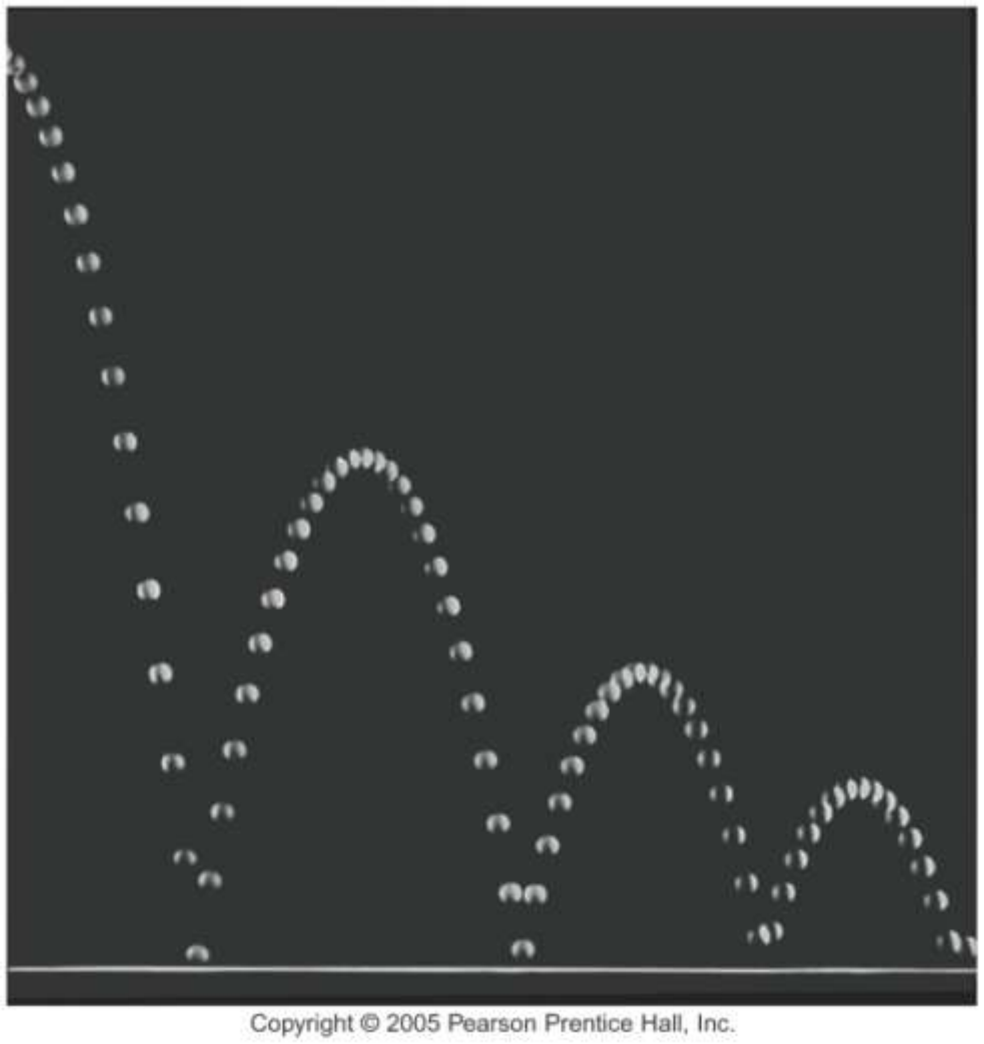
$$\cos\theta = \frac{V_x}{V}$$

$$\tan\theta = \frac{V_y}{V_x}$$

$$V^2 = V_x^2 + V_y^2$$

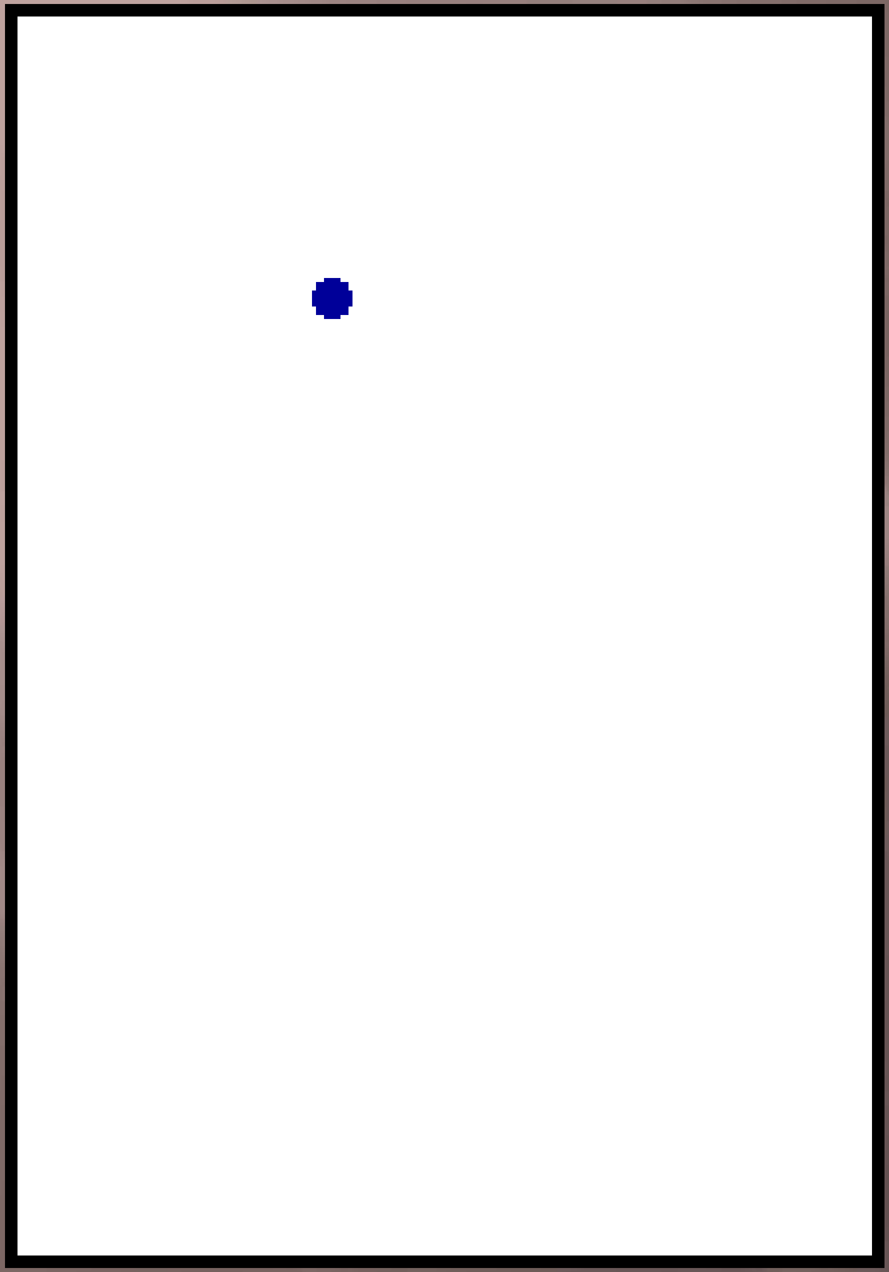
If the components are perpendicular, they can be found using trigonometric functions.

3.5 Projectile Motion



A **projectile** is an object moving in two dimensions under the influence of Earth's **gravity**; its path is a **parabola**.

