4 Momentum

4-1 Impulse and Momentum

Vocabulary **Momentum:** A measure of how difficult it is to stop a moving object.

momentum = (mass)(velocity) or $p = mv$

If the momentum of an object is changing, as it is when a force is exerted to start it or stop it, the change in momentum can be found by looking at the change in mass and velocity during the interval.

change in momentum = change in [(mass)(velocity)] or $\Delta p = \Delta(mv)$

For all the exercises in this book, assume that the mass of the object remains constant, and consider only the change in velocity, Δv, which is equal to $v_f - v_o$. Momentum is a vector quantity. Its direction is in the direction of the object's velocity.

The SI unit for momentum is the **kilogram·meter/second (kg·m/s).**

Vocabulary **Impulse:** The product of the force exerted on an object and the time interval during which it acts.

impulse = (force)(elapsed time) or $J = F\Delta t$

The SI unit for impulse is the **newton·second (N·s).**

The impulse given to an object is equal to the change in momentum of the object.

$$F\Delta t = m\Delta v$$

The same change in momentum may be the result of a large force exerted for a short time, or a small force exerted for a long time. In other words, impulse is the thing that you *do,* while change in momentum is the thing that you *see.*

The units for impulse and momentum are equivalent. Remember, $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$. Therefore, $1\ \text{N}\cdot\text{s} = 1\ \text{kg}\cdot\text{m/s}$.

Solved Examples

Example 1: Tiger Woods hits a 0.050-kg golf ball, giving it a speed of 75 m/s. What impulse does he impart to the ball?

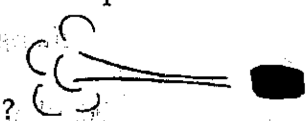
Solution: Because the impulse equals the change in momentum, you can reword this exercise to read, "What was the ball's change in momentum?" It is understood that the ball was initially at rest, so its initial speed was 0 m/s.

Given: $m = 0.050$ kg
$\Delta v = 75$ m/s

Unknown: $\Delta p = ?$
Original equation: $\Delta p = m\Delta v$

Solve: $\Delta p = (0.050 \text{ kg})(75 \text{ m/s}) = \mathbf{3.8\ kg \cdot m/s}$

Example 2: Wayne hits a stationary 0.12-kg hockey puck with a force that lasts for 1.0×10^{-2} s and makes the puck shoot across the ice with a speed of 20.0 m/s, scoring a goal for the team. With what force did Wayne hit the puck?



Given: $m = 0.12$ kg
$\Delta v = 20.0$ m/s
$\Delta t = 1.0 \times 10^{-2}$ s

Unknown: $F = ?$
Original equation: $F\Delta t = m\Delta v$

Solve: $F = \dfrac{m\Delta v}{\Delta t} = \dfrac{(0.12 \text{ kg})(20.0 \text{ m/s})}{1.0 \times 10^{-2} \text{ s}} = 240 \text{ kg} \cdot \text{m/s}^2 = \mathbf{240\ N}$

Example 3: A tennis ball traveling at 10.0 m/s is returned by Venus Williams. It leaves her racket with a speed of 36.0 m/s in the opposite direction from which it came. a) What is the change in momentum of the tennis ball? b) If the 0.060-kg ball is in contact with the racket for 0.020 s, with what average force has Venus hit the ball?

Solution: In this exercise, the tennis ball is coming toward Venus with a speed of 10.0 m/s in one direction, but she hits it back with a speed of 36.0 m/s in the opposite direction. Therefore, you must think about velocity vectors and call one direction positive and the opposite direction negative.

a. *Given:* $v_o = -10.0$ m/s
$v_f = 36.0$ m/s
$m = 0.060$ kg

Unknown: $\Delta p = ?$
Original equation: $\Delta p = m\Delta v = m(v_f - v_o)$

Solve: $\Delta p = m(v_f - v_o) = (0.060 \text{ kg})[36.0 \text{ m/s} - (-10.0 \text{ m/s})] = \mathbf{2.8\ kg \cdot m/s}$

b. *Given:* $m = 0.060$ kg
$\Delta v = 46.0$ m/s
$\Delta t = 0.020$ s

Unknown: $F = ?$
Original equation: $F\Delta t = m\Delta v$

Solve: $F = \dfrac{m\Delta v}{\Delta t} = \dfrac{(0.060 \text{ kg})(46.0 \text{ m/s})}{(0.020 \text{ s})} = \mathbf{140\ N}$

Example 4: To demonstrate his new high-speed camera, Flash performs an experiment in the physics lab in which he shoots a pellet gun at a pumpkin to record the moment of impact on film. The 1.0-g pellet travels at 100. m/s until it embeds itself 2.0 cm into the pumpkin. What average force does the pumpkin exert to stop the pellet?

