CHAPTER 6

Work and Energy

The motion of an object can be evaluated in terms of its energy. This can prove advantageous, because energies are scalar rather than vector quantities and because energy is conserved in certain situations.

Work Done by a Constant Force

When a constant force is applied to an object, the work done on it is equal to the product of the parallel component of force and the magnitude of displacement it causes, $W = Fd\cos\theta$, where θ represents the angle of force with respect to the displacement vector. The units of work are **joules,** equivalent to N·m.

- The perpendicular component of force does not affect work done to cause the displacement.
- Since friction opposes motion, all work done on the object by frictional forces will be negative. This can be thought of as work done *by* the object rather than work done *on* the object. It results in a *loss* of energy for the object.

Work Done by a Varying Force

When a varying force is applied to an object, the work done between two points of displacement can be determined from a distance vs. force graph, and the work is equal to the area under the curve between the two points of displacement.

Kinetic Energy, and the Work-Energy Principle

Though there are several types of energy, the total energy of a closed system remains constant. Energy is a scalar quantity expressing the capacity to do work.

- **Kinetic energy** refers to the energy associated with motion, an action which has a capacity to do work.
- An object's **translational kinetic energy** is defined as $KE = 1/2(mv^2)$.
- The **work-energy principle** defines the work done by all forces on an object as equivalent to change in the object's kinetic energy, $W_{net} = \Delta KE$.
- The increase or decrease of a body's kinetic energy depends on the sign of work done on that body.

Potential Energy

Potential energy describes the forces on a body that are a function of its position with regard to other bodies.

- **Gravitational potential energy** refers to the capacity of an object to do work based on the force of gravity acting on it, and it is proportional to the height of the object, y, where y represents a vertical distance, PE $= mgy$. The value of y is relative to the reference level where PE $= 0$. This reference level may or may not be chosen as the ground.

- Only an object's change in potential energy can be calculated as a function of the vertical distance between two points in space. More generally, the change in a body's potential energy is defined as the negative of work done by gravity to move the body between two points.
- A spring not at its normal length has potential energy that has a capacity to do work when the dislocating force of compression or stretching is removed.
- The force of a spring is given by **Hooke's law,** or the **spring equation,** $F_s = -kx$, where k is a constant particular to a spring, and x is the displacement from its normal length.
- The elastic potential energy of a spring is given by PE $= 1/2(kx^2)$.

Conservative and Nonconservative Forces

- **Conservative forces** are forces for which the work done is independent of the path taken. The work depends only on the starting and finishing positions. For **nonconservative forces** the work done depends on the path taken.
- Accounting for potential energy, the work-energy principle can be defined in terms of the changes in potential and kinetic energies, and the work done by nonconservative forces is given by $W_{NC} = \Delta KE + \Delta PE$.

Mechanical Energy and Its Conservation

The **total mechanical energy** of a system, E, is **conserved,** or constant, when only conservative forces are acting on it. This means that the sum of the kinetic and potential energies remains constant $KE_1 + PE_1 = KE_2 + PE_2$. The **principle of conservation of mechanical energy** states that the total mechanical energy of a system remains constant in the absence of nonconservative forces.

Problem Solving Using Conservation of Mechanical Energy

The conservation of mechanical energy can be applied using the appropriate definitions of kinetic and potential energies.

- When gravity is the only force in a system, $1/2(mv_1^2) + mgy_1 = 1/2(mv_2^2) + mgy_2$.
- When the only force involved comes from a massless spring, $1/2(mv_1^2) + 1/2(kx_1^2) = 1/2(mv_2^2) + 1/2(kx_2^2)$.

Other Forms of Energy; Energy Transformations and the Law of Conservation of Energy

All forms of energy, including chemical, nuclear, electrical and thermal, are essentially kinetic or potential energy at a microscopic or atomic level, and all forms of energy can be converted to others. This conversion may involve the energy transfer between bodies.

- The total energy remains constant during the transfer or transformation of energy involving both conservative and nonconservative forces, which is a statement of **the law of conservation of energy.**

Energy Conservation with Dissipative Forces: Solving Problems

Some mechanical energy can be transformed into thermal energy during force-related processes.