3.5 Projectile Motion



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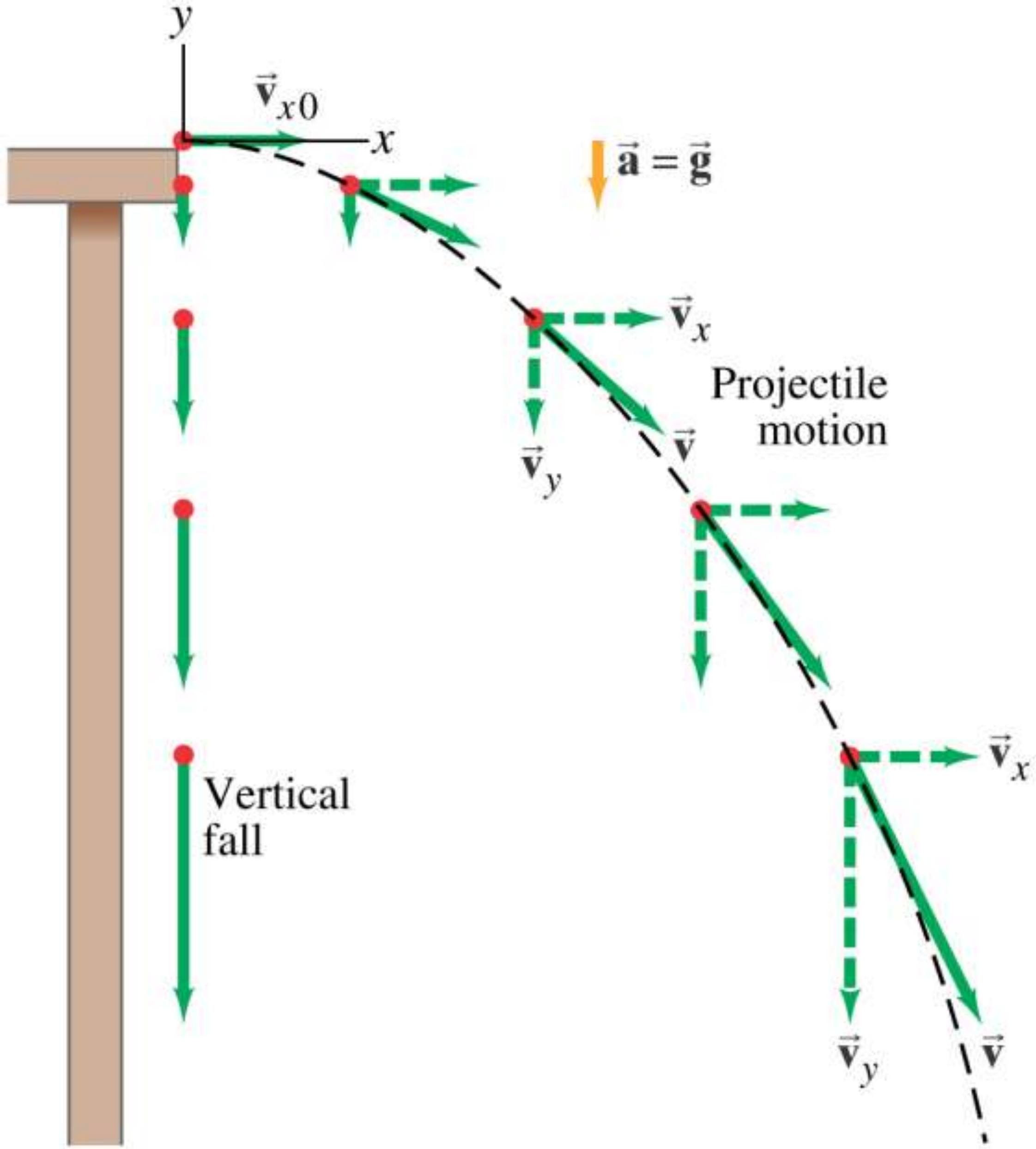
3.5 Projectile Motion

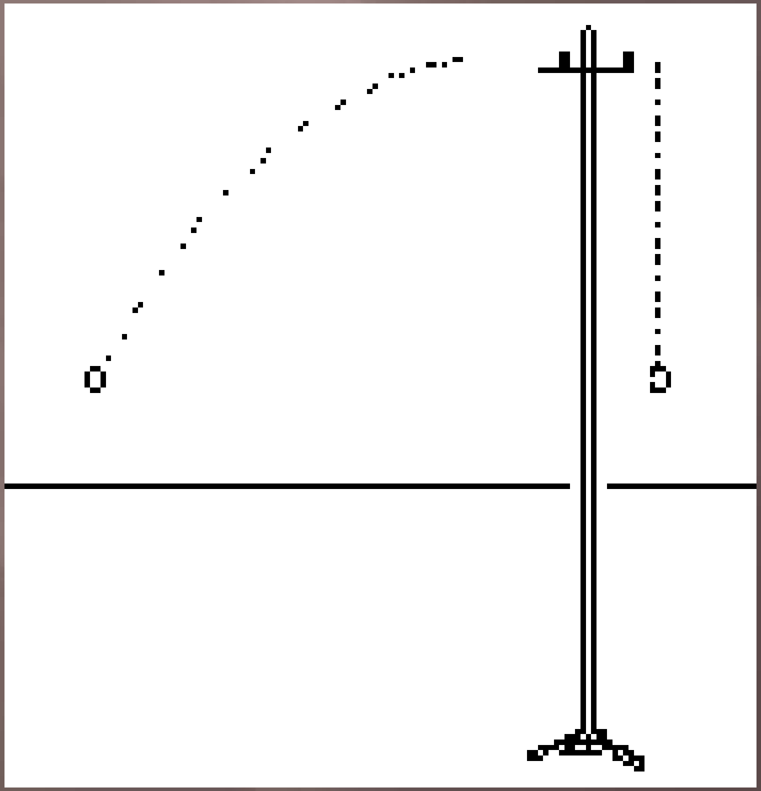


It can be understood by analyzing the horizontal and vertical motions separately.

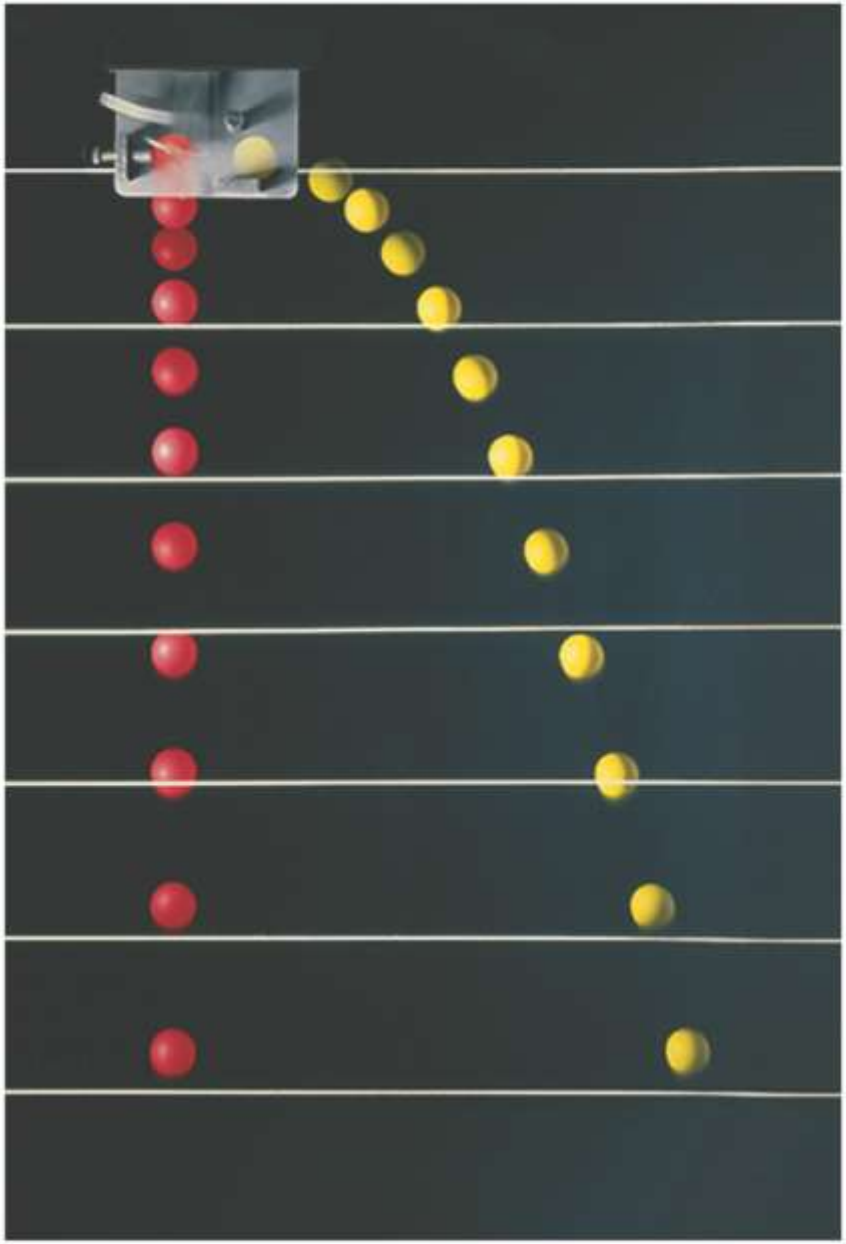
3.5 Projectile Motion

Galileo found…

An object projected horizontally will reach the ground in the same time as an object dropped.