Solution: First, convert g to kg and cm to m.

$$1.0 \text{ g} = 0.0010 \text{ kg} \qquad 2.0 \text{ cm} = 0.020 \text{ m}$$

Before you can solve for the force in the exercise, you must first know how long the force is being exerted. Remember, in order to find the time, you must use the average velocity, v_{av}.

$$v_{av} = \frac{v_f + v_o}{2} = \frac{0 \text{ m/s} + 100. \text{ m/s}}{2} = 50.0 \text{ m/s}$$

Given: $v = 50.0$ m/s
$\Delta d = 0.020$ m

Unknown: $\Delta t = ?$
Original equation: $\Delta d = v\Delta t$

Solve: $\Delta t = \frac{\Delta d}{v} = \frac{0.020 \text{ m}}{50.0 \text{ m/s}} = 0.00040 \text{ s}$

Now we can solve for the force the pumpkin exerts to stop the pellet.

Given: $m = 0.0010$ kg
$\Delta v = 100.$ m/s
$\Delta t = 0.0040$ s

Unknown: $F = ?$
Original equation: $F\Delta t = m\Delta v$

Solve: $F = \frac{m\Delta v}{\Delta t} = \frac{(0.0010 \text{ kg})(100. \text{ m/s})}{(0.00040 \text{ s})} = \mathbf{250\ N}$

Practice Exercises

Exercise 1: On April 15, 1912, the luxury cruiseliner *Titanic* sank after running into an iceberg. a) What momentum would the 4.23×10^8-kg ship have imparted to the iceberg if it had hit the iceberg head-on with a speed of 11.6 m/s? (Actually, the impact was a glancing blow.) b) If the captain of the ship had seen the iceberg a kilometer ahead and had tried to slow down, why would this have been a futile effort?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 2: Auto companies frequently test the safety of automobiles by putting them through crash tests to observe the integrity of the passenger compartment. If a 1000.-kg car is sent toward a cement wall with a speed of 14 m/s and the impact brings it to a stop in 8.00×10^{-2} s, with what average force is it brought to rest?

Answer: ____________________

Exercise 3: Rhonda, who has a mass of 60.0 kg, is riding at 25.0 m/s in her sports car when she must suddenly slam on the brakes to avoid hitting a dog crossing the road. She is wearing her seatbelt, which brings her body to a stop in 0.400 s. a) What average force did the seatbelt exert on her? b) If she had not been wearing her seatbelt, and the windshield had stopped her head in 1.0×10^{-3} s, what average force would the windshield have exerted on her? c) How many times greater is the stopping force of the windshield than the seatbelt?

Answer: **a.** ____________________

Answer: **b.** ____________________

Answer: **c.** ____________________

Exercise 4: If 270 million people in the United States jumped up in the air simultaneously, pushing off Earth with an average force of 800. N each for a time of 0.10 s, what would happen to the 5.98×10^{24} kg Earth? Show a calculation that justifies your answer.

Answer: ____________________

Exercise 5: In Sharkey's Billiard Academy, Maurice is waiting to make his last shot. He notices that the cue ball is lined up for a perfect head-on collision, as shown. Each of the balls has a mass of 0.0800 kg and the cue ball comes to a complete stop upon making contact with the 8 ball. Suppose Maurice hits the cue ball by exerting a force of 180. N for 5.00×10^{-3} s, and it knocks head-on into the 8 ball. Calculate the resulting velocity of the 8 ball.

Answer: ____________________

Exercise 6: During an autumn storm, a 0.012-kg hail stone traveling at 20.0 m/s made a 0.20-cm-deep dent in the hood of Darnell's new car. What average force did the car exert to stop the damaging hail stone?

Answer: ____________________

4-2 Conservation of Momentum

According to the **law of conservation of momentum,** the total momentum in a system remains the same if no external forces act on the system. Consider the two types of collisions that can occur.

Vocabulary **Elastic collision:** A collision in which objects collide and bounce apart with no energy loss.

In an elastic collision, because momentum is conserved, the *mv* before a collision for each of the two objects must equal the *mv* after the collision for each of the two objects. This is written as

$$m_1 v_{1o} + m_2 v_{2o} = m_1 v_{1f} + m_2 v_{2f}$$

The subscripts 1 and 2 refer to objects 1 and 2, respectively.

