CHAPTER 7

Linear Momentum

Collisions between objects can be evaluated using the laws of conservation of energy and of linear momentum. Often these objects can be regarded as particles.

Momentum and Its Relation to Force

An object's linear momentum for a particular instant is given by the product of its mass and its velocity vector at that moment, such that $\mathbf{p} = m\mathbf{v}$. The unit of momentum is kg·m/s.

- For an object's momentum to change, force is required such that the net force on an object is equal to the ratio of its change in momentum in a time interval to that time interval, $\Sigma\mathbf{F} = \Delta\mathbf{p}/\Delta t$.
- The **law of conservation of momentum** states that net momentum is conserved in collisions when no external forces are acting on the system.
- In equation form, $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_1' + m_2\mathbf{v}_2'$, or more generally, $\Sigma m_n\mathbf{v}_n = \Sigma m_n\mathbf{v}_n'$.
- When there are no external forces on a system, it is considered an **isolated system.**

Collisions and Impulse

In collisions, the force changes considerably over the course of the interaction.

- **Impulse** is defined as the product of force and a time interval, which is equal to the change in momentum, such that impulse $= \mathbf{F}\Delta t = \Delta\mathbf{p}$.
- In **elastic collisions,** net kinetic energy is conserved. Elastic collisions occur in objects that are hard enough for no thermal energy to be produced from any structural deformation. At the instant of collision, all energy is elastic potential energy.
- For an elastic collision, $1/2(m_1v_1^2) + 1/2(m_2v_2^2) = 1/2(m_1v_1'^2) + 1/2(m_2v_2'^2)$, or more generally, $\Sigma 1/2(m_nv_n^2) = \Sigma 1/2(m_nv_n'^2)$. Elastic collisions occur on the atomic scale, but they are an idealization for larger bodies.
- By combining the quantitative expressions for conservation of momentum and conservation of kinetic energy, it is found that the **magnitude of relative speed** of two objects is the same before and after a head-on elastic collision, $v_1 - v_2 = -(v_1' - v_2')$.

Inelastic Collisions

Inelastic collisions lose some kinetic energy during the interaction of objects.

- For inelastic collisions, $KE_1 + KE_2 = KE_1' + KE_2'$ + **thermal and other non-kinetic energies.**

- When objects do not separate after colliding, the collision is described as **completely inelastic,** such that $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}_3$.

Collisions in Two or Three Dimensions

Conservation of momentum can be applied to multidimensional situations, such that momentum for each coordinate axes is conserved.

- In two dimensions, the vector sum relies on trigonometry, in which the positive x axis of the frame of reference can be defined in the direction of one object's motion. For p_x, $p_{1x} + p_{2x} = p'_{1x} + p'_{2x}$ and for p_y, $p_{1y} + p_{2y} = p'_{1y} + p'_{2y}$.

For Additional Review

Without using the relative speed relation for elastic collisions, solve an elastic collisions question involving conservation of linear momentum and kinetic energy using the quadratic formula.

Multiple-Choice Questions

1. In an inelastic collision in which there are no net nonzero external forces, which of the following are NOT true?
   I. Total vector momentum is conserved.
   II. Kinetic energy is conserved.
   III. Total energy is conserved.
   (A) I only
   (B) II only
   (C) III only
   (D) I and II
   (E) I, II, and III

2. Two equally weighted objects of mass m are moving in opposite directions, one with half the velocity of the other. If they collide and stick together, what is the resultant velocity in terms of the faster object's velocity, v?
   (A) $m/2v$
   (B) $mv/2$
   (C) $v/4$
   (D) $4/mv$
   (E) $2/v$

3. What is the ratio of the momentum and the kinetic energy for a body traveling horizontally at a constant velocity?
   (A) v^2
   (B) m/v
   (C) v^2/m
   (D) $2/v$
   (E) $4v/m$