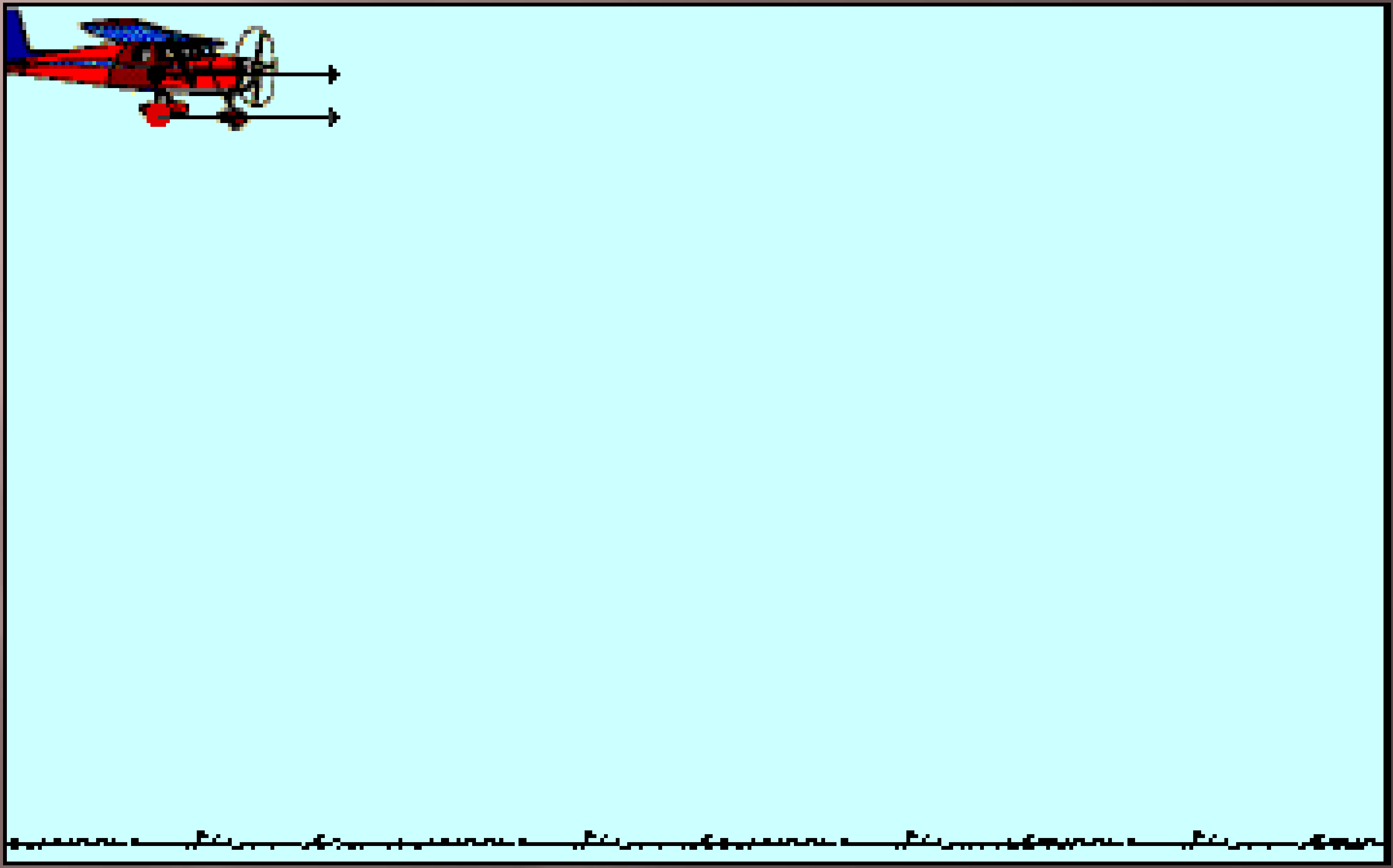
3.5 Projectile Motion



The **speed** in the *x*-direction is constant; in the *y*-direction the object moves with constant **acceleration** *g*.

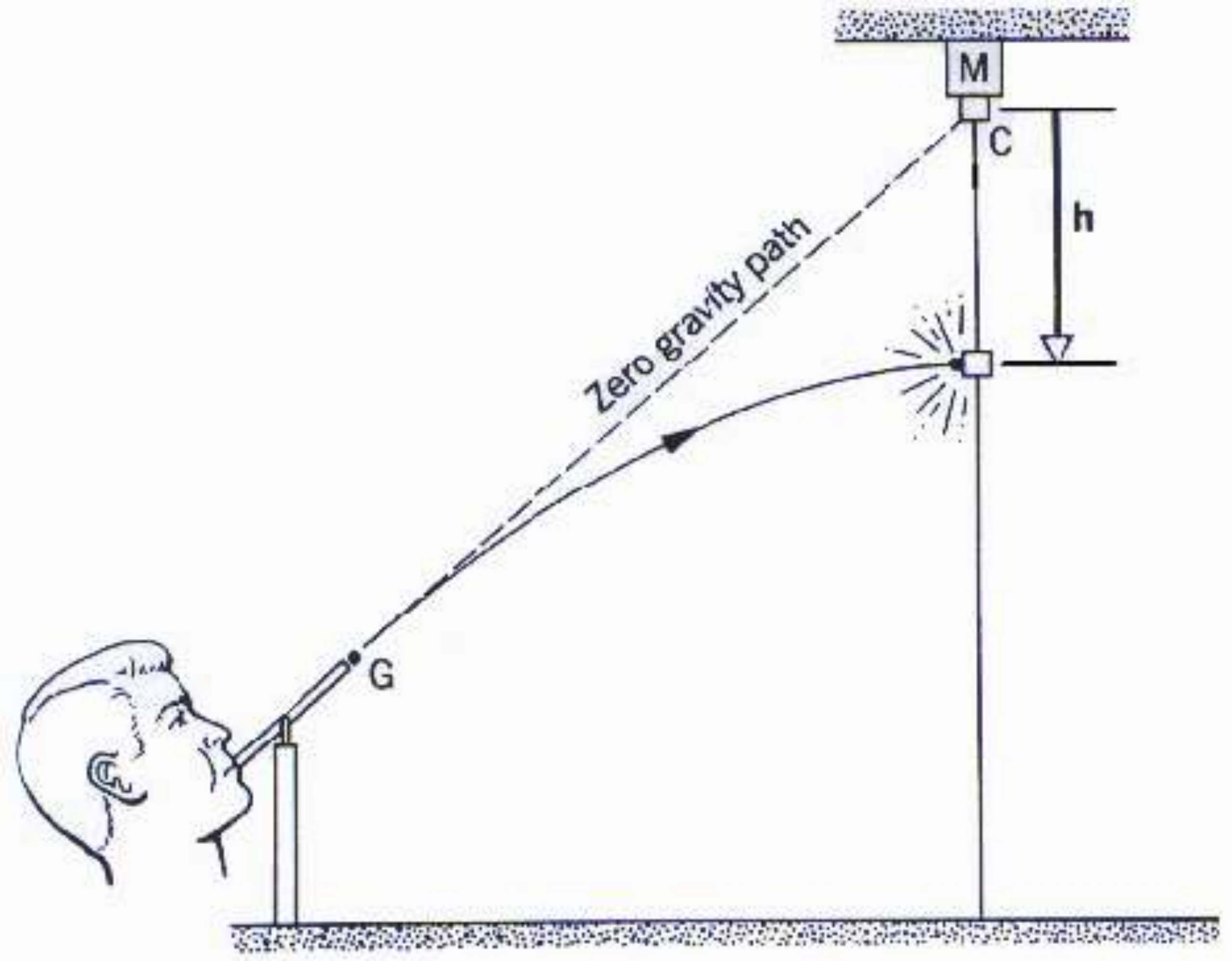
This photograph shows two balls that start to fall at the same time. The one on the right has an initial speed in the *x*-direction. It can be seen that vertical positions of the two balls are identical at identical times, while the horizontal position of the yellow ball increases linearly.

