CHAPTER 4

Motion and Force: Dynamics

The study of forces necessary for certain aspects of motion, including changing the rate of linear movement or maintaining circular motion, constitutes **dynamics.**

Force; Newton's First Law of Motion

- **Force** is the push or pull experienced by an object. Because it has magnitude and direction, force is a vector. The magnitude of force can be measured by a spring scale.
- Overturning the prevailing view (expounded by Aristotle) that a body's natural state is at rest, Galileo posited that a body not subject to friction moving horizontally at a constant velocity would continue at this velocity. This viewpoint expresses **Newton's first law of motion,** or **law of inertia,** in which the motion of a body at rest or a body moving linearly with a constant velocity is maintained unless subjected to a net nonzero force.
- The **inertia** of a body refers to its ability to maintain its state of motion.
- **Inertial reference frames** are reference frames in which the first law of motion is valid—that is, either at rest or moving at a constant velocity in a straight line. Otherwise, reference frames are called **noninertial.**

Mass; Newton's Second Law of Motion

- A body's inertia is quantified in its **mass,** which measures its ability to resist a change in motion. The standard unit for mass is the **kilogram. Weight** is the force of gravity on a mass.
- When a net nonzero force acts on a body, the magnitude and/or direction of its velocity will be affected. A net nonzero force will cause a body to accelerate at a rate inversely proportional to the body's mass. The direction of acceleration and net force will be the same.
- These statements, embodied in the equation $\Sigma F = ma$, express **Newton's second law of motion.** This relationship is also true for the vector components for force and acceleration, where $\Sigma F_x = ma_x$, $\Sigma F_y = ma_y$, and $\Sigma F_z = ma_z$. The unit of force is the **newton,** equivalent to $kg \cdot m/s^2$.

Newton's Third Law of Motion

- When one body applies a force on another body, a force equal in magnitude but opposite in direction is applied back on the first body by the second. This statement is **Newton's third law of motion.**

- The prepositions used in describing such interactions are precise, so the force *by* one body *on* another body is equal in magnitude but opposite in direction from the force *on* the first body *by* the second body.

Weight—The Force of Gravity; and the Normal Force

- Galileo's assertion that all objects fall at the same acceleration can be applied to the definition of gravitational force, such that $F_G = mg$, where $g = 9.8\ m/s^2$. The magnitude of this gravitational force is called **weight,** and its direction points to the Earth's center.
- The weight of an object at rest on the surface of the Earth is opposed by an equal and opposite **contact force,** so the object has a net force of zero. This force is called a **normal force** when it is perpendicular to the contact surface.

Solving Problems with Newton's Laws: Vector Forces and Free-Body Diagrams

The sum of all force vector components makes up the components of the **net force.**

- **Free-body diagrams** depict the forces on a body and angles at which they are acting, enabling the forces to be broken into components. Free-body diagrams may or may not include friction, depending on how the problem is specified.
- The force of **tension** in a massless flexible cord is uniform throughout its length.

Applications Involving Friction, Inclines

- **Kinetic friction** is present when one body slides across a surface and is a vector force acting in the opposite direction as the velocity vector. The magnitude of the force of friction is proportional to the normal force such that $F_{fr} = \mu_k F_N$, where μ_k is a constant whose value depends on the surfaces in contact.
- **Static friction** is the force parallel to the surfaces of contact that opposes force insufficient to set one of the bodies in contact in motion if that force is less than the maximum static friction, where $F_{max} = \mu_s F_N$. The **coefficient of static friction,** μ_s, is a constant whose value depends on the surfaces in contact. Immediately after motion is initiated, kinetic friction is applicable, where $F_{fr} \leq \mu_s F_N$.

For Additional Review

Determine the forces involved for one block placed atop another block that is, in turn, placed on a surface, when any combination of these objects are in motion. Assume the two blocks and the surface are composed of the same material.