- The nonconservative forces (such as friction) that cause this are called **dissipative forces.**
- When gravity and friction are present in a process, the mechanical energy equation must be amended to account for the latter, such that $1/2(mv_1^2) + mgy_1 = 1/2(mv_2^2) + mgy_2 + F_{fr}d$.

Power

Power is the rate at which work is done or energy is transformed, $P_{avg} = W/t = Fv_{avg}$. The units for power are **watts,** equivalent to Joules/second.

For Additional Review

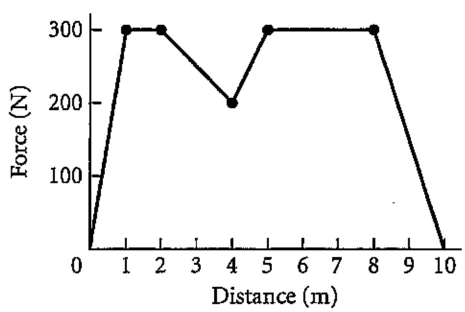
Analyze how the conservation of mechanical energy would be applied to situations in which gravitational force and the elastic force of a spring are present.

Multiple-Choice Questions

1. What is the work done by friction on a 15 kg object pulled horizontally in a straight line for 15 meters, if the coefficient of friction between the object and the surface is given by $\mu_k = 0.4$?
   (A) −59 J
   (B) −91 J
   (C) −145 J
   (D) −590 J
   (E) −890 J

2. How much work must be done on a 27.5 kg object to move it 18 m up a 30° incline?
   (A) −4800 J
   (B) −2400 J
   (C) 0 J
   (D) 2400 J
   (E) 4800 J

3. According to an object's distance vs. force (parallel) graph, what is the work done in this process as the object moves from 2 m to 8 m?



   (A) −1650 J
   (B) 1650 J
   (C) 3300 J
   (D) −3300 J
   (E) 0 J

4. What is the work done to slow a 1.8×10^5 kg train car from 60 m/s to 20 m/s?
   (A) 2.9×10^6 J
   (B) 6.1×10^4 J
   (C) 3.1×10^1 J
   (D) -1.3×10^3 J
   (E) -2.9×10^8 J

5. A 45 kg object slides down an uneven frictionless incline from rest without rotating. What is its speed after it travels 2.5 meters vertically?
   (A) 3.0 m/s
   (B) 4.3 m/s
   (C) 5.7 m/s
   (D) 7.0 m/s
   (E) 11 m/s

6. Which of the following are equivalent units for the spring constant, k?
   I. $N{\cdot}m^2$
   II. kg/s^2
   III. J/m^2
   (A) I only
   (B) II only
   (C) III only
   (D) I and II only
   (E) II and III only

7. For an object sliding down a frictionless uneven incline without rolling, which of the following represents the change in height necessary for its velocity to double in terms of its initial velocity V?
   (A) $2V^2g$
   (B) $3V/g^2$
   (C) $3V^2/2g$
   (D) $3V/2g$
   (E) $2g/3V^2$

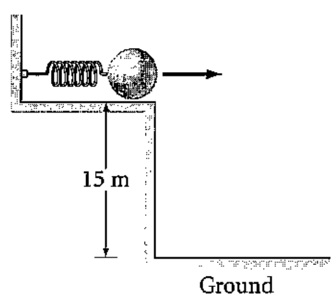
8. A 2.0 kg ball compresses a spring 0.10 meters and is released from rest. If the ball leaves the spring at a speed of 0.20 m/s, what is the value of the spring constant?
   (A) 5.0 N/m
   (B) 8.0 N/m
   (C) 12 N/m
   (D) 16.0 N/m
   (E) 18.0 N/m

9. What is the force of friction as a 12 kg object slides for 30 meters down a 30° incline at a constant velocity?
   (A) 59 N
   (B) 98 N
   (C) 280 N
   (D) 540 N
   (E) 1100 N

10. What is the average power necessary to move a 35 kg block up a frictionless 30° incline at 5 m/s?
    (A) 68 W
    (B) 121 W
    (C) 343 W
    (D) 430 W
    (E) 860 W

Free-Response Questions

1. A 6 kg ball is pressed against a spring with a k value of 23 N/m, compressing it by 0.5 m. The spring is horizontal with a ledge. The length of the spring in its resting position equals the length of the ledge. The ledge is 15 m above the ground.



