7 Law of Universal Gravitation

7-1 Gravitational Force

Vocabulary

Law of Universal Gravitation: Every particle attracts every other particle with a force that is proportional to the mass of the particles and inversely proportional to the square of the distance between them.

$$F \propto \frac{mM}{d^2}$$

The sign $\propto$ means "proportional to." To make an equation out of the above situation, insert a quantity called the **universal constant of gravitation, *G*.**

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

Now the magnitude of this gravitational force can be represented as

$$\textbf{Force} = \frac{\textbf{(universal constant of gravitation)(mass 1)(mass 2)}}{\textbf{(distance)}^2}$$

or $$F = \frac{GmM}{d^2}$$

Like all other forces, the gravitational force of attraction between two objects is measured in newtons.

Solved Examples

Example 1: The gravitational force of attraction between Earth and the sun is 1.6×10^{23} N. What would this force have been if Earth were twice as massive?

Solution: The gravitational force of attraction between two bodies is proportional to the mass of each of the two bodies. As one mass increases, the gravitational force between the two bodies increases proportionally. Therefore, if Earth's mass were doubled, the gravitational force between the sun and Earth would double as well.

Therefore, $F = 2F_o = 2(1.6 \times 10^{23}\ \mathrm{N}) = \mathbf{3.2 \times 10^{23}\ N}$

Example 2: The gravitational force of attraction between Earth and the sun is 1.6×10^{23} N. What would this gravitational force have been if Earth had formed twice as far away from the sun?

Solution: The gravitational force of attraction between two bodies is inversely proportional to the square of the distance between them. In this case, if the distance is twice as great, the force between Earth and the sun would be 1/4 as much.

$$\text{Therefore, } F \propto \frac{1}{d^2} \quad \text{or} \quad F = \frac{F_o}{4} = \frac{(1.6 \times 10^{23}\text{ N})}{4} = \mathbf{4.0 \times 10^{22}\ N}$$

Example 3: Oliver, whose mass is 65 kg, and Olivia, whose mass is 45 kg, sit 2.0 m apart in their physics classroom. a) What is the force of gravitational attraction between Oliver and Olivia? b) Why don't Oliver and Olivia drift toward each other?

a) *Given:* $m_{\text{Oliver}} = 65$ kg
$M_{\text{Olivia}} = 45$ kg
$d = 2.0$ m
$G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$

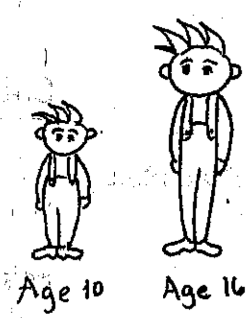
Unknown: $F = ?$
Original equation: $F = \frac{GmM}{d^2}$

$$\textit{Solve: } F = \frac{GmM}{d^2} = \frac{(6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2)(65\text{ kg})(45\text{ kg})}{(2.0\text{ m})^2} = \mathbf{4.9 \times 10^{-8}\ N}$$

b) Because the gravitational force of Earth is much greater than the force Oliver and Olivia exert on each other.

Practice Exercises

Exercise 1: When Royce was 10 years old, he had a mass of 30 kg. By the time he was 16 years old, his mass increased to 60 kg. How much larger is the gravitational force between Royce and Earth at age 16 compared to age 10?



Answer: ______________________

Exercise 2: If John Glenn weighed 640 N on Earth's surface, a) how much would he have weighed if his Mercury spacecraft had (hypothetically) remained at twice the distance from the center of Earth? b) Why is it said that an astronaut is never truly "weightless?"

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 3: Mr. Gewanter, whose mass is 60.0 kg, is doing a physics demonstration in the front of the classroom. a) How much gravitational force does he exert on 55.0-kg Martha in the front row, 1.50 m away? b) How does this compare to what he exerts on 65.0-kg Lester, 4.00 m away in the back row?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 4: Astrologers claim that your personality traits are determined by the positions of the planets in relation to you at birth. Scientists argue that these gravitational effects are so small that they are totally insignificant. Compare the gravitational attraction between you and Mars to the gravitational attraction between you and your 70.0-kg doctor at the moment of your birth, if the doctor stands 0.500 m away. (Note: $M_M = 6.42 \times 10^{23}$ kg, $d_{\text{E to M}} = 7.83 \times 10^{10}$ m. This is the average distance between Earth and Mars. This distance varies as the two planets orbit the sun.)