Multiple-Choice Questions

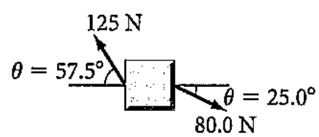
1. What is the net force required for a 1.5×10^5 kg train car to move from 20 km/h to 60 km/h over 100 meters?
   (A) 9.3×10^4 N
   (B) 1.8×10^5 N
   (C) 4.4×10^5 N
   (D) 9.3×10^5 N
   (E) 4.5×10^6 N

2. What is the weight of 55 kg object on a planet that has twice the force of gravity present on the Earth?
   (A) 500 N
   (B) 710 N
   (C) 920 N
   (D) 1100 N
   (E) 1300 N

3. What is the normal force for a 500 kg object resting on a horizontal surface if a massless rope with a tension of 150 N is acting at a 45° angle to the normal force?
   (A) 4800 N
   (B) −4800 N
   (C) 4900 N
   (D) −4900 N
   (E) 0 N

Questions 4–6

A 6 kg weight is pulled from two massless ropes as shown below.



4. What is the magnitude of the resultant force?
   (A) 5.31 N
   (B) 29.1 N
   (C) 53.2 N
   (D) 71.8 N
   (E) 98.5 N

5. At what angle with respect to the x axis is the resultant force?
   (A) 41.1°
   (B) 62.1°
   (C) 85.8°
   (D) 94.2°
   (E) 112°

6. What is the magnitude of acceleration at which the object moves?
   (A) 4 m/s^2
   (B) 6 m/s^2
   (C) 8 m/s^2
   (D) 10 m/s^2
   (E) 12 m/s^2

7. What is the coefficient of friction for a 50 kg block descending a 60° incline at a constant velocity?
   (A) 0.1
   (B) $\sqrt{3}$
   (C) 0.3
   (D) π
   (E) Not enough information is presented to determine the coefficient of friction.

8. For a 7.5 kg object on a horizontal surface that has a coefficient of static friction where $\mu_s = 1.0$, and a coefficient of kinetic friction where $\mu_k = 0.8$, the maximum parallel force which will not set the object in motion is
   (A) 23.5 N
   (B) 51.2 N
   (C) 60.0 N
   (D) 73.5 N
   (E) 81.3 N

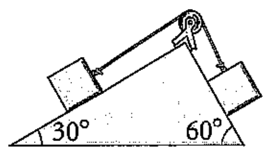
9. For a 5.0 kg object on a horizontal surface that has a coefficient of static friction where $\mu_s = 0.15$ and a coefficient of kinetic friction where $\mu_k = 0.07$, what is the parallel force necessary to accelerate the object at 12 m/s^2?
   (A) 53.6 N
   (B) 56.6 N
   (C) 60.0 N
   (D) 63.4 N
   (E) 67.4 N

10. Which of the following statements is NOT true?
    I. $a = \Sigma F/m$
    II. $F_{fr} = \mu_k F_N$
    III. $F_{fr} > \mu_s F_N$
    (A) I only
    (B) II only
    (C) III only
    (D) I and II only
    (E) II and III only

Free-Response Questions

1. A mass at the origin is pulled by four perpendicular ropes, with a force of 10 N in the direction of the positive y axis, 8 N to the positive x axis, 15 N to the negative x axis, and 18 N to the negative y axis.
   (a) Find the magnitude of the resultant force.
   (b) Find the angle at which the force is acting.
   Now each of the ropes is pulled at 30° counterclockwise from the given axes.
   (c) Draw a free-body diagram and find the horizontal and vertical components of the new resultant force.
   (d) At what angle is the resultant force acting with respect to the x axis?
   (e) How do you account for these results?

2. Two 2 kg blocks are joined by a massless cord and pulley as shown below.



   (a) Ignoring friction, determine each block's acceleration using a free-body diagram for each.
   (b) If the coefficient of kinetic friction is given by $\mu_k = 0.1$, find each block's acceleration using a free-body diagram for each.

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. (B) is correct. Converting km/h to m/s,
(20 km/h)(1000 m/1 km)(1 h/60 min)(1 min/60 s) = 5.6 m/s and
(60 km/h)(1000 m/1 km)(1 h/60 min)(1 min/60 s) = 16.7 m/s.
Since $v^2 = v_0^2 + 2a(x - x_0)$, $a = [(16.7\text{ m/s})^2 - (5.6\text{ m/s})^2]/2(100\text{ m}) = 1.2\text{ m/s}^2$.
Using the definition of force, $\Sigma F = ma = (1.5 \times 10^5\text{ kg})(1.2\text{ m/s}^2)$
$= 1.8 \times 10^5$ N.

2. (D) is correct. Weight, or the force of the planet's gravitation, is given by $W = F_G = mg_p = (55\text{ kg})(19.6\text{ m/s}^2) = 1100$ N.