3.5 Projectile Motion



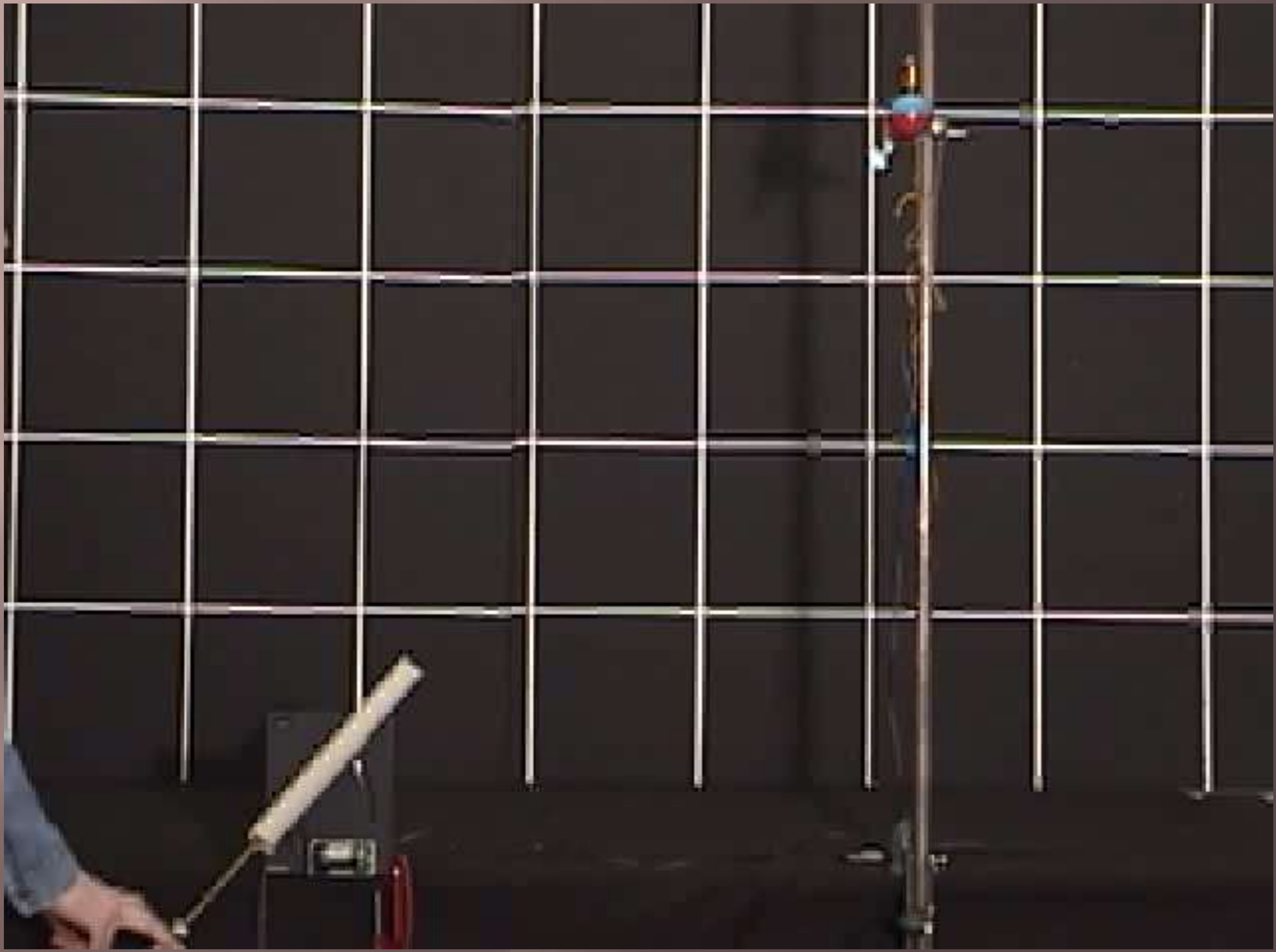
3.5 Projectile Motion

MONKEY HUNTER

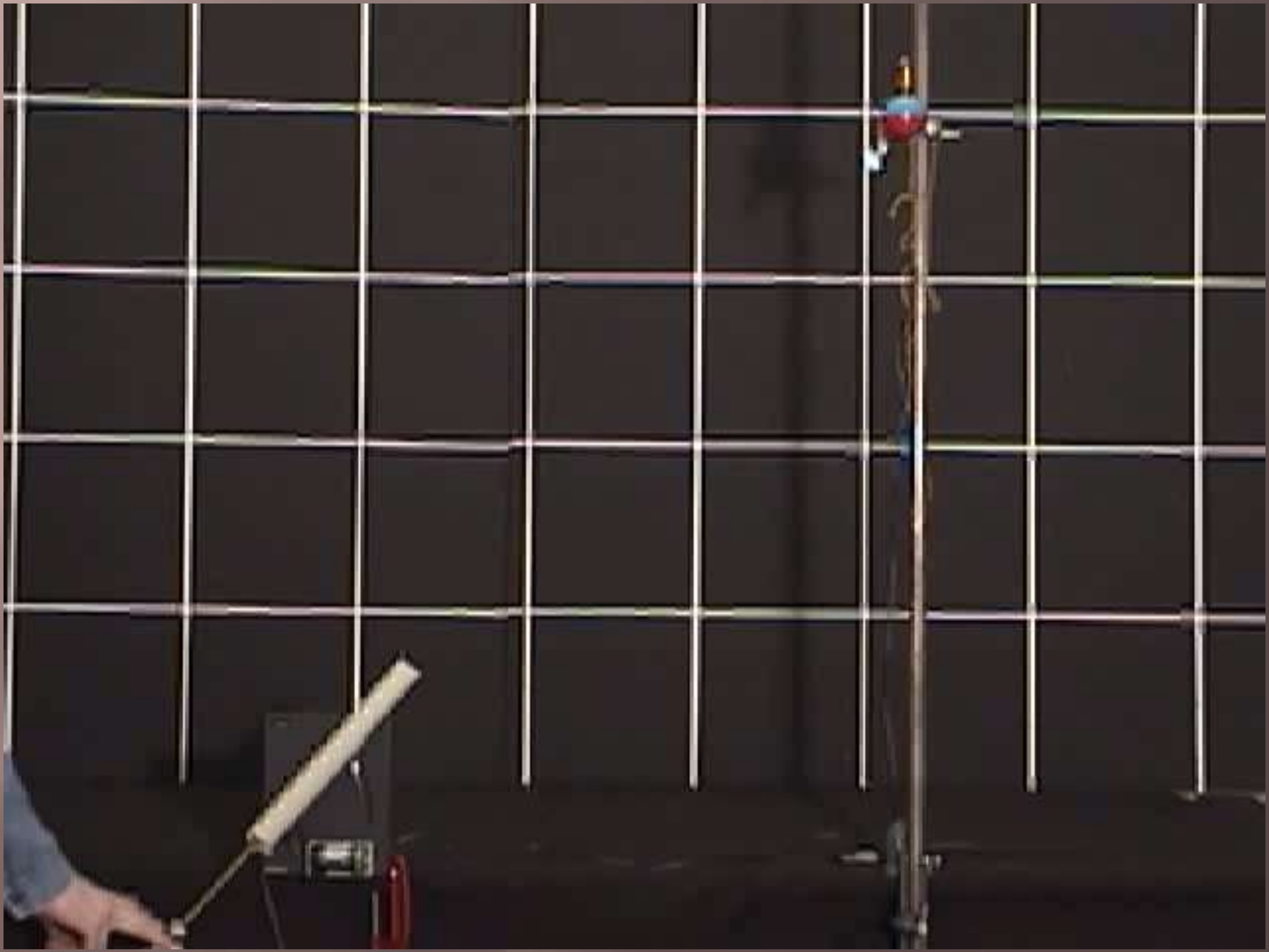


Monkey Hunter Gizmo

Monkey Hunter Video



Monkey Hunter Video Slow Motion



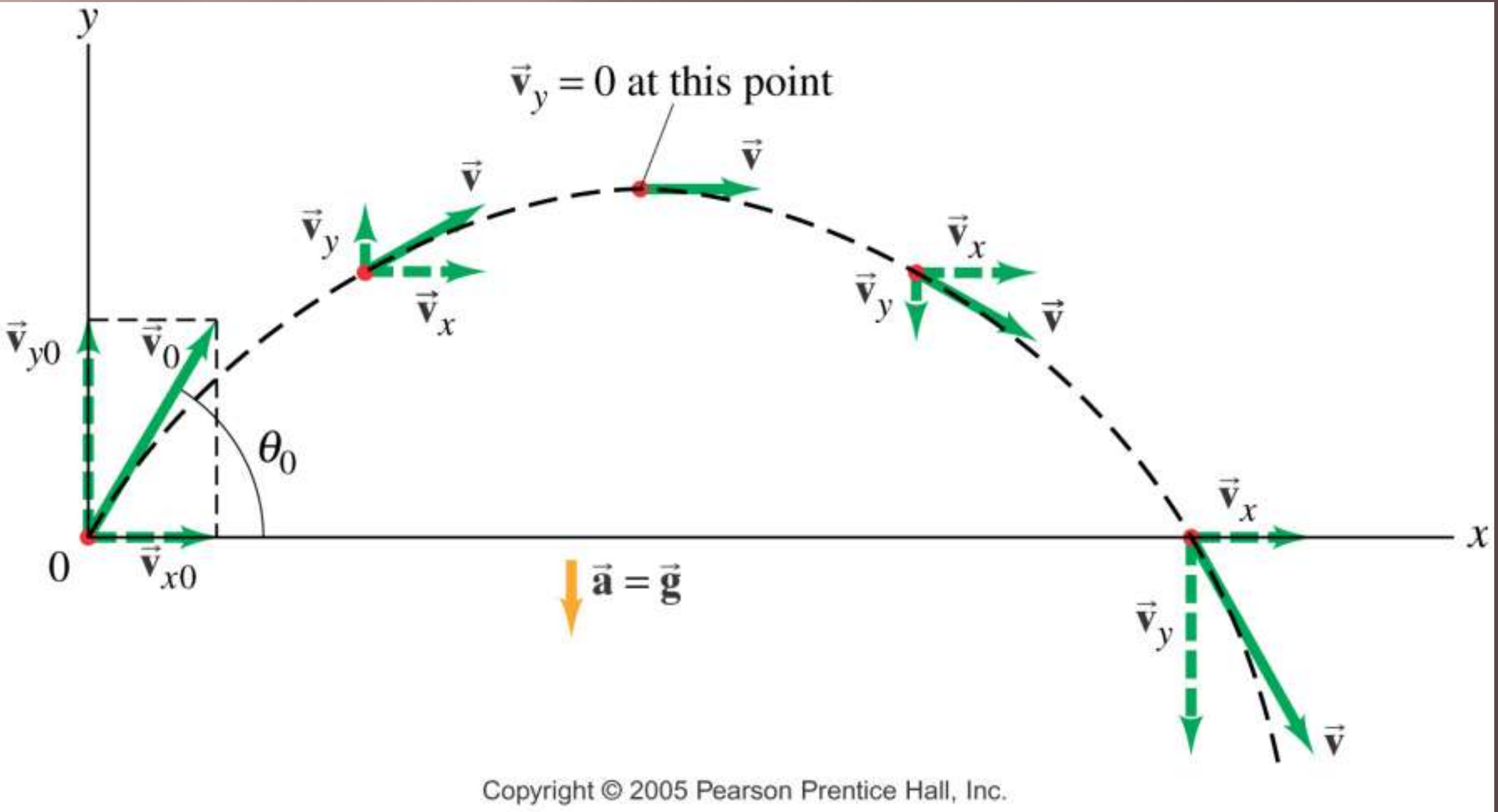
3.5 Upwardly Launched Projectile Motion

Galileo also found…