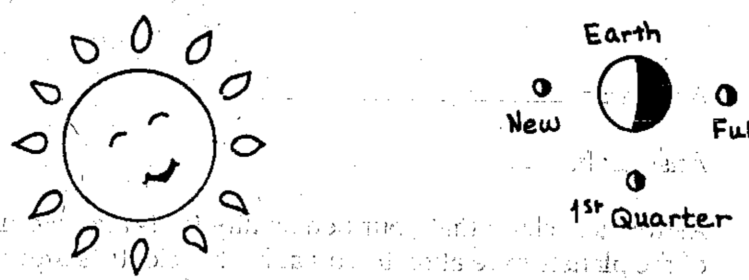
Answer: ____________________

Answer: ____________________

Exercise 5: Our galaxy, the Milky Way, contains approximately 4.0×10^{11} stars with an average mass of 2.0×10^{30} kg each. How far away is the Milky Way from our nearest neighbor, the Andromeda Galaxy, if Andromeda contains roughly the same number of stars and attracts the Milky Way with a gravitational force of 2.4×10^{30} N?

Answer: ____________________

Exercise 6: Tides are created by the gravitational attraction of the sun and moon on Earth. Calculate the net force pulling on Earth during a) a new moon, b) a full moon, c) a first quarter moon. The diagram is intended to help your understanding of the situation but is *not* drawn to scale. ($m_M = 7.35 \times 10^{22}$ kg, $m_E = 5.98 \times 10^{24}$ kg, $m_S = 1.99 \times 10^{30}$ kg, $d_{E-M} = 3.84 \times 10^8$ m, $d_{E-S} = 1.50 \times 10^{11}$ m)



Answer: **a.** ____________________

Answer: **b.** ____________________

Answer: **c.** ____________________

7-2 Gravitational Acceleration

You can use the law of universal gravitation to find the gravitational acceleration, g, of any body if you know that body's mass and radius. For example, let's look at the situation on Earth. The weight of an object on Earth's surface is equal to the gravitational force between that object and Earth:

$$mg = \frac{GmM}{d^2}$$

The m on the left represents the mass of an object, such as a human being. The m on the right side of the equation stands for this same mass, so the term cancels out of the equation. The M on the right represents the mass of Earth or other celestial body on which the person is standing. The d in the denominator is equal to the radius of the celestial body. So the equation becomes

$$g = \frac{GM}{d^2}$$

In this equation, g is the acceleration due to gravity on the celestial body in question. On Earth you already know that this value is 10.0 m/s^2.

Solved Examples

Example 4: Temba is standing in the lunch line 6.38×10^6 m from the center of Earth. Earth's mass is 5.98×10^{24} kg. a) What is the acceleration due to gravity? b) When Temba eats his lunch and his mass increases, does this change the acceleration due to gravity?

a. *Given:* $M = 5.98 \times 10^{24}$ kg
$d = 6.38 \times 10^6$ m
$G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$

Unknown: $g = ?$
Original equation: $g = \frac{GM}{d^2}$

Solve: $g = \frac{GM}{d^2} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{(6.38 \times 10^6\ \text{m})^2} = \mathbf{9.80\ m/s^2}$

b. No, his acceleration due to gravity does not change because it is not dependent on his mass.

Example 5: The sun has a mass that is 333 000 times Earth's mass and a radius 109 times Earth's radius. What is the acceleration due to gravity on the sun?

Solution: One way to solve this exercise is to actually multiply the given values by the mass and radius of Earth. However, there is an easier and much neater way to come up with the correct answer. By working with ratios, you can find an answer without any information about Earth.

Given: $M_S = 333\ 000\ M_E$ *Unknown:* $g = ?$

$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$ *Original equation:* $g = \frac{GM}{d^2}$

$d_s = 109\ d_E$

Solve: Set up the above equation as a ratio of sun to Earth before substituting numbers.

$$\frac{g_S}{g_E} = \frac{\frac{GM_S}{d_S^2}}{\frac{GM_E}{d_E^2}}$$

Simplifying gives $$\frac{g_S}{g_E} = \frac{M_S d_E^2}{M_E d_S^2} = \frac{(333\ 000\ M_E)(d_E^2)}{(M_E)(109 d_E)^2} = \frac{(333\ 000)}{(109)^2} = 28.0$$

Therefore, $g_S = 28.0\ g_E$ so the acceleration due to gravity on the sun is 28.0 times what it is on Earth. In other words, it is 28.0 times 10.0 m/s^2, or **280. m/s^2.**

Practice Exercises

Exercise 7: In *The Little Prince*, the Prince visits a small asteroid called B612. If asteroid B612 has a radius of only 20.0 m and a mass of 1.00×10^4 kg, what is the acceleration due to gravity on asteroid B612?

Answer: ____________________

Exercise 8: In Exercise 5 in the previous section, what is the Andromeda Galaxy's acceleration rate toward the Milky Way?