   (a) What is the elastic potential energy initially?
   (b) What is the gravitational potential energy when the ball leaves the spring?
   (c) What is the gravitational potential energy when the ball hits the ground?
   (d) Find the magnitude of the velocity vector when the ball hits the ground.
   (e) How far from the base of the ledge will the ball land?

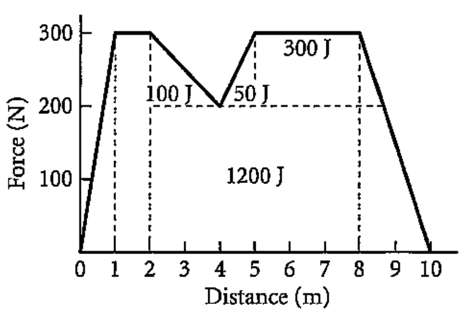
ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. (E) is correct. The force of friction is given by $F_{fr} = \mu_k F_N$
$= (0.4)(15\text{ kg})(9.8\text{ m/s}^2) = 59$ N, so the work done is
$W_{fr} = (59\text{ N})(15\text{ m})(\cos 180°) = -890$ J.

2. (D) is correct. The height is given by $h = 18\text{ m} \sin 30° = 9$ m. $W = mgh =$ $(27.5\text{ kg})(9.8\text{ m/s}^2)(9\text{ m}) = 2400$ J.

3. (B) is correct. The work done by a varying force can be ascertained by the area under the curve across the given distance interval. This can be accomplished by breaking the area under the curve into a series of rectangles and triangles whose areas can be evaluated relatively simply.



The work done is given by 100 J + 50 J + 300 J + 1200 J = 1650 J.

4. (E) is correct. The work is equal to the change in kinetic energy,
$W = KE_2 - KE_1 = 1/2(mv_2^2) - 1/2\,(mv_1^2) = 1/2m[v_2^2 - v_1^2]$
$= 1/2(1.8 \times 10^5\text{ kg})\,(20\text{ m/s})^2 - (60\text{ m/s}^2)$
$= 1/2(1.8 \times 10^5\text{ kg})(-3200\text{ m}^2/\text{s}^2)$
$= -2.9 \times 10^8$ J.

5. (D) is correct. Initially the object is at rest, and its energy is all potential, $PE_i = KE_f + PE_f$. Although full height is not known, it is clear by filling in the values $mg(H + 2.5\text{ m}) = 1/2(mv^2) + mgH$, and $mg(2.5\text{ m}) = 1/2(mv^2)$ that it is not a necessary quantity. $(45\text{ kg})(9.8\text{ m/s}^2)(2.5\text{ m}) = 1/2(45\text{ kg})(v^2)$, so $v = \sqrt{(45\text{ kg})(9.8\text{ m/s}^2)(2.5\text{ m})/1/2(45\text{ kg})} = 7$ m/s.

6. (E) is correct. The spring constant, k, is written as N/m, which can be deduced from the restoring force equation $F = -kx$. This eliminates item I. Similarly, from the equation $PE = 1/2(kx^2)$, k would have the units J/m^2. Since newtons are kg·m/s^2, k = N/m = kg/s^2.

7. (C) is correct. If the final height is H, the initial height is $H + \Delta h$, which will enable the relevant conservation of energy equation to be solved in terms of Δh. Also, the $v_f = 2V$.
Then $1/2(mv_i^2) + mgh_i = (mv_f^2) + mgh_f$ becomes $1/2(mV^2) + mg(H + \Delta h)$ $= 1/2(m(2V)^2) + mg(H)$.
This is equivalent to $1/2(V)^2 + g(H + \Delta h) = 1/2((2V)^2) + g(H)$.
So $\Delta h = 3V^2/2g$.