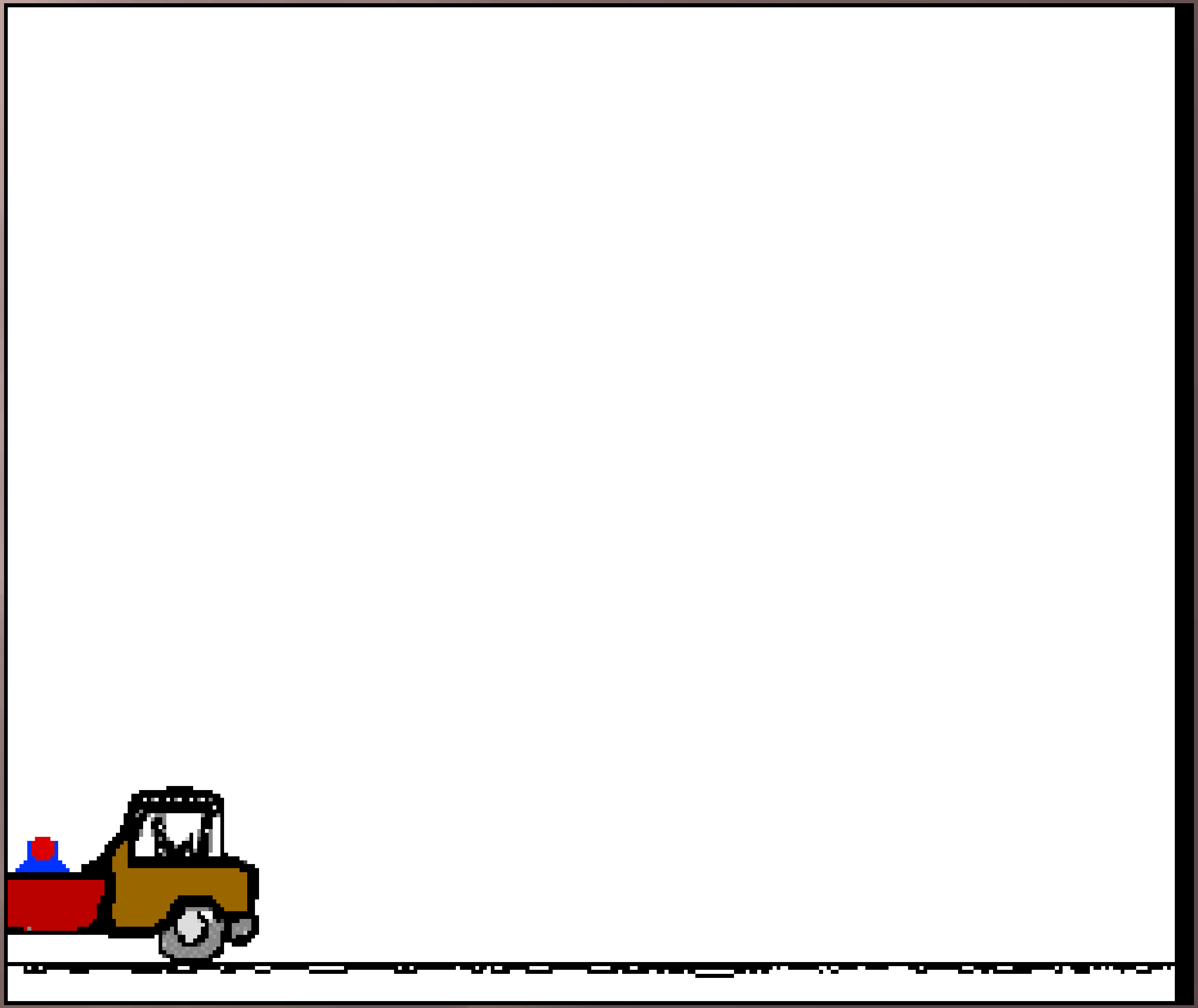
If an object projected at an angle, the object will land with the same magnitude and angle as take off.

3.5 Upwardly Launched Projectile Motion

If an object is **launched** at an initial angle of θ_0 with the horizontal, the analysis is similar except that the initial velocity has a **vertical** component.



