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CAN HE SWING FROM A THREAD?—
CENTRIPETAL ACCELERATION

I WOULD LIKE TO MAKE one more point about forces, in particular as they relate to Spidey's web-swinging abilities. Practically every issue of the *Amazing Spider-Man* features scenes of him using his webbing to swing from building to building through the canyons of New York City. But is this realistic? Specifically, is Spider-Man's webbing strong enough to support his own weight, as well as the weight of any falling criminal, victim, or innocent bystander whom he happens to catch mid-flight as he swings in his parabolic trajectory? As Spider-Man swings in an arc, there is an extra force in addition to his weight that the webbing must supply. Let's now consider why.

Remember that Newton's second law of motion, $\mathbf{F} = \mathbf{ma}$, told us that a force is needed to change an object's motion. A change in motion, or acceleration, refers to either a change in magnitude (either speeding up or slowing down) or to a change in direction. If no force acts on the object, then it persists in "uniform motion," which means constant motion in a straight line. Any change in motion, whether in magnitude or direction, can only come about if a force acts on the object. When an automobile negotiates a hairpin turn, an external force (friction between the tires and the road) changes the auto's direction, even if the speed remains unchanged.

In order to change the direction of motion, an external force is needed—and the corollary of this is that a force can only produce an acceleration (a change in the motion) in the direction that it acts. For example, gravity pulls an object toward the ground, regardless of its initial motion. More importantly, gravity can *only* pull an object toward the ground, because that is the only direction it acts. If the Golden Age, pre-flying Superman runs off the edge of a cliff with a steady horizontal speed, he will start falling due to gravity. Since gravity does not act in the horizontal direction, his horizontal speed will not change as he falls! No force, no change, after all. His vertical velocity does increase, becoming greater the longer he falls, just as in the case of Gwen Stacy considered a moment ago, because there *is* a force acting on him in the vertical direction. The net effect of his constant horizontal speed plus an ever-increasing vertical speed is a parabolic trajectory that becomes steeper the longer he plummets. Put another way, a 90 mph fastball, thrown without spin perfectly parallel to the ground, falls to the ground at the exact same rate as a ball simply dropped out of the pitcher's hand at the same moment. Both balls would strike the ground at the same instant (if released from the same height), because the only force changing their motion is gravity, in the vertical direction. Any change in either the direction or magnitude of an object's motion can only arise from an external force acting in the direction of the change.

As Spider-Man swings from building to building, his trajectory is a semicircular arc rather than a straight line. Therefore, even if the magnitude of his speed does not change during his swing, his direction of motion is continually being altered, which can only be accomplished by an external force. It should be obvious that this force comes from the tension in the webbing. The webbing therefore has to do double duty and supply *two* forces: (1) supporting Spider-Man's weight, which it would have to maintain even if he were simply hanging from the vertical line, and (2) a second force to divert him in a circular trajectory. If the webbing line were to snap in mid-swing, then the only external force acting on Spider-Man is gravity, and his motion at this point would not differ at all from that of a ball tossed with the same velocity Spider-Man possessed at the instant the webbing broke.

The acceleration that this additional force in the webbing provides as Spidey swings in a circular arc is identical to the acceleration experienced by the moon as it makes its circular orbit about the Earth. In one case the force arises from the tension in the webbing, while in the other it is Newton's force of gravitational attraction, but for both it changes straight-line motion into circular motion. Gravity is the moon's "webbing," causing its direction to change.

If the tension in the webbing or gravity were suddenly to disappear, both Spidey and the moon would depart from their circular trajectories, and continue moving with the velocity they had at the moment the external forces were removed. With a little bit of geometry or calculus one can show that the acceleration **a** of an object being constantly deflected onto a circular orbit with a velocity **v** is $\mathbf{a} = (\mathbf{v} \times \mathbf{v})/\mathbf{R} = \mathbf{v}^2/\mathbf{R}$, where **R** is the radius of the circle.

Spider-Man's webbing has to supply a force **mg**, in order to support his weight, and an additional force $\mathbf{mv}^2/\mathbf{R}$ in order to change his direction as he swings. The faster he swings (the larger his velocity **v**) or the tighter his arc (that is, the smaller the radius **R**), the greater will be this centripetal acceleration $\mathbf{v}^2/\mathbf{R}$. When Spidey swings from a web strand 200 feet long, at a speed of 50 mph, the centripetal acceleration is 27 feet/sec^2, in addition to the acceleration due to gravity of 32 feet/sec^2. If Spider-Man's mass in the metric system is approximately 73 kilograms, then his weight **mg** is 160 pounds, and the additional force the webbing must supply just to change his trajectory from straight-line motion into a circular arc is roughly 135 pounds. The total tension in the webbing is nearly 300 pounds, and will be more if Spidey is carrying someone as he swings.

Three hundred pounds or greater of tension may seem to be more than a thin strand of fiber can withstand, but if Spider-Man's webbing is anything like real spider silk, he has nothing to worry about. Dragline silk webbing, which spiders use for their webs and while rapidly fleeing predatory birds, is actually five times stronger per pound than steel cable and more elastic than nylon. The webbing's properties result from thousands of rigid filaments only a few billionths of a meter wide (providing great redundancy so that no one filament is crucial for the integrity of the webbing), interspersed with fluid-filled channels that distribute the tensile force along the length of the webbing. Spider-Man is able to alter the material properties of his webbing by adjusting its chemical composition as it jets from his web-shooters. Similarly, real spiders can control the tensile strength of their webs by varying the relative concentration of crystallizing and noncrystallizing proteins.

There is considerable interest in commercial applications of webbing, which would require large quantities of spider silk. As it is not practical to harvest spiders for their silk (they are too territorial to farm in a conventional manner), recent genetic engineering experiments have inserted a spider's web-making genes into goats, so that the goats' milk will contain webbing that can be more easily sieved and acquired. While the development of web-producing goats has hit some snags,* other scientists have reported preliminary success with infecting spider cells in the laboratory with a genetically engineered virus that induces the cell to directly manufacture the proteins found in spiderwebs. The silk-producing gene from spiders has also been successfully introduced into E. coli and plant cells. Such research could have far-reaching practical applications. As Jim Robbins discussed in his article "Second Nature" in the July 2002 issue of *Smithsonian*: "In theory, a braided spider silk rope the diameter of a pencil could stop a fighter jet landing on an aircraft carrier. The combination of strength and elasticity allows it to withstand an impact five times more powerful than Kevlar, the synthetic fiber used in bulletproof vests."

The high tensile strength of real spider silk enables it to support a weight of more than 20,000 pounds per square centimeter. That is, if the cross section of the webbing was a circle with a diameter of 1 cm (a little bit less than a half inch), then the webbing could hold a weight of 8 tons before breaking. Even a webbing strand with a diameter of only a quarter inch could support more than 6,000 pounds safely, well below the 300 pounds of weight and centripetal force we estimated earlier. Unless Spider-Man is trying to carry both the Hulk and the Blob simultaneously, his webbing should be more than able to do the job.

Therefore, according to Newton's laws of motion, it is entirely plausible that Spider-Man can swing from building to building, stop a runaway elevated train (as in the 2004 film *Spider-Man 2*), or weave a bulletproof shield out of very narrow lines of webbing. So, to answer the question posed in the title of this chapter: Simply take a look overhead!

* Sorry.