8. (B) is correct. The energy relation is given by
$1/2(mv_i^2) + 1/2(kx_i^2) = 1/2(mv_f^2) + 1/2(kx_f^2)$,
where $x_i = 0.1$ m, $x_f = 0.0$ m, $v_i = 0.0$ m/s and $v_f = 0.20$ m/s,
so $1/2(2.0\text{ kg})(0\text{ m/s})^2 + 1/2(k(0.10\text{ m})^2)$
$= 1/2(2.0\text{ kg})(0.20\text{ m/s})^2 + 1/2(k)(0.0^2)$.

Solving for k, $k = \dfrac{(2.0\text{ kg})(0.20\text{ m/s})^2}{(0.10\text{ m})^2} = 8.0$ N/m.

9. (A) is correct. In this case, the energy relation is given by
$1/2(mv_1^2) + mgy_1 = 1/2(mv_2^2) + mgy_2 + F_{fr}d$.
The vertical height is given by 30 m sin 30° = 15 m,
so $1/2(12\text{ kg})(v_1^2) + (12\text{ kg})(9.8\text{ m/s}^2)y_1$
$= 1/2(12\text{ kg})(v_2^2) + (12\text{ kg})(9.8\text{ m/s}^2)(y_2) + F_{fr}(30\text{ m})$.
Solving for F_{fr}, the KE components cancel because $v_1 = v_2$,

$$F_{fr} = \frac{(12\text{ kg})(9.8\text{ m/s}^2)(15\text{ m})}{(30\text{ m})} = 59\text{ N}.$$

10. (E) is correct. Average power is given by $P_{avg} = Fv_{avg}$
$= (mg\sin 30°)(5\text{ m/s}) = (35\text{ kg})(9.8\text{ m/s}^2)(1/2)(5\text{ m/s}) = 860$ W.

Free-Response Questions

1. (a) PE $= 1/2(kx^2) = 1/2(23\text{ N/m})(0.5\text{ m})^2 = 2.9$ J
   (b) PE $= mgh = (6\text{ kg})(9.8\text{ m/s}^2)(15\text{ m}) = 882$ J
   (c) PE = 0 J. When it hits, the energy is entirely kinetic.
   (d) For the period of time where the ball's motion is directly influenced by the spring, $1/2mv_1^2 + 1/2kx_1^2 = 1/2mv_2^2 + 1/2kx_2^2$, where $v_1 = 0$, $x_1 = 0.5$, $k = 23$ N/m, $x_2 = 0$. To find the velocity at the instant the ball leaves the spring
   $1/2(6\text{ kg})(0\text{ m/s})^2 + 1/2(23\text{ N/m})(0.5\text{m})^2 = 1/2(6\text{ kg})v_2^2 + 1/2(23\text{ N/m})(0\text{ m})^2$,

   $$\sqrt{\frac{(23\text{ N/m})(0.5\text{ m})^2}{6\text{ kg}}} = 0.98\text{ m/s} = v_2.$$

   Next, $1/2(mv_1^2) + mgy_1 = 1/2(mv_2^2) + mgy_2$ using the $y_1 = 15$ m.
   $1/2(6\text{ kg})(0.98\text{ m/s})^2 + (6\text{ kg})(9.8\text{ m/s}^2)(15\text{ m})$
   $= 1/2(6\text{ kg})v_2^2 + (6\text{ kg})(9.8\text{ m/s}^2)(0\text{ m})$.
   Here, $1/2(0.98\text{ m/s})^2 + (9.8\text{ m/s}^2)(15\text{ m}) = 1/2v_2^2$, and $v_2 = 17$ m/s.
   (e) Using the relation $y = y_0 + v_{y0}t + 1/2at^2$, $0 = (15\text{ m}) + (0\text{ m/s})t - 1/2gt^2$, the ball will be in the air for 1.75 seconds. When the spring is released, its elastic PE is converted to KE to launch the ball horizontally:
   $2.9\text{ J} = 0.5(6\text{ kg})(V^2)$, $V_{XO} = 0.98$ m/s.
   Next, $x = v_{x0}t = (0.98\text{ m/s})(1.75\text{ s}) = 1.7$ m.

The response correctly identifies the potential energies and kinetic energies at both stages in the motion. The conservation of energy is applied in two steps—initially when the potential energy is elastic, and then when the potential energy is gravitational.