3.5 Upwardly Launched Projectile Motion



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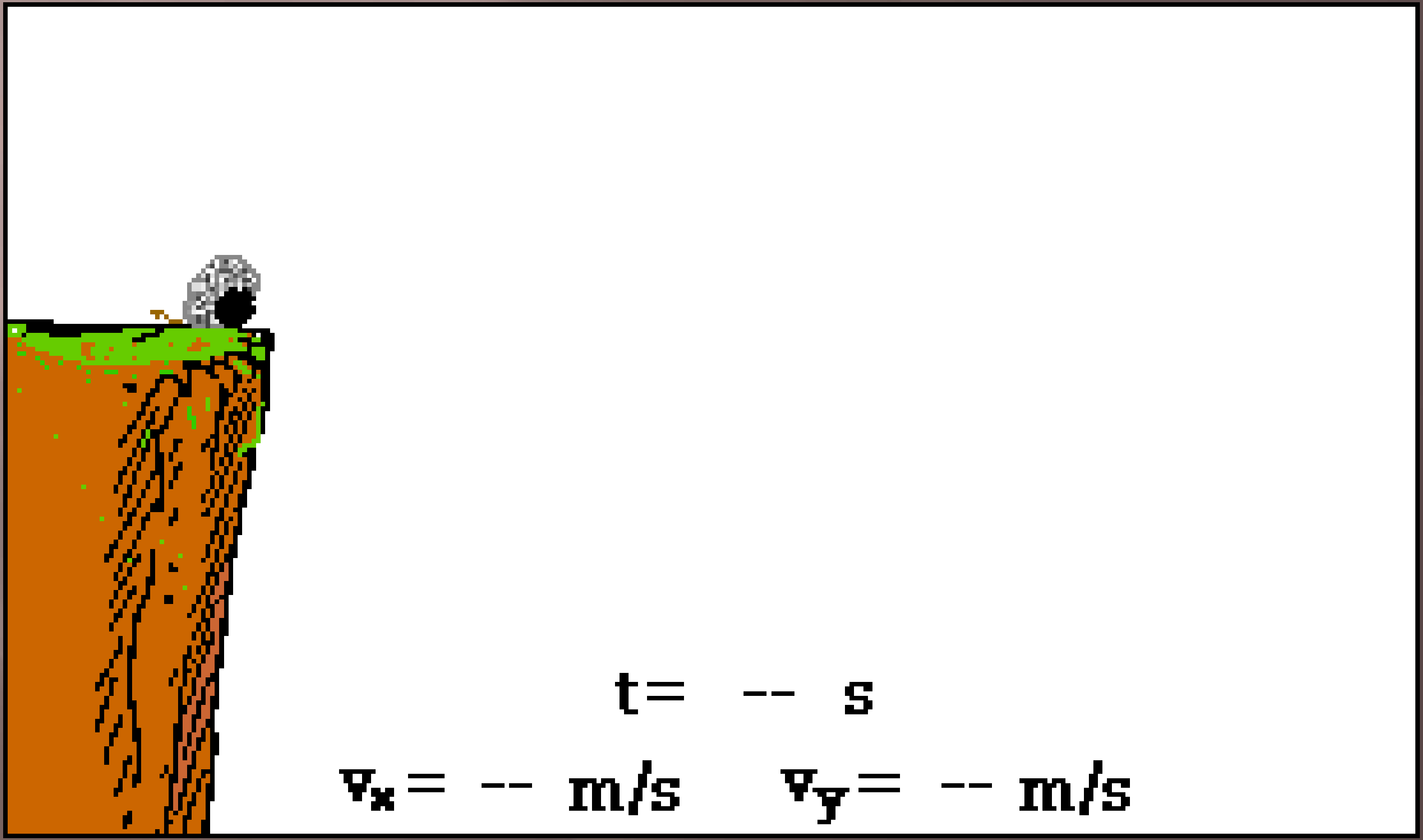


3.5 Upwardly Launched Projectile Motion

Horizontal Equations:

$$v_{2x} = v_{1x} + a_x t$$

$$x_2 = x_1 + v_{1x} t + \tfrac{1}{2} a_x t^2$$

v_{1x} = initial velocity in x direction

v_{2x} = final velocity in x direction

a_x = acceleration in x direction

x_1 = initial position in x direction

x_2 = final position in x direction

t = time

3.5 Upwardly Launched Projectile Motion

Vertical Equations:

$$y_2 = y_1 + v_{1y}t + \tfrac{1}{2}a_y t^2$$

$$v_{2y} = v_{1y} + a_y t$$

$$v_{2y}^2 = v_{1y}^2 + 2a_y(y_2 - y_1)$$

v_{1y} = initial velocity in y direction

v_{2y} = final velocity in y direction

a_y = acceleration in y direction

y_1 = initial position in y direction

y_2 = final position in y direction

t = time

3.5 Upwardly Launched Projectile Motion

Range is the distance in the x direction an object can be projected if the initial and final y positions are the same.

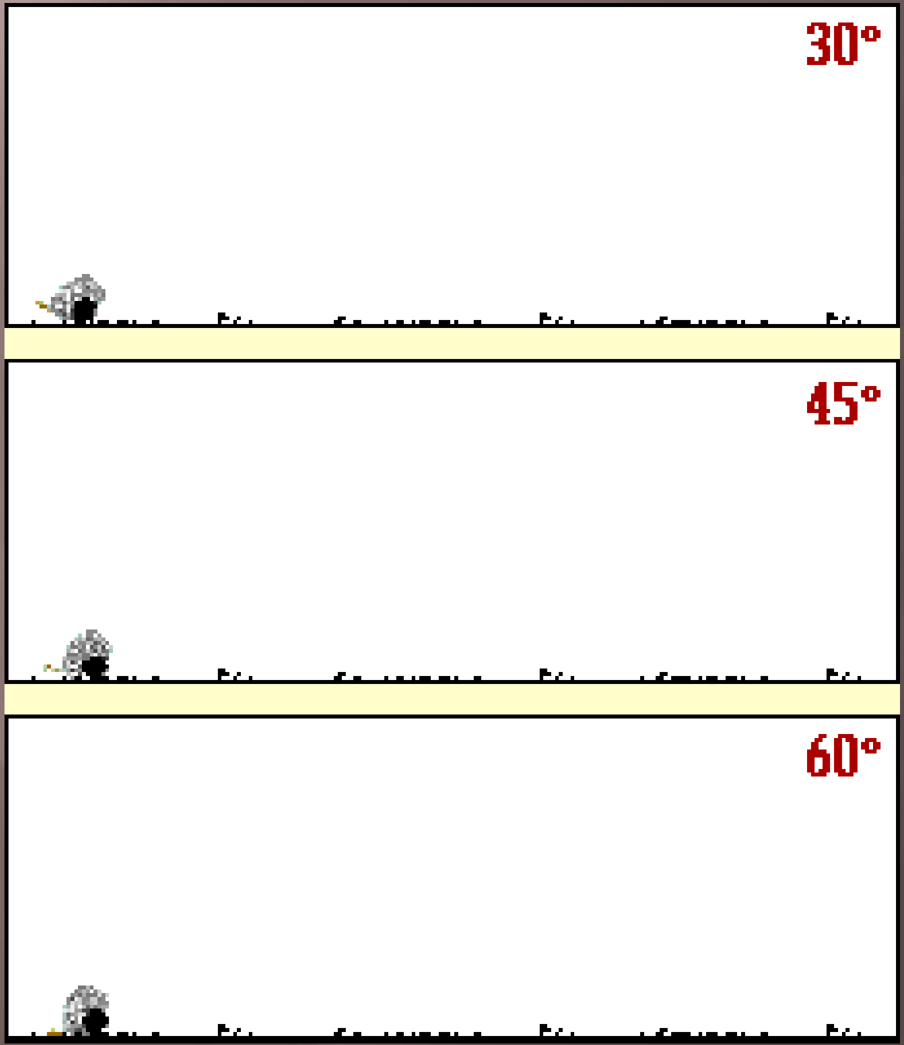
$$R = \frac{V_o^2 \sin 2\theta}{g}$$

R = range $\quad$ V_o = initial velocity

g = gravity $\quad$ θ = angle

3.5 Upwardly Launched Projectile Motion

Maximum Range is when an object is launched at an angle of 45 degrees.

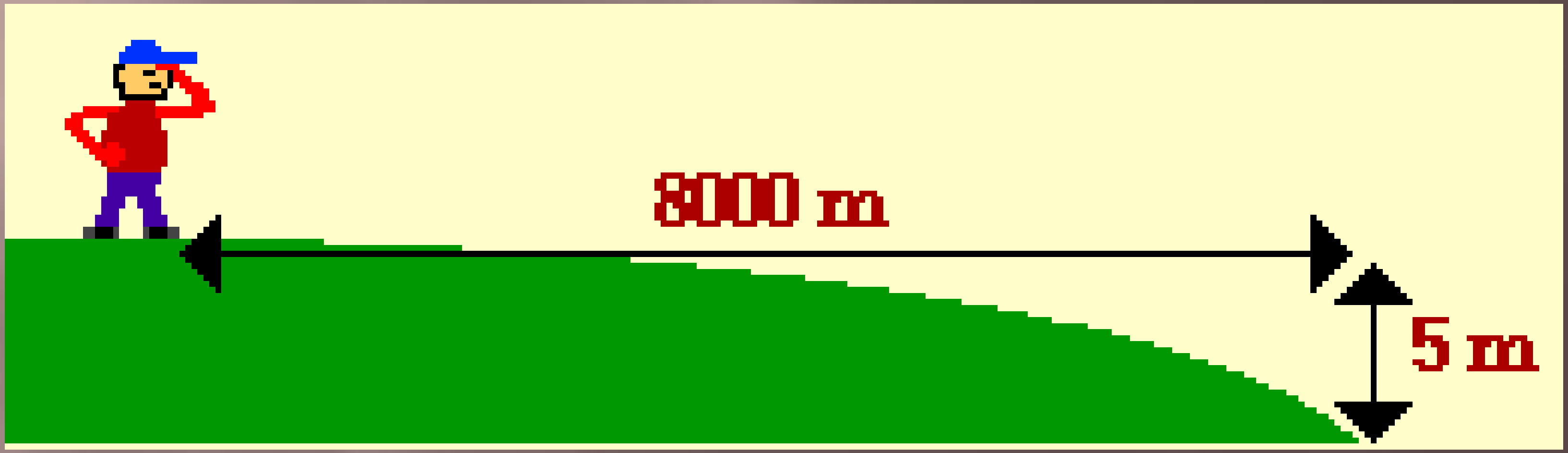


3.5 Upwardly Launched Projectile Motion

Time for a Gizmo!

