Name: _____ Period: _____

Projectile Motion

For a projectile, the object is moving in two directions with two separate velocities, v_x and v_y.

An object projected horizontally will reach the ground in the same time as an object dropped.

If an object projected at an angle, the object will land with the same magnitude and angle as take off.

EQUATIONS:

Horizontal

$$V_{2x} = V_{1x} + a_x t$$

$$x_2 = x_1 + v_{1x} t + \tfrac{1}{2} a_x t^2$$

Vertical

$$V_{2y} = V_{1y} + a_y t$$

$$y_2 = y_1 + v_{1y} t + \tfrac{1}{2} a_y t^2$$

Trigonometric

$$v_{2y}^2 = v_{1y}^2 + 2a_y (y_2 - y_1)$$

$$\sin\theta = \tfrac{opp}{hyp}$$

$$\cos\theta = \tfrac{adj}{hyp}$$

$$\tan\theta = \tfrac{opp}{adj}$$

Range

$$R = \frac{V^2 \sin 2\theta}{g}$$

Sample Problem:

A cannon ball is shot from a cannon with a speed of 50 m/s. The cannon is pointing above the ground at an angle of 30 degrees. The ground is level everywhere around the cannon. How far from the cannon will the ball land?

Solution:

Since the ground is level, we can use the range equation to solve this problem. The equation is

$$R = \frac{V_o^2 \sin 2\theta}{g}$$

Here, θ = 30 degrees and V_o = 50 m/s, g = 9.8 m/s^2. Plugging in the numbers, we get

$$R = \frac{(50)^2 \sin(2 \times 30)}{9.8}$$

or R = 220.92 meters. The cannon ball will land 220.92 meters from the cannon.

Note that using the range equation only works if the launch position is at the same level as the landing position. It is absolutely wrong to use it in any other case!

Sample Problem:

A marble with speed 0.2 m/s rolls off the edge of a table 0.8 m high. How long does it take the marble to hit the floor, and how far from the edge of the table does the marble hit the floor?

Solution:
As soon as the marble is over the edge of the table, it's in free-fall, meaning that it begins accelerating straight down, under the influence of the Earth's gravitational acceleration, 9.8 m/s^2. We will define towards the ground as negative so a_y = -9.8 m/s^2

This means, that even though it has the horizontal speed of 0.2 m/s, the time it takes to hit the floor can be found by basically ignoring this speed for the moment.

As soon as it leaves the edge of the table, it's vertical speed, v_{yo} = 0 m/s, and it must fall a distance 0.8 m before it hits the floor. How long will this take? Use

$$y_2 = y_1 + v_{1y}t + \tfrac{1}{2}a_y t^2$$

where y_o = 0.8 m , y = 0 meters (the floor), v_{yo} = 0 m/s, and a_y = - 9.8 m/s^2.
Plugging these numbers in we get

$0 = 0.8 - 1/2(9.8)\ t^2$

Solve for t and t = 0.4 seconds. The marble will hit the floor in 0.4 seconds.

How far will the marble travel? Well, we just figured out that after 0.4 seconds, the marble is on the floor and its "flight" is over. So, to find how far from the edge of the table it traveled, we can use this equation

$$x_2 = x_1 + v_{1x}t + \tfrac{1}{2}a_x t^2$$

We'll set x_o = 0 m, right at the table's edge, so x will be our landing distance. What about v_{xo}? That's the 0.2 m/s, the speed at which it left the table. The time, t, will be the 0.4 seconds from above. This is the total time the marble has to fly. So plugging in the numbers, we'll get

$x_2 = 0 + 0.2\ (0.4) + 1/2(0)(.4)^2$

or, the marble will land 0.08 meters from the edge of the table.

1. An enemy ship is anchored in the sea 560 m from a island defending itself. The maximum velocity a defense cannon fire a cannon ball is 82 m/s.
   a) At what angle, with respect to sea level, should the cannon be elevated to hit the enemy ship?
   b) How far away from the island should the ship move to be out of range of the island's defense cannon?

a)

b)

2. A person is trapped in the snow, and a rescue plane wants to drop them some emergency supplies. The plane is flying level at an altitude of 1200 meters at a speed of 180 m/s.

a) How long would it take the package to fall from that height?

b) How far in front of the person should the pilot drop the supplies, so that they land right on top of the trapped person?

a)

b)

3. A ball is shot upward from the level ground at an angle of 50 degrees with respect to the horizontal. It is given an initial speed of 40 m/s.
   a) How far did the ball travel horizontally?
   b) How long will it take before the ball hits the ground?

a)

b)

4. A tennis ball is launched at an angle of 40 degrees and it stays in the air for 15 seconds.

a) What was its initial velocity in the Y direction?
b) What was its initial velocity is the X direction?
c) How far did it travel in the X direction?

a)

b)

c)