4. A 10 kg object at rest explodes into four pieces. Each of three of these pieces has a mass of 2.0 kg, and the pieces travel due south, due east, and due west, respectively, at 3.0 m/s. What is the magnitude of velocity of the remaining piece?
   (A) 1.0 m/s
   (B) 1.5 m/s
   (C) 3.0 m/s
   (D) 4.5 m/s
   (E) 6.0 m/s

5. Two equally weighted objects are moving in opposite directions, one at 4 m/s and the other at 3 m/s. They collide inelastically and stick together. What percentage of the kinetic energy is lost in the collision?
   (A) 22%
   (B) 43%
   (C) 52%
   (D) 81%
   (E) 98%

6. A 5 kg object has a momentum of 15 kg m/s. What is the net force required to accelerate the object to 8 m/s over 15 seconds?
   (A) 0.35 N
   (B) 1.7 N
   (C) 5.1 N
   (D) 11 N
   (E) 23 N

7. What is the linear momentum of a 15 kg object traveling at a constant velocity that has 270 joules of kinetic energy?
   (A) 60 kg·m/s
   (B) 75 kg·m/s
   (C) 90 kg·m/s
   (D) 110 kg·m/s
   (E) 125 kg·m/s

8. How much energy is lost when a 0.1 kg projectile traveling at 120 m/s becomes imbedded in a 2 kg block initially at rest?
   (A) 34 J
   (B) 90 J
   (C) 440 J
   (D) 690 J
   (E) 720 J

9. What is the magnitude of the average impulse of a wall on a 525 gram ball that strikes horizontally at 25 m/s and rebounds horizontally at 25 m/s?
   (A) 20 kg·m/s
   (B) 26 kg·m/s
   (C) 45 kg·m/s
   (D) 78 kg·m/s
   (E) 99 kg·m/s

10. A 15 kg mass moving at 8 m/s collides elastically with a 5 kg mass at rest. What is the speed of the 15 kg mass after the collision?
    (A) 4.0 m/s
    (B) 8.0 m/s
    (C) 12 m/s
    (D) 20 m/s
    (E) 36 m/s

Free-Response Questions

1. A 3 kg mass moving laterally at 5 m/s collides with a 5 kg mass at rest. As a result, the 5 kg mass travels at 2 m/s at a 26° angle counterclockwise from the direction of the motion of the initial mass. The 5 kg mass then collides with a 4 kg mass at rest. The 4 kg mass then leaves at 1 m/s at a 30° angle counterclockwise from the direction of motion of the 5 kg mass.
   (a) Find the final magnitude and direction for the velocity of each of the first two masses.
   (b) Are these collisions elastic? Why or why not?

2. An electron, mass 1.66×10^{-27} kg fired at 100 m/s, collides elastically head-on with a gold nucleus of mass 3.27×10^{-25} kg at rest.
   (a) Find each of the resulting velocities.
   (b) How much kinetic energy is lost in the collision?
   (c) What would the resulting velocity be if the collision had been completely inelastic?

ANSWERS AND EXPLANATIONS

Multiple-Choice Questions

1. **(B) is correct.** By definition, an inelastic collision is one in which kinetic energy is not conserved. Both total energy and total vector momentum are always conserved.

2. **(C) is correct.** If total linear momentum is conserved and the final velocity will be given by V, $mv + m(v/2) = (m + m)V$, so $mv - mv/2 = 2mV$, and $V = (mv/4m) = v/4$.

3. **(D) is correct.** Momentum is given by mv, and kinetic energy is given by $1/2(mv^2)$. The ratio $mv/1/2(mv^2) = 2/v$.

4. **(B) is correct.** The last piece must have a mass of 4.0 kg. When linear momentum is broken into x (east-west) and y (north-south) components, it is clear that the x axis momentum is conserved by the two 2.0 kg pieces traveling east and west at the same velocity, so the remaining piece must be traveling along the y axis. Using conservation of momentum, $0 = P_{1y} + P_{2y} + P_{3y} + P_{4y} = m_1v_1 + m_2v_2 + m_3v_3 + m_4v_4 = 0 + 0 + (2.0\text{ kg})(-3.0\text{ m/s}) + (4.0\text{ kg})(v_4)$, so its velocity must be 1.5 m/s in the positive y (or north) direction.