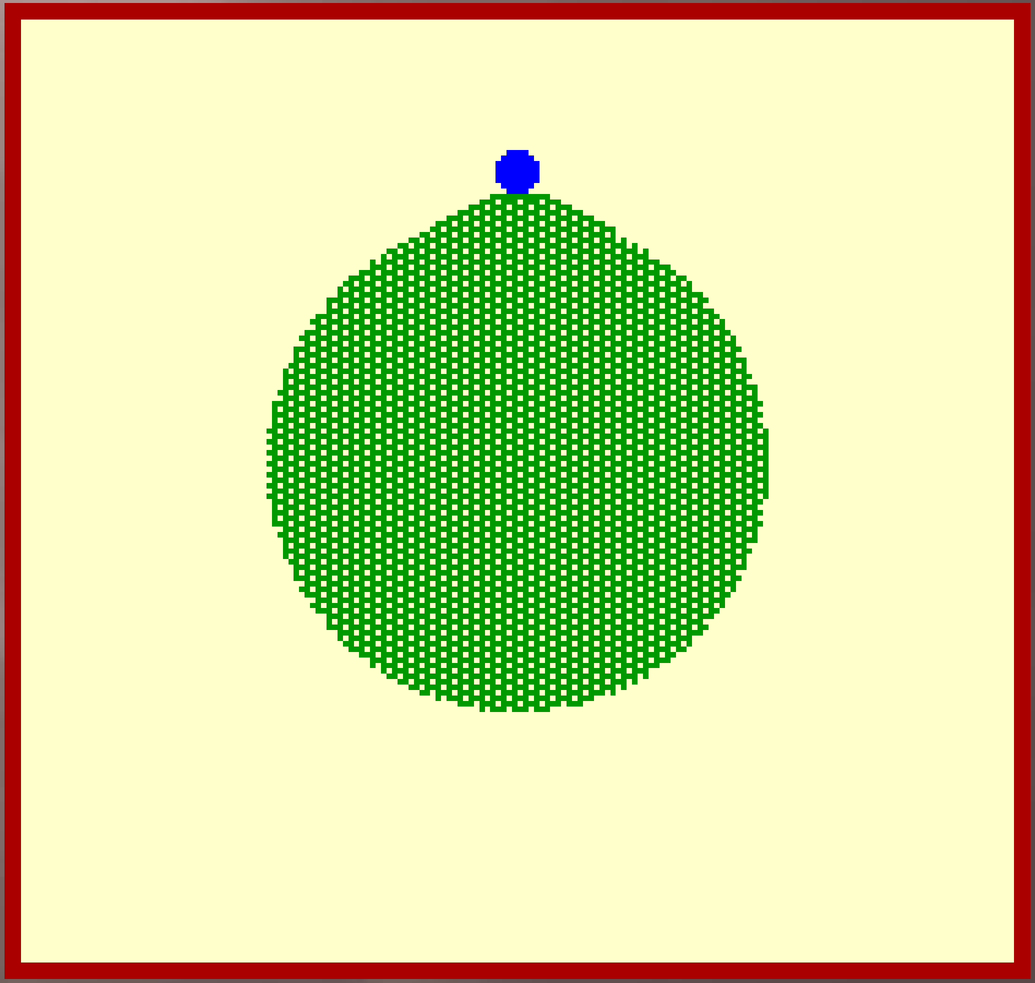
3.6 Fast Moving Projectiles-Satellites

If an object is thrown with a very fast velocity, it will orbit the Earth.



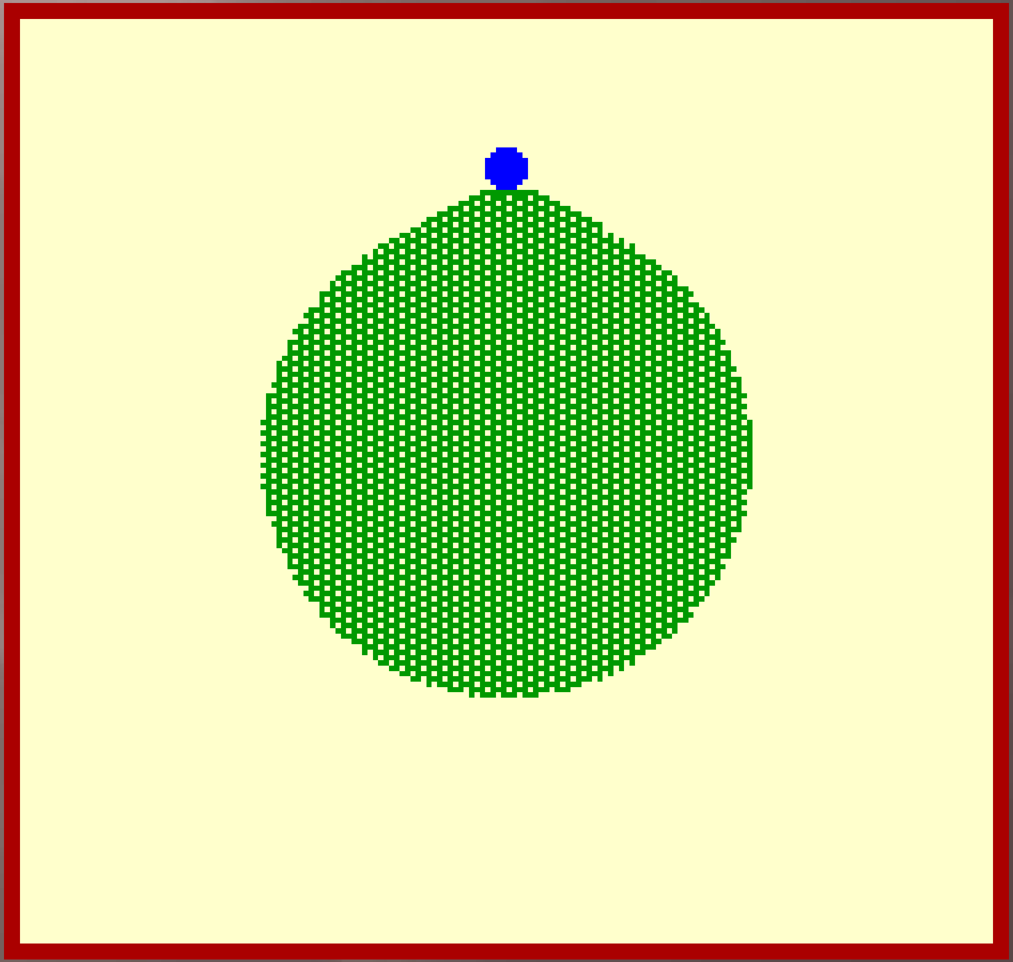
3.6 Fast Moving Projectiles-Satellites

In the absence of gravity a satellite would move in a straight line path tangent to the Earth.