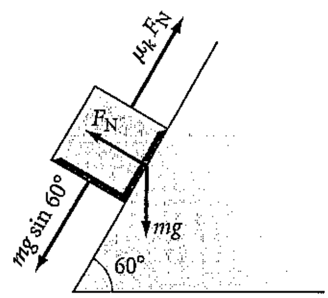
3. (A) is correct. Because the weight of the object is clearly too great—$F_G = (500\text{ kg})(9.8\text{ m/s}^2) = 4900$ N—to be moved by the vertical component of a 150 N force, $\Sigma F_y = 0$, where $\Sigma F_y = F_N - mg + (150\text{ N})(\sin 45°)$.
Since $0 = F_N - (500\text{ kg})(9.8\text{ m/s}^2) + (150\text{ N})(\sin 45°)$, $F_N = 4800$ N.

4. (D) is correct. Breaking the force into components,
$\Sigma F_x = 80\text{ N} \cos 25° - 125\text{ N} \cos 57.5° = 5.3$ N and
$\Sigma F_y = 125\text{ N} \sin 57.5° - 80\text{ N} \sin 25° = 71.6$ N. The resultant force is given by $F_R = \sqrt{(5.3\text{ N})^2 + (71.6\text{ N})^2} = 71.8$ N.

5. (C) is correct. Using the components of force, where
$\Sigma F_x = 80\text{ N} \cos 25° - 125\text{ N} \cos 57.5° = 5.3$ N and
$\Sigma F_y = 125\text{ N} \sin 57.5° - 80\text{ N} \sin 25° = 71.6$ N,
the angle is given by $\tan\theta = F_y/F_x$. $\theta = \tan^{-1} 71.6/5.3$
$= 85.8°$ with respect to the x axis.

6. **(E) is correct.** Using the resultant force vector (again, from $\Sigma F_x = 80$ N $\cos 25° - 125$ N $\cos 57.5° = 5.3$ N and $\Sigma F_y = 125$ N $\sin 57.5° - 80$ N $\sin 25° = 71.6$ N) $F_R = \sqrt{(5.3\text{ N})^2 + (71.6\text{ N})^2} = 71.8$ N. By Newton's second law of motion, $a = \Sigma F/m = 71.8\text{ N}/6\text{ kg} = 12\text{ m/s}^2$ in the direction of force.

7. **(B) is correct.** The reference frame can be defined such that the x axis is affixed to the incline. Then, $\Sigma F_y = F_N - mg\cos 60° = 0$, so $F_N = mg\cos 60°$ and $\Sigma F_x = mg\sin 60° - \mu_k F_N = ma_x = 0$ because $a_x = 0$ since it is traveling at a constant velocity. Therefore, $\mu_k = mg\sin 60°/F_N = mg\sin 60°/mg\cos 60° = \tan 60° = \sqrt{3}$.



8. **(D) is correct.** The normal force is given by $F_N = mg = (7.5\text{ kg})(9.8\text{ m/s}^2) = 73.5$ N. If $\mu_s = 1.0$, the maximum $F_{sfr} = \mu_s F_N = 73.5$ N, which a force must overcome to initiate motion in the object. So the force necessary must exceed 73.5 N; if the force is less than 73.5 N, it will not initiate motion in the object.

9. **(D) is correct.** The relationship for the force sought, $F_?$, and the information provided is given by $\Sigma F = F_? - \mu_s F_N = ma$, so $F_? - (0.07)(5\text{ kg})(9.8\text{ m/s}^2) = (5\text{ kg})(12\text{ m/s}^2)$, so $F_? = 63.4$ N.

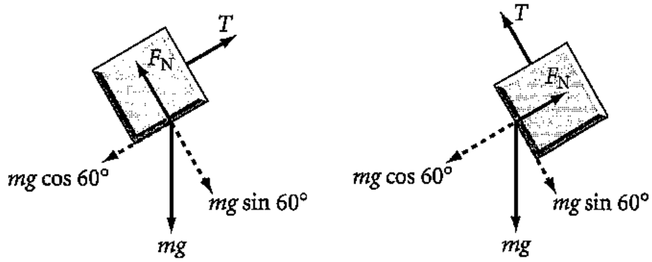
10. **(C) is correct.** Statement I is Newton's second law of motion, and Statement II is the relationship between frictional force and the product of the coefficient of kinetic friction and the normal force. Statement III misstates the relationship between frictional force and static friction, which is $F_{fr} \leq \mu_s F_N$.