5. **(E) is correct.** Because of the inelastic collision, the final velocity of the conjoined mass, v_3, can be determined from the conservation of linear momentum, $mv_1 + mv_2 = (m + m)v_3$, $(4\text{ m/s})m + (-3\text{ m/s})m = 2m(v_3)$, so $(1/2)\text{ m/s} = v_3$. The initial kinetic energy $1/2(mv^2) + 1/2(mv^2)$ $= 1/2(m)(4\text{ m/s})^2 + 1/2(m)(3\text{ m/s}^2)^2 = 12.5\text{ m J}$, while the final kinetic energy is $1/2(2m)(1/2\text{ m/s})^2 = 1/4\text{ m J}$. Therefore, independent of mass, 98% of KE is lost.

6. **(B) is correct.** A 5 kg object at 8 m/s has a momentum of 40 kg m/s. Force is given by $F = \Delta p/\Delta t$, such that $(40\text{ kg m/s} - 15\text{ kg m/s})/15\text{ s} = 1.7\text{ N}$.

7. **(C) is correct.** The relationship between mass, velocity and kinetic energy is $1/2(mv^2) = 270\text{ J}$, so $v = 6\text{ m/s}$. The linear momentum mv $= (15\text{ kg})(6\text{ m/s}) = 90\text{ kg m/s}$.

8. **(D) is correct.** This is a perfectly inelastic collision. The conservation of linear momentum states that $m_1v_1 + m_2v_2 = (m_1 + m_2)v_f$,
so $(0.1\text{ kg})(120\text{ m/s}) + (2\text{ kg})(0\text{ m/s}) = (2.1)v_f$ and $v_f = 5.7\text{ m/s}$.
The initial kinetic energy is $1/2m_1v_1^2 = 1/2(0.1\text{ kg})(120\text{ m/s})^2 = 720\text{ J}$,
while the final kinetic energy is $1/2(m_1 + m_2)(v_f)^2$
$= 1/2(2.1\text{ kg})(5.7\text{ m/s})^2 = 34\text{ J}$,
so the energy loss is 690 J.

9. **(B) is correct.** Since impulse is given by Δp, where the initial momentum is mv_1 and the final momentum is mv_2, $\Delta p = mv_2 - mv_1$
$= (0.525\text{ kg})(-25\text{ m/s}) - (0.525\text{ kg})(25\text{ m/s}) = -26.25\text{ kg m/s}$.
The magnitude, to significant figures, is 26 kg·m/s.

10. **(A) is correct.** Two equations are applicable, since both kinetic energy and linear momentum are conserved in an elastic collision. For the former, in a direct elastic head-on collision, $v_1 - v_2 = v_2' - v_1'$. For the latter, $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$, $(15\text{ kg})(8\text{ m/s}) = (15\text{ kg})v_1' + (5\text{ kg})v_2'$.
By substitution, $8\text{ m/s} + v_1' = v_2'$, $(15\text{ kg})(8\text{ m/s})$
$= (15\text{ kg})v_1' + (5\text{ kg})(8\text{ m/s} + v_1')$
and $v_1' = 4\text{ m/s}$.

Free-Response Questions

1. (a) For the first collision, the linear momentum is conserved in the direction of motion, which is defined as the positive x-direction:
$m_1v_1 = m_1v_1' \cos\theta_1 + m_2v_2' \cos\theta_2$.