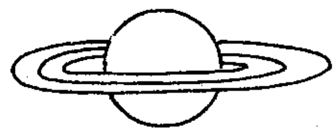
Answer: ____________________

Exercise 9: Black holes are suspected when a visible star is being noticeably pulled by an invisible partner that is more than 3 times as massive as the sun. a) If a red giant (a dying star) is gravitationally accelerated at 0.075 m/s^2 toward an object that is 9.4×10^{10} m away, how large a mass must the unseen body possess? b) How many times more massive is the object than the sun? ($M_s = 1.99 \times 10^{30}$ kg)

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 10: The planet Saturn has a mass that is 95 times Earth's mass and a radius that is 9.4 times Earth's radius. What is the acceleration due to gravity on Saturn?



Answer: ____________________

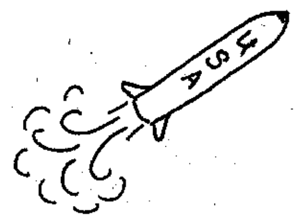
7-3 Escape Speed

Vocabulary **Escape Speed:** The minimum speed an object must possess in order to escape from the gravitational pull of a body.

In Chapter 5, you worked with gravitational potential energy and kinetic energy. When an object moves away from Earth, its gravitational potential energy increases. Since its total energy is conserved, its kinetic energy decreases. When the object is close to Earth, the gravitational force on it is a fairly constant *mg*. However, as you know, the gravitational force drops rapidly as you get farther from Earth. If an object moves upward from Earth with enough speed, it will never run out of kinetic energy and will escape from Earth.

The escape speed for an object leaving the surface of any celestial body of mass M and radius d is

$$v = \sqrt{\frac{2GM}{d}}$$



Notice that the mass of the escaping object does not affect the escape speed.

Solved Examples

Example 6: Earth has a mass of 5.98×10^{24} kg and a radius of 6.38×10^6 km. What is the escape speed of a rocket launched on Earth?

Given: $M = 5.98 \times 10^{24}$ kg
$d = 6.38 \times 10^6$ m
$G = 6.67 \times 10^{-11}$ N·m²/kg²

Unknown: $v = ?$
Original equation: $v = \sqrt{\frac{2GM}{d}}$

Solve: $v = \sqrt{\frac{2GM}{d}} = \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{6.38 \times 10^6\ \text{m}}}$

$= \mathbf{11\ 200\ m/s}$

Any rocket trying to escape Earth's gravitational pull must be going at least 11 200 m/s before engine cut-off, in order to get away.

Example 7: Compare Example 6 with the escape speed of a rocket launched from the moon. The mass of the moon is 7.35×10^{22} kg and the radius is 1.74×10^6 m.

Given: $M = 7.35 \times 10^{22}$ kg
$d = 1.74 \times 10^6$ m
$G = 6.67 \times 10^{-11}$ N·m²/kg²

Unknown: $v = ?$
Original equation: $v = \sqrt{\frac{2GM}{d}}$

Solve: $v = \sqrt{\dfrac{2GM}{d}} = \sqrt{\dfrac{2(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(7.35 \times 10^{22}\ \mathrm{kg})}{1.74 \times 10^{6}\ \mathrm{m}}} = \mathbf{2370\ m/s}$

Notice that you can escape from the moon by traveling much more slowly than you must travel to escape the gravitational pull of Earth. This is why launching a Lunar Module from the moon's surface was so much easier than launching an *Apollo* spacecraft from Earth.

Practice Exercise

Exercise 11: How fast would you need to travel a) to escape the gravitational pull of the sun? ($M_S = 1.99 \times 10^{30}$ kg, $d_S = 6.96 \times 10^{8}$ m) b) As the sun begins to die, it will become a red giant. This means that its mass will remain the same but its diameter will increase substantially (perhaps even out as far as Earth's orbit!). When the sun becomes a red giant, will its escape speed be greater than, less than, or the same as, it is now?

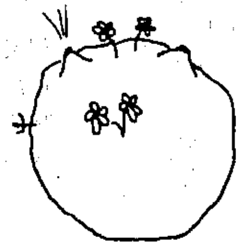
Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 12: How fast would the moon need to travel in order to escape the gravitational pull of Earth, if Earth has a mass of 5.98×10^{24} kg and the distance from Earth to the moon is 3.84×10^{8} m?

Answer: ____________________

Exercise 13: What is the escape speed needed a) to escape the gravitational pull of Asteroid B612 (see Exercise 7)? b) What would happen if you jumped up on Asteroid B612?



Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 14: Scotty finds it difficult to play catch on planet Apgar because the planet's escape speed is only 5.00 m/s, and if Scotty throws the ball too hard, it flies away. If planet Apgar has a mass of 1.56×10^{15} kg, what is its radius?

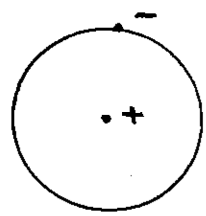
Answer: ______________________

Additional Exercises

A-1: Halley's Comet orbits the sun about every 75 years due to the gravitational force the sun provides. Compare the gravitational force between Halley's Comet and the sun when the comet is at aphelion (its greatest distance from the sun) and d is about 4.5×10^{12} m to the force at perihelion (or closest approach), where d is about 5.0×10^{10} m.