Vocabulary **Inelastic collision:** A collision in which objects collide and some mechanical energy is transformed into heat energy.

A common kind of inelastic collision is one in which the colliding objects stick together, or start out stuck together and then separate. However, in an inelastic collision the objects need not remain stuck together but may instead deform in some way.

Because momentum is also conserved in an inelastic collision, the *mv* before the collision for each of the two objects must equal the *mv* after the collision for each of the two objects. When objects are stuck together after the collision (assuming mass does not change), this equation becomes

$$m_1v_{1o} + m_2v_{2o} = (m_1 + m_2)v_f$$

where v_f is the combined final velocity of the two objects.

Solved Examples

Example 5: Tubby and his twin brother Chubby have a combined mass of 200.0 kg and are zooming along in a 100.0-kg amusement park bumper car at 10.0 m/s. They bump Melinda's car, which is sitting still. Melinda has a mass of 25.0 kg. After the elastic collision, the twins continue ahead with a speed of 4.12 m/s. How fast is Melinda's car bumped across the floor?

Solution: Notice that you must add the mass of the bumper car to the mass of the riders.

Given: $m_1 = 300.0$ kg
$m_2 = 125.0$ kg
$v_{1o} = 10.0$ m/s
$v_{2o} = 0$ m/s
$v_{1f} = 4.12$ m/s

Unknown: $v_{2f} = ?$
Original equation:
$m_1v_{1o} + m_2v_{2o} = m_1v_{1f} + m_2v_{2f}$

Solve: $v_{2f} = \dfrac{m_1v_{1o} + m_2v_{2o} - m_1v_{1f}}{m_2}$

$$= \frac{(300.0\text{ kg})(10.0\text{ m/s}) + (125.0\text{ kg})(0\text{ m/s}) - (300.0\text{ kg})(4.12\text{ m/s})}{125.0\text{ kg}}$$

$$= \frac{3000\text{ kg·m/s} + 0\text{ kg·m/s} - 1236\text{ kg·m/s}}{125.0\text{ kg}} = \frac{1764\text{ kg·m/s}}{125.0\text{ kg}}$$

$= \mathbf{14.1\ m/s}$

Example 6: Sometimes the curiosity factor at the scene of a car accident is so great that it actually produces secondary accidents as a result, while people watch to see what is going on. If an 800.-kg sports car slows to 13.0 m/s to check out an accident scene and the 1200.-kg pick-up truck behind him continues traveling at 25.0 m/s, with what velocity will the two move if they lock bumpers after a rear-end collision?

Solution: Since the two vehicles lock bumpers, both objects have the same final velocity.

Given: $m_1 = 800.\text{ kg}$
$m_2 = 1200.\text{ kg}$
$v_{1o} = 13.0\text{ m/s}$
$v_{2o} = 25.0\text{ m/s}$

Unknown: $v_f = ?$
Original equation:
$m_1v_{1o} + m_2v_{2o} = (m_1 + m_2)v_f$

$$\textit{Solve: } v_f = \frac{m_1v_{1o} + m_2v_{2o}}{(m_1 + m_2)} = \frac{(800.\text{ kg})(13.0\text{ m/s}) + (1200.\text{ kg})(25.0\text{ m/s})}{(800.\text{ kg} + 1200.\text{ kg})}$$

$$= \frac{10\,400\text{ kg·m/s} + 30\,000\text{ kg·m/s}}{2000.\text{ kg}} = \mathbf{20.2\text{ m/s forward}}$$

Example 7: Charlotte, a 65.0-kg skin diver, shoots a 2.0-kg spear with a speed of 15 m/s at a fish who darts quickly away without getting hit. How fast does Charlotte move backwards when the spear is shot?

Solution: To start, Charlotte and the spear are together and both are at rest.

Given: $m_1 = 65.0\text{ kg}$
$m_2 = 2.0\text{ kg}$
$v_o = 0\text{ m/s}$
$v_{2f} = 15.0\text{ m/s}$

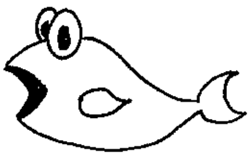
Unknown: $v_{1f} = ?$
Original equation:
$(m_1 + m_2)v_o = m_1v_{1f} + m_2v_{2f}$

$$\textit{Solve: } v_{1f} = \frac{(m_1 + m_2)v_o - m_2v_{2f}}{m_1}$$

$$= \frac{(65.0\text{ kg} + 2.0\text{ kg})(0\text{ m/s}) - (2.0\text{ kg})(15\text{ m/s})}{65.0\text{ kg}}$$

$$= \frac{-30.\text{kg·m/s}}{65.0\text{ kg}} = \mathbf{-0.46\text{ m/s}}$$

Remember, the minus sign here is indicating direction. Therefore, Charlotte would travel with a speed of 0.46 m/s in a direction opposite to that of the spear.