$(3 \text{ kg})(5 \text{ m/s}) = (3 \text{ kg})v_1' \cos \theta_1 + (5 \text{ kg})(2 \text{ m/s}) \cos 26°$.
In the y-direction, $0 = (3 \text{ kg})v_1' \sin \theta_1 + (5 \text{ kg})(2 \text{ m/s}) \sin 26°$.
$2 \text{ m/s} = v_1' \cos \theta$, and $v_1' \sin \theta_1 = -1.46$, so $\theta = -36°$, or 36° below the direction of the first object, and $v_1' = 2.5$ m/s. For the second collision, the linear momentum is again conserved in the direction of motion, which can be redefined as the positive x-direction:
$m_1 v_1 = m_1 v_1' \cos \theta_1 + m_2 v_2' \cos \theta_2$.
$(5 \text{ kg})(2.0 \text{ m/s}) = (5 \text{ kg})v_1' \cos \theta_1 + (4 \text{ kg})(1 \text{ m/s}) \cos 30°$.
In the y-direction, $0 = (5 \text{ kg})\, v_1' \sin \theta_1 + (4 \text{ kg})(1 \text{ m/s}) \sin 30°$.
$1.3 \text{ m/s} = v_1' \cos \theta$, and $v_1' \sin \theta_1 = .4$, so $\theta = -17°$, or 17° below the direction of the 5 kg object, and $v_1' = 1.4$ m/s. Since the positive x-direction was redefined for the second collision, the final velocity of the 5 kg object is $26^0 - 17^0 = 9^0$ above (ccw) from the original velocity of the 3 kg object.

(b) Neither is elastic. The first collision is not elastic because
$1/2(3 \text{ kg})(5 \text{ m/s})^2 > 1/2(3 \text{ kg})(2.5 \text{ m/s})^2 + 1/2(5 \text{ kg})(2 \text{ m/s})^2$,
$37.5 \text{ J} > 9.4 \text{ J} + 10 \text{ J}$.
The second collision is also not elastic because
$1/2(5 \text{ kg})(2.0 \text{ m/s})^2 > 1/2(5 \text{ kg})(1.3 \text{ m/s})^2 + 1/2(4 \text{ kg})(1 \text{ m/s})^2$,
$10 \text{ J} > 4.2 + 2 \text{ J}$.

This response correctly applies the conservation of linear momentum, resolving the vectors into their perpendicular components for part a. The frame of reference is also successfully shifted for ease in calculating the details of the second collision. For the response to part b, the energy loss is quantified to express that the collisions are not elastic.

2. (a) Based on the conservation of linear momentum,
$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$,
so $(1.66 \times 10^{-27} \text{ kg})(100 \text{ m/s}) + (3.27 \times 10^{-25} \text{ kg})(0 \text{ m/s})$
$= (1.66 \times 10^{-27} \text{ kg})v_1' + (3.27 \times 10^{-25} \text{ kg})v_2'$.
In a direct elastic collision, $v_1 - v_2 = v_2' - v_1'$.
Here, $v_2 = 0$, so $v_1 + v_1' = 100 \text{ m/s} + v_1' = v_2'$. Combining these results,
$(1.66 \times 10^{-25} \text{ kg m/s}) = (1.66 \times 10^{-27} \text{ kg})v_1' + (3.27 \times 10^{-23} \text{ kg m/s}) +$
$(3.27 \times 10^{-25} \text{ kg})v_1'$
so $v_1' = -99$ m/s and $v_2' = 1$ m/s.

(b) In a perfectly elastic collision, 0 J of energy would be lost, since kinetic energy is conserved.

(c) Again, from the conservation of linear momentum,
$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$,
$(1.66 \times 10^{-27} \text{ kg})(100 \text{ m/s}) + (3.27 \times 10^{-25} \text{ kg})(0 \text{ m/s})$
$= (1.66 \times 10^{-27} \text{ kg} + 3.27 \times 10^{-25} \text{ kg})v_f$,
so $v_f = .51$ m/s.

The conservation of linear momentum is correctly applied, creating two equations with two unknowns, allowing for linear combination or substitution for the response to part a. For the response to part b, the definition of an elastic collision is presented to account for kinetic energy. For the response to part c, the definition of an inelastic collision is applied to the conservation of linear momentum to determine the final velocity.