Free-Response Questions

1. (a) $\Sigma F_x = 8\text{ N} - 15\text{ N} = -7\text{ N}$, $\Sigma F_y = 10\text{ N} - 18\text{ N} = -8\text{ N}$, $F_R = \sqrt{(-7\text{ N})^2 + (-8\text{ N})^2} = 10.6$ N.
   (b) $\tan\theta = F_y/F_x = -8\text{ N}/-7\text{ N} = 49°$ below the negative x axis, or 229° with the x axis.
   (c) $\Sigma F_x = 8\text{ N}\cos 30° + 18\text{ N}\sin 30° - 15\text{ N}\cos 30° - 10\text{ N}\sin 30° = -2.1$ N and $\Sigma F_y = 8\text{ N}\sin 30° + 10\text{ N}\cos 30° - 15\text{ N}\sin 30° - 82\text{ N}\cos 30° = -10.4$ N.
   (d) $\tan\theta = F_x/F_y = (-10.4)/(-2.1) = 79°$ below the negative x axis, or 259° with respect to the x axis.
   (e) The two circumstances are almost the same. The axes of the second reference frame are essentially shifted by 30°, so the magnitude remains the same, and the angle is shifted by 30°.

This response correctly demonstrates what the question is seeking: that when all forces are shifted by the same angle, the resultant force is the same but is shifted by the same angle. The responses to parts a and b use straightforward calculation, demonstrating vector summation and trigonometric determination of angle. The responses to parts c and d are a more complicated variation of the answers to parts a and b, requiring the correct sign after resolving the vectors into components.

2. (a)



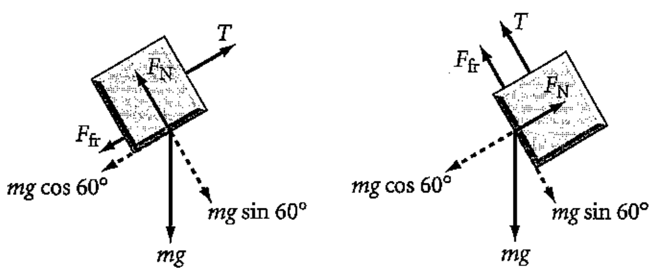
For each block, the x axis can be placed on the incline. Because the right incline is steeper, both blocks will move to the right along the inclines. This is taken as the positive x direction for each.

$\Sigma F_{x1} = T - mg\cos 60° = T - (2\text{ kg})(9.8\text{ m/s}^2)(1/2)$

$\Sigma F_{x2} = mg\sin 60° - T = (2\text{ kg})(9.8\text{ m/s}^2)(\sqrt{3}/2) - T$

$\Sigma F = T - (2\text{ kg})(9.8\text{ m/s}^2)(1/2) + (2\text{ kg})(9.8\text{ m/s}^2)(\sqrt{3}/2) - T = 7.2\text{ N} = ma = (2\text{ kg} + 2\text{ kg})(a) = (4\text{ kg})a$, so $a = 1.8\text{ m/s}^2$ in each of their respective positive x directions.

(b)



For each block, the x axis can be placed on the incline, and the direction of the frictional force is always opposite the direction of motion.

$\Sigma F_{x1} = T - mg\cos 60° - \mu_k F_N = T - (2\text{ kg})(9.8\text{ m/s}^2)(1/2) - (0.1)(2\text{ kg})(9.8\text{ m/s}^2)(\sin 60°)$

$\Sigma F_{x2} = mg\sin 60° - T - \mu_k F_N = (2\text{ kg})(9.8\text{ m/s}^2)(\sqrt{3}/2) - T - (0.1)(2\text{ kg})(9.8\text{ m/s}^2)(\cos 60°)$

$\Sigma F = T - (2\text{ kg})(9.8\text{ m/s}^2)(1/2) - 1.7\text{ N} + (2\text{ kg})(9.8\text{ m/s}^2)(\sqrt{3}/2) - T - 0.98\text{ N} = 4.5\text{ N} = ma = (2\text{ kg} + 2\text{ kg})(a) = (4\text{ kg})a$, so $a = 1.1\text{ m/s}^2$ in each of their respective positive x directions.

This response correctly accounts for vector components of force for parts a and b and of friction for part b. The vector summations and free-body diagrams are presented and combined to determine acceleration.