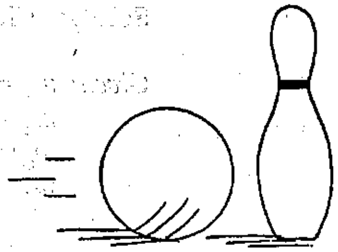
Practice Exercises

Exercise 7: Jamal is at the state fair playing some of the games. At one booth he throws a 0.50-kg ball forward with a velocity of 21.0 m/s in order to hit a 0.20-kg bottle sitting on a shelf, and when he makes contact the bottle goes flying forward at 30.0 m/s. a) What is the velocity of the ball after it hits the bottle? b) If the bottle were more massive, how would this affect the final velocity of the ball?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 8: Jeanne rolls a 7.0-kg bowling ball down the alley for the league championship. One pin is still standing, and Jeanne hits it head-on with a velocity of 9.0 m/s. The 2.0-kg pin acquires a forward velocity of 14.0 m/s. What is the new velocity of the bowling ball?



Answer: ____________________

Exercise 9: Running at 2.0 m/s, Bruce, the 45.0-kg quarterback, collides with Biff, the 90.0-kg tackle, who is traveling at 7.0 m/s in the other direction. Upon collision, Biff continues to travel forward at 1.0 m/s. How fast is Bruce knocked backwards?

Answer: ____________________

Exercise 10: Anthony and Sissy are participating in the "Roll-a-Rama" rollerskating dance championship. While 75.0-kg Anthony rollerskates backwards at 3.0 m/s, 60.0-kg Sissy jumps into his arms with a velocity of 5.0 m/s in the same direction. a) How fast does the pair roll backwards together? b) If Anthony is skating toward Sissy when she jumps, would their combined final velocity be larger or smaller than your answer to part a? Why?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 11: To test the strength of a retainment wall designed to protect a nuclear reactor, a rocket-propelled F-4 Phantom jet aircraft was crashed head-on into a concrete barrier at high speed in Sandia, New Mexico on April 19, 1988. The F-4 phantom had a mass of 19100 kg, while the retainment wall's mass was 469000 kg. The wall sat on a cushion of air that allowed it to move during impact. If the wall and F-4 moved together at 8.41 m/s during the collision, what was the initial speed of the F-4 Phantom?

Answer: ____________________

Exercise 12: Valentina, the Russian Cosmonaut, goes outside her ship for a spacewalk, but when she is floating 15 m from the ship, her tether catches on a sharp piece of metal and is severed. Valentina tosses her 2.0-kg camera away from the spaceship with a speed of 12 m/s. a) How fast will Valentina, whose mass is now 68 kg, travel toward the spaceship? b) Assuming the spaceship remains at rest with respect to Valentina, how long will it take her to reach the ship?

Answer: a. ____________________

Answer: b. ____________________

Exercise 13: A 620.-kg moose stands in the middle of the railroad tracks, frozen by the lights of an oncoming 10 000.-kg locomotive that is traveling at 10.0 m/s. The engineer sees the moose but is unable to stop the train in time and the moose rides down the track sitting on the cowcatcher. What is the new combined velocity of the locomotive and the moose?

Answer: ____________________

Exercise 14: Lee is rolling along on her 4.0-kg skateboard with a constant speed of 3.0 m/s when she jumps off the back and continues forward with a velocity of 2.0 m/s relative to the ground. This causes the skateboard to go flying forward with a speed of 15.5 m/s relative to the ground. What is Lee's mass?

Answer: ____________________

Additional Exercises

A-1: Bernie, whose mass is 70.0 kg, leaves a ski jump with a velocity of 21.0 m/s. What is Bernie's momentum as he leaves the ski jump?

A-2: Ethel is sitting on a park bench feeding the pigeons when a child's ball rolls toward her across the grass. Ethel returns the ball to the child by hitting it with her 2.0-kg pocketbook with a speed of 20 m/s. If the impact lasts for 0.4 s, with what force does Ethel hit the ball?

A-3: When Reggie stepped up to the plate and hit a 0.150-kg fast ball traveling at 36.0 m/s, the impact caused the ball to leave his bat with a velocity of 45.0 m/s in the opposite direction. If the impact lasted for 0.002 s, what force did Reggie exert on the baseball?