A-2: In Exercise A-1, what is the comet's acceleration a) at aphelion? b) at perihelion? ($M_S = 1.99 \times 10^{30}$ kg)

A-3: An early planetary model of the hydrogen atom consisted of a 1.67×10^{-27}-kg proton in the nucleus and a 9.11×10^{-31}-kg electron in orbit around it at a distance of 5.0×10^{-11} m. In this model, what is the gravitational force between a proton and an electron?



A-4: At what height above Earth would a 400.0-kg weather satellite have to orbit in order to experience a gravitational force half as strong as that on the surface of Earth?

A-5: It is said that people often behave in unusual ways during a full moon. a) Calculate the gravitational force that the moon would exert on a 50.0-kg student in your physics class. The moon is 3.84×10^8 m from Earth and has a mass of 7.35×10^{22} kg. b) Does the moon attract the student with a force that is greater than, less than, or the same as the force with which the student attracts the moon?

A-6: The tiny planet Mercury has a radius of 2400 km and a mass of 3.3×10^{23} kg. a) What would be the gravitational acceleration of an astronaut standing on the surface of Mercury? b) Compare the motion of a ball dropped on the surface of Mercury to that of a ball dropped on Earth.

A-7: The acceleration due to gravity on Venus is 0.89 that of Earth. a) If the radius of Venus is 6.05×10^6 m, what is Venus' mass? b) How does this compare to Earth's mass? c) If you were on a diet and had to "weigh in," would you rather stand on a scale on Venus or on Earth in order to appear as if you had lost the most weight?

A-8: The planet Mars has a mass that is 0.11 times Earth's mass and a radius that is 0.54 times Earth's radius. a) How much would a 60.0-kg astronaut weigh if she were to stand on the surface of Mars? b) Although Mercury is much smaller than Mars, it has almost the same gravitational acceleration. Describe how you might explain this phenomenon.

A-9: On October 26, 2000, the NEAR Shoemaker spacecraft swooped within 3 miles of the asteroid Eros, taking images and collecting data from a distance closer than any spacecraft has ever come to an asteroid. Eros has a mass of 6.69×10^{15} kg. The strange potato-like shape of Eros makes its diameter difficult to determine. If the NEAR spacecraft is orbiting a distance of 18 300 m from Eros' center of mass, what gravitational acceleration does Eros provide on NEAR?

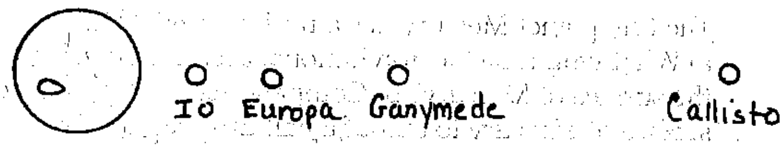
A-10: Find the NEAR spacecraft's escape speed from Eros, using the information given in A-9.

A-11: NASA has announced that a mission to Mars to return rock samples to Earth could come as early as 2011. If NASA landed a 360.-kg spacecraft on the surface of Mars a) what would be the weight of the spacecraft on the planet's surface b) what escape speed would be needed for the craft to leave the planet and head back to Earth with its rock samples. ($M_m = 6.42 \times 10^{23}$ kg, $d_M = 3.39 \times 10^6$ m)

Challenge Exercises for Further Study

B-1: At what distance from Earth's center must a spacecraft be in order to experience the same gravitational attraction from both Earth and the moon when directly between the two? ($M_E = 5.98 \times 10^{24}$ kg, $M_M = 7.35 \times 10^{22}$ kg $d_{E-M} = 3.84 \times 10^8$ m)

B-2: Jupiter's innermost Galilean satellite, Io, is covered with active volcanoes, which exist because of the immense gravitational tugging on the satellite by Jupiter and the other moons near Io. Io orbits 4.2×10^8 m from the center of Jupiter. The other Galilean satellites are located as follows from Jupiter's center. Europa: 6.7×10^8 m, Ganymede: 1.0×10^9 m, and Callisto: 1.9×10^9 m. If Jupiter and its satellites are lined up as shown, what gravitational force does the satellite Io experience? ($M_I = 8.9 \times 10^{22}$ kg, $M_E = 4.9 \times 10^{22}$ kg, $M_G = 1.5 \times 10^{24}$ kg, $M_C = 1.1 \times 10^{23}$ kg, $M_J = 1.9 \times 10^{27}$ kg)



B-3: Saturn's satellite, Titan, orbits the planet in a little less than 16 days. Titan orbits Saturn at an average distance of 1.216×10^9 m from the center of the planet. Use this information to find the mass of Saturn.