3.6 Fast Moving Projectiles-Satellites

Launch Speed less than 8000 m/s
Projectile falls to Earth



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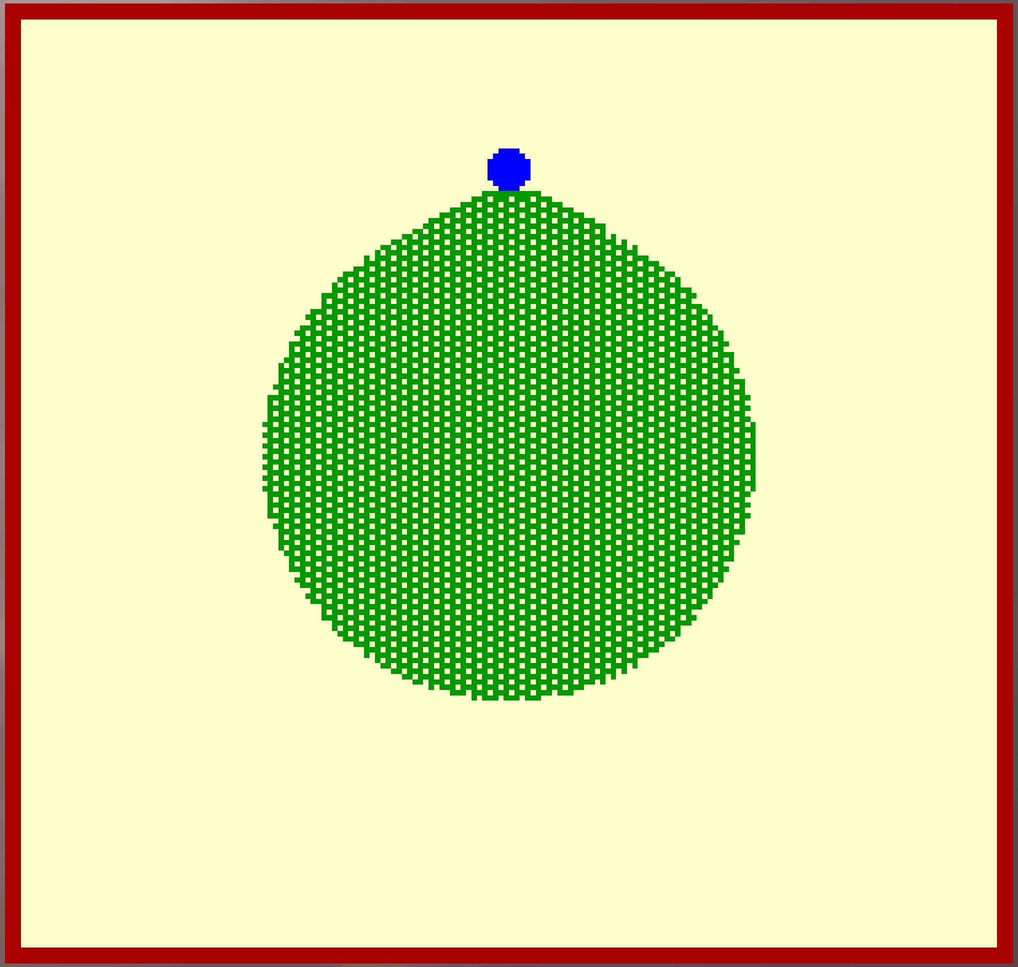
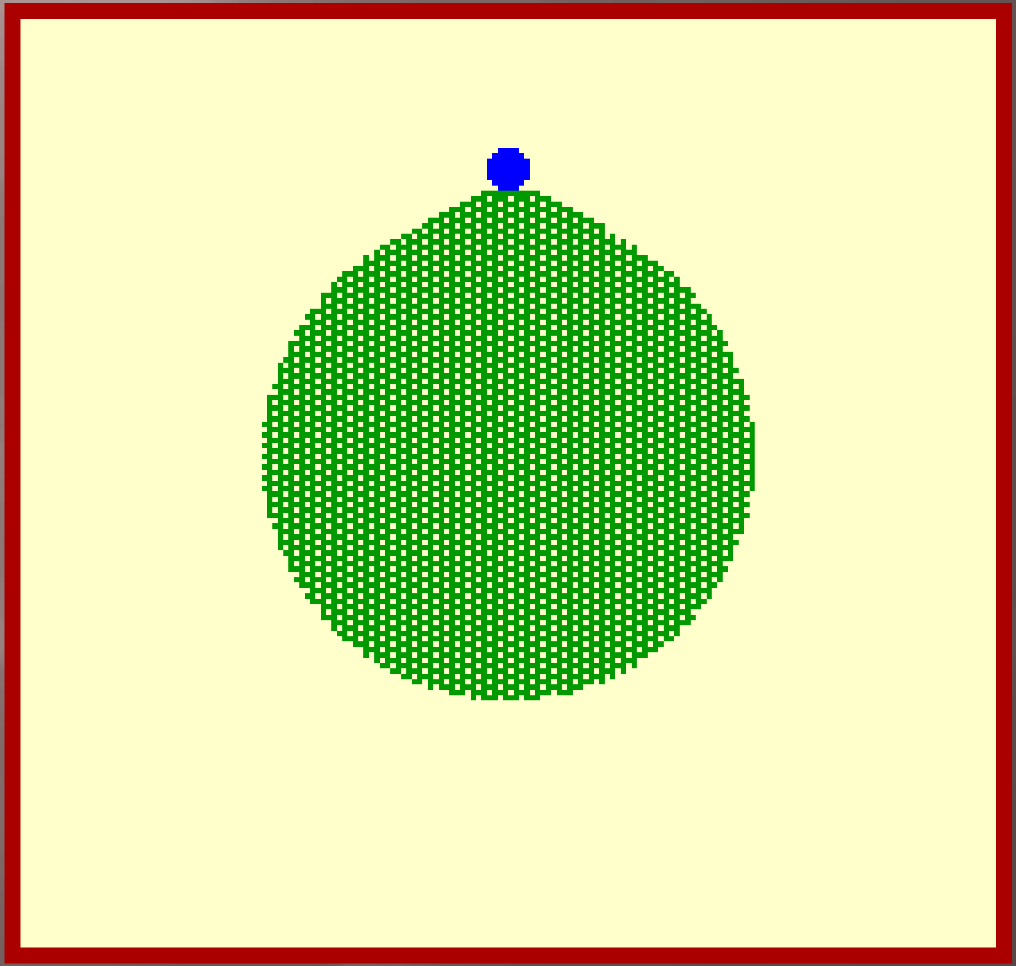


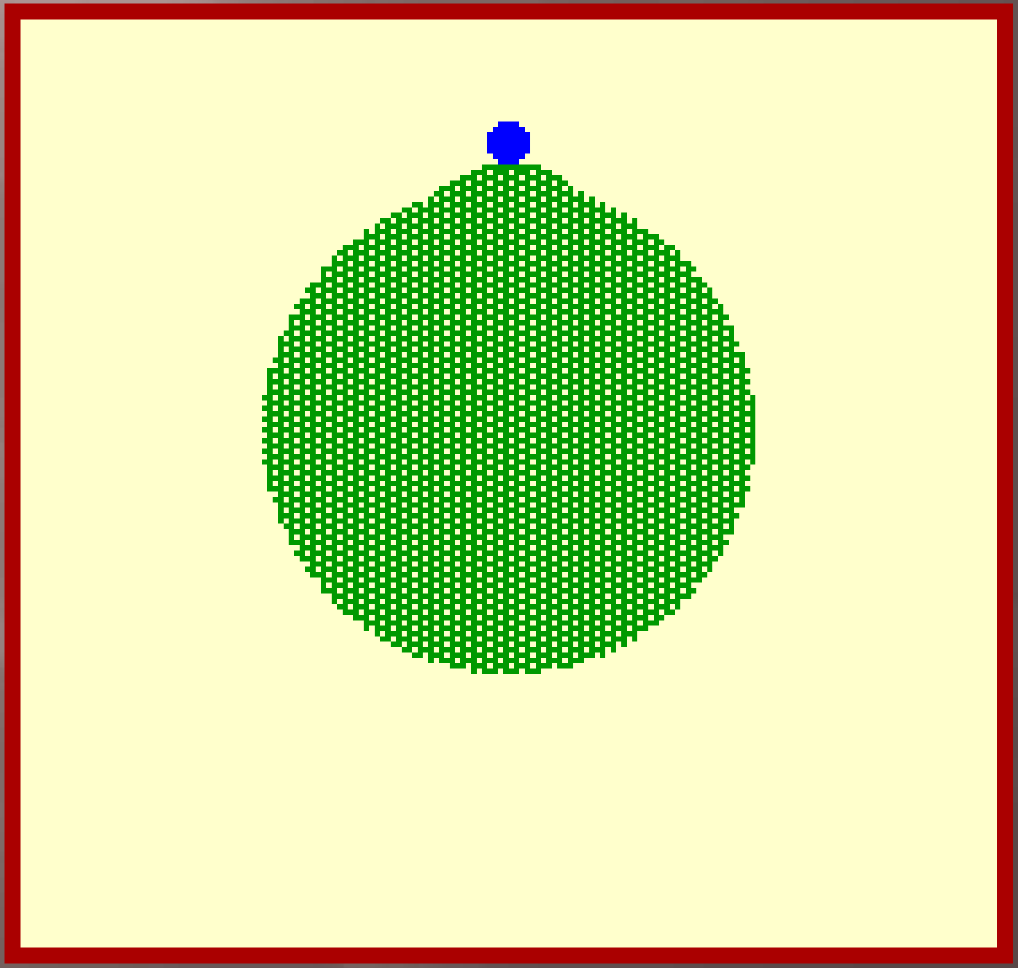
3.6 Fast Moving Projectiles-Satellites

Launch Speed equal to 8000 m/s
Projectile orbits Earth - Circular Path



3.6 Fast Moving Projectiles-Satellites

Launch Speed greater than 8000 m/s
Projectile orbits Earth - Elliptical Path



3.6 Fast Moving Projectiles-Satellites

Read Page 201 for Satellite Physics Comic

3.7 Projectile Motion Is Parabolic