A-4: The U.S. Army's parachuting team, the Golden Knights, are on a routine jumping mission over a deserted beach. On a jump, a 65-kg Knight lands on the beach with a speed of 4.0 m/s, making a 0.20-m deep indentation in the sand. With what average force did the parachuter hit the sand?

A-5: The late news reports the story of a shooting in the city. Investigators think that they have recovered the weapon and they run ballistics tests on the pistol at the firing range. If a 0.050-kg bullet were fired from the handgun with a speed of 400 m/s and it traveled 0.080 m into the target before coming to rest, what force did the bullet exert on the target?

A-6: About 50 000 years ago, in an area located outside Flagstaff, Arizona, a giant 4.5×10^7-kg meteor fell and struck the earth, leaving a 180-m-deep hole now known as Barringer crater. If the meteor was traveling at 20 000 m/s upon impact, with what average force did the meteor hit the earth?

A-7: Astronaut Pam Melroy, history's third woman space shuttle pilot, flew the space shuttle *Discovery* to the International Space Station to complete construction in October of 2000. To undock from the space station Pilot Melroy released hooks holding the two spacecraft together and the 68 000-kg shuttle pushed away from the space station with the aid of four large springs. a) If the 73 000-kg space station moved back at a speed of 0.50 m/s, how fast and in what direction did the space shuttle move? b) What was the relative speed of the two spacecraft as they separated?

A-8: Tyrrell throws his 0.20-kg football in the living room and knocks over his mother's 0.80-kg antique vase. After the collision, the football bounces straight back with a speed of 3.9 m/s, while the vase is moving at 2.6 m/s in the opposite direction. a) How fast did Tyrrell throw the football? b) If the football continued to travel at 3.9 m/s in the same direction it was thrown, would the vase have to be more or less massive than 0.80 kg?

A-9: A 300.-kg motorboat is turned off as it approaches a dock and it coasts in toward the dock at 0.50 m/s. Isaac, whose mass is 62.0 kg, jumps off the front of the boat with a speed of 3.0 m/s relative to the boat. What is the velocity of the boat after Isaac jumps?

A-10: Miguel, the 72.0-kg bullfighter, runs toward an angry bull at a speed of 4.00 m/s. The 550.-kg bull charges toward Miguel at 12.0 m/s and Miguel must jump on the bull's back at the last minute to avoid being run over. What is the new velocity of Miguel and the bull as they move across the arena?

A-11: A space shuttle astronaut is sent to repair a defective relay in a 600.00-kg satellite that is traveling in space at 17 000.0 m/s. Suppose the astronaut and his Manned Maneuvering Unit (MMU) have a mass of 400.00 kg and travel at 17 010.0 m/s toward the satellite. What is the combined velocity when the astronaut grabs hold of the satellite?

A-12: The U.S.S. *Constitution*, the oldest fully commissioned war ship in the world, is docked in Boston, Massachusetts. Also known as "Old Ironsides" for her seemingly impenetrable hull, the frigate houses 56 pieces of heavy artillery. Mounted on bearings that allow them to recoil at a speed of 1.30 m/s are 20 carronades, each with a mass of 1000. kg. If a carronade fires a 14.5-kg cannonball straight ahead, with what muzzle velocity does the cannonball leave the cannon?

Challenge Exercises for Further Study

B-1: On a hot sumer afternoon, Keith and Nate are out fishing in their rowboat when they decide to jump into the water and go for a swim. Keith, whose mass is 65.0 kg, jumps straight off the front of the boat with a speed of 2.00 m/s relative to the boat, while Nate propels his 68.0-kg body simultaneously off the back of the boat at 4.00 m/s relative to the boat. If the 100.-kg boat is initially traveling forward at 3.00 m/s, what is its velocity after both boys jump?

B-2: Lilly, whose mass is 45.0 kg, is ice skating with a constant speed of 7.00 m/s when she hits a rough patch of ice with a coefficient of friction of 0.0800. How long will it take before Lilly coasts to a stop?

B-3: In a train yard, train cars are rolled down a long hill in order to link them up with other cars as shown. A car of mass 4000. kg starts to roll from rest at the top of a hill 5.0 m high, and inclined at an angle of 5.0° to the horizontal. The coefficient of rolling friction between the train and the track is 0.050. What velocity would the car have if it linked up with 3 identical cars sitting on flat ground at the bottom of the track? (Hint: The equation for rolling friction is just like the one for